

trigonometric functions are the same type of thing.<sup>11</sup> Actually, this is none other than ‘the plane angle’ that we know so well.

That is to say, taking the principal values,

$$\text{rad} = \frac{2}{\pi} \sin^{-1} 1 \tag{26}$$

$$\text{degree} = \frac{1}{90} \sin^{-1} 1 \tag{27}$$

It is clear that these relationships are completely parallel to Eq. (25).

In this way, it is possible to assume units that cannot be reduced to pure numbers for entirely mathematical quantities as well. Of course, no conflict results if ‘mathematical’ units are regarded as pure numbers as they are ordinarily considered. This does not mean that one viewpoint or the other is the correct one, but rather that this is a problem that should be settled by agreement. In this paper, we take the position that ‘mathematical’ units are a kind of unit that cannot be reduced to a pure number without being divided by the same type dimensioned quantities.

In the International System of Units (SI), the issue of whether plane angle is a base unit or a derived unit involving length divided by length (pure number) had not been settled. Previously, a separate classification referred to as supplementary units was established. I quote from the SI document, “Le Système International d’Unités ” (2<sup>nd</sup> Ed., 1973), as translated by the Japanese National Research Laboratory of Metrology.<sup>12</sup>

“Although it is possible to consider an SI unit to be either a base unit or a derived unit, the 11<sup>th</sup> Conférence Générale des Poids et Mesures (1960) recognized a third class of units referred to as supplementary units. It was thus not settled whether supplementary units are base units or derived units”.

“The General Conference on Weights and Measures did not settle (or, rather, has not yet settled) the matter of whether a number of SI units belong to the category of base units or the category of derived units. Those SI units are placed in a third class of units that is referred to as ‘supplementary units’. One is free to choose whether to treat the supplementary units as base units or derived units”.

### A.3 Coherent unit system

A unit system in which a number of base units and equations that express relationships that describe natural laws are used to define (‘derive’, in the terminology of units) all other units in the system is called a coherent unit system.

In a coherent unit system, there is only one unit for each quantity. Thus, a coherent unit system is one for which that group of defining relationship equations is the simplest (specifically, this is to say that the equations have the simplest set of coefficients). As can also be understood from the fact that “the concept of quantity is defined by the formulas that represent natural laws” as described in the previous section, the term coherence begins with the specification of the set of relationship equations that describe natural laws.

For example, denoting the area of a triangle as  $S$ , the length of the base as  $a$ , and the height as  $h$ , we

---

<sup>11</sup>  $\sin^{-1} x = -i \log(ix \pm \sqrt{1-x^2})$ .

<sup>12</sup> In a later revision, the category of supplementary units was removed.