

3.1 Natural units

Units of the following four dimensions are base units for which natural units are used just as they are. Concerning impedance, see Appendix B, “A method of organizing the dimensions of electromagnetic quantities”; concerning plane angle and logarithmic quantity, see Appendix A.2, “‘Mathematical’ units”.

Natural unit of impedance	—	$\Omega_n = \text{sr} \sqrt{\frac{\mu_0}{\epsilon_0}} = 29.9792458\Omega(\text{strict})$
Natural unit of plane angle	—	$\text{rad} = \frac{2}{\pi} \sin^{-1} 1$
Natural unit of logarithmic quantity	—	$\text{neper} = \log e$
Natural unit of amount of substance	—	$\text{mol}_n = N_A^{-1}(\text{inverse of the Avogadro constant})$

3.2 Supplementary constants

The following four series of quantities are not coherent with respect to natural units, but they may be positioned as supplementary constants and used as units.

Elementary electrical quantity	$e = \sqrt{\alpha \hbar / \Omega_n}$	$= 1.0374\text{-}43\text{B}6 \times 10_{(12)}^{-14} \text{C}_u$
Total solid angle of the surface of a hypersphere	$\Omega_k (k = 1, 2, \dots)$	$= \frac{2\pi^{\frac{k+1}{2}}}{\Gamma(\frac{k+1}{2})} \text{rad}^k$
Logarithm of an integer	$B_k (k = 1, z, \dots)$	$= \log_e 2^k \text{neper}$
Universal mole	$\text{mol}_u = 10_{(12)}^{20} (N_A^{-1})$	$= 10_{(12)}^{20} \text{mol}_n$

3.2.1 The elementary electrical quantity

Because the fine structure constant, α , is a dimensionless quantity, when the natural unit of impedance, Ω_n , is taken as a base unit, we cannot construct a coherent unit system with both the quantum of action, \hbar , and the elementary electrical quantity e as defining constants. The reason for not using the elementary electrical quantity e in place of the quantum of action, \hbar , in the defining constants is that the fine structure constant, α , should appear only in various quantities that represent the nature of an electron.⁸

To make it possible to use the elementary electrical quantity e as a unit, the elementary electrical quantity e is positioned as a supplementary constant. We take the elementary electrical quantity to be the positive value of the electrical quantity of an electron, so the sign is the opposite that of the International System of Units (SI).

3.2.2 Total solid angle of hyperspherical surfaces

The solid angle of a hyperspherical surface is an extension of the concept of a plane angle into a multi-dimensional space, so the ‘area’ for when a section of unit ‘area’ on the surface of a sphere of unit radius is ‘seen’ from the center of the sphere is expressed as rad^k (the International System of Units (SI) sr when $k = 2$). Written as $r\text{mrad}^2$, it should be spoken as ‘steradian’. Because the surface area of a sphere of radius r is $4\pi r^2$, the total solid angle of a sphere is $4\pi \text{sr} (= \Omega_2)$.

In the Universal System of Units Standard, a hyperspherical solid angle is regarded as an integer power of a plane angle. The area, S , of a spherical square (a figure on the surface of a sphere that has four

⁸ Of the supplementary constants, only the elementary electrical quantity contains in its definition a quantity that is obtained through measurement. I expect that the fine structure constant, too, may sooner or later become a mathematical constant that can be calculated strictly.