SOCREAL2022

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Abstracs of Keynote Lectures and Accepted Papers

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About SOCREAL2022

Since the last years of the 20th century, a number of attempts have been made in order to model various aspects of social interaction among agents including individual agents, organizations, and individuals representing organizations. The aim of SOCREAL Workshop is to bring together researchers working on diverse aspects of such interaction in logic, philosophy, ethics, computer science, cognitive science and related fields in order to share issues, ideas, techniques, and results.

The earlier editions of SOCREAL Workshop were held in March 2007, March 2010, October 2013, October 2016, and November 2022. Building upon the success of these editions, its 6th edition will be held from 28 February till 1 March 2022 under the auspices of Philosophy and Ethics Laboratory at Faculty of Humanities and Human Sciences, Hokkaido University, CAEP (Center for Applied Ethics and Philosophy) at Faculty of Humanities and Human Sciences, Hokkaido University, and LOG-UCI (An interdisciplinary study of the logical dynamics of the interaction between utterances and social contexts), a research project funded by JSPS (JSPS KAKENHI Grant Number JP 17H02258).

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Dynamic Logic of Relation Changers Meets Brouwer

Ryo Hatano, Tokyo University of Science
Katsuhiko Sano, Hokkaido University

This talk proposes an intuitionistic generalization of van Benthem and Liu (2007)'s dynamic logic of relation changers, where relation changers are dynamic operators which rewrite each agentfs accessibility relation. We employ Nishimura (1982)'s Kripke semantics for a constructive propositional dynamic logic to define the semantics of relation changers. A sound and complete axiomatization of the constructive dynamic logic of relation changers is provided. Moreover, we follow Hatano and Sano (2020)'s approach to provide a different semantics for dynamic logic of relation changers, where relation changers are regarded as bounded morphisms. This alternative semantics leads us to a semantic completeness proof of the axiomatization for the original semantics, which does not require a reduction strategy based on recursion axioms.

Epistemic Infinite-Regress Logics:
the Surface to Deeper Layers and Latent Infinity

by

Tai-Wei Hu, University of Bristol, England

Mamoru Kaneko, University of Tsukuba, Waseda University

Abstract

Common knowledge/belief is an important component in game theory, but its infinitary nature often hinders progress of game theory as a part of social science. The state of affairs behind a game situation may include such an infinite structure as latent. Formally, we consider an infinite-regress logic IR_β with two agents, which is a fixed-point logic. The subscript β is be a bound on the nested depths of beliefs and fixed-point (infinite-regress) operators; the limit case $\beta = \omega$ is unbounded. A proof system IR_β is constructed within the bound β, but the corresponding Kripke semantics already includes an infinite valuation, which is uniform over different bounds β up to ω. The smallest meaningful case is $\beta = 3$. The soundness-completeness theorem connecting IR_β with its semantics is provided; the proof theoretical part is interpreted as going from the surface to deeper layers as β becomes larger. In logic IR_β, each's basic beliefs may be different from the other's, in order to capture the feature that the individual beliefs are in the mind of each agent. Nevertheless, they have external interactions through their social world, after each's individual logical calculation, and after external interactions, each may revise his internal basic beliefs. Then, the situation starts again.

Plurivalent Logic for Multi-Agent Systems

Satoshi Tojo

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Dynamic Epistemic Logic is versatile in knowledge representation, however, its Kripke semantics requires a huge number of possible worlds, and furthermore, the combinatorial number of access relations complicates the description and is not intelligible. On the contrary, sometimes we need more to express; e.g, we want to distinguish between legible information and illegible one, and so on. To solve such problems, we employ many-valued logic to the multi-agent system. We extend the semantics of epistemic logic to 4-valued one to distinguish the public propositions and private propositions. Plurivalent Logic provides multiple valuation functions; one strictly refers to logical truth and so do others to various agent's epistemic states. Therefore the logic simply simulates epistemic logic with a pair of truth values. Furthermore, paired semantics can simulate classical logic, weak Kleene logic, and paraconsistent Kleene logic, with simple designated-value changes.

Agents, Actions, and Social Reality

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In this presentation, we propose *BDOI*-model of atomic agents. *BDOI*-model characterizes mental states of an atomic agent through triple ⟨*belief*, *desire*, *normative belief*⟩ and explains actions based on mental states and intention. Parts of mental states and interpretation of terms can be shared among atomic agents. An aim of this presentation is to explain the construction of social reality based on an analysis of agents, shared mental states, and actions.

1. Model of Atomic Agents

Donald Davidson provided the standard theory of action (Davidson 1980). The core of this theory can be expressed by the following three theses (Schlosser 2021, Sect. 2 and Sect. 2.1).

(1a) The notion of *intentional action* is more fundamental than the notion of *action*.
(1b) There is a close connection between intentional action and acting for a reason.
(1c) [Theory of agency] A being has the capacity to act intentionally just in case it has the right functional organization: just in case the instantiation of certain mental states and events (such as desires, beliefs, and intentions) would cause the right events (such as certain movements) in the right way.

In this presentation, we accept the first two theses and modify (1c). We propose that mental states can be characterized by beliefs, desires, and normative beliefs and that an intention leads an agent, based on the given mental states, to performance of an action. John Searle pointed out that an action may have *desire-based reason* or *desire-independent reason* (Searle 2010: Chap. 6 Sect. 1). Modifying Searle's position, we propose that an action may have reason that is based on both desires and normative beliefs.

Now, we start our proposal with the following description of atomic agents and call it *BDOI-model of atomic agents* (Nakayama 2017a, 2021).

(2a) [Atomic agent] An atomic agent can perform some actions. A part of mental states of an atomic agent can be characterized through triple ⟨*belief*, *desire*, *normative* *belief*⟩, where each of the three components of the triple is a set of First-Order sentences (*FO*-sentences). I call this triple *BDO*-system. This *BDO*-system can be updated when an agent obtains new information.

(2b) [Transparency] Mental states described in (2a) is transparent in the following sense:

(i) [Belief] If A believes that φ, then A knows that A believes that φ.

(ii) [Desire] If A desires that φ, then A knows that A desires that φ.

(iii) [Normative belief] If A believes that it is obligated that φ, then A knows that A believes that it is obligated that φ.

(2c) [Intention as decision making] An *atomic agent* chooses an action type based on her/his *BDO*-system and performs it. In such a case, we say that this agent *intentionally performed* this action.

We use **not**, **&**, ***or***, ⇒, and ⇔ as meta-language expressions of logical connectives. Pair ⟨*BB*, *OB*⟩ which is a subsystem of *BDO*-system ⟨*BB*, *DB*, *OB*⟩ is called a *BO-system*. Let *cons* be an abbreviation of consistent and $Cn(X)$ be an abbreviation of *the deductive closure of X*.

(3a) [Belief] $\mathbf{B}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} (cons(BB)\ \&\ \varphi \in Cn(BB))$

(3b) [Possibility] $\mathbf{M}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} cons(BB \cup \{\varphi\})$

(3c) [Obligation] $\mathbf{O}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} (cons(BB \cup OB)\ \boldsymbol{\&}\ \varphi \in Cn(BB \cup OB)\ \boldsymbol{\&}\ \boldsymbol{not}\ (\varphi \in Cn(BB)))$

(3d) [Prohibition] $\mathbf{F}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} \mathbf{O}_{BDO}\,\neg\varphi$

(3e) [Permission] $\mathbf{P}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} (cons(BB \cup OB \cup \{\varphi\})\boldsymbol{\&}\ \boldsymbol{not}\ (\varphi \in Cn(BB)))$

(3f) [Desire] $\mathbf{D}_{BDO}\,\varphi \Leftrightarrow_{\mathrm{def}} (cons(BB \cup DB)\ \boldsymbol{\&}\ \varphi \in Cn(BB \cup DB)\ \boldsymbol{\&}\ \boldsymbol{not}\ (\varphi \in Cn(BB)))$

(3g) *BDO* is consistent $\Leftrightarrow_{\mathrm{def}} (cons(BB \cup OB)\ \boldsymbol{\&}\ cons(BB \cup DB))$

(3h) [Respect] Atomic agent A with *BDO*-system ⟨*BB*, *DB*, *OB*⟩ respects *BO*-system $\langle BB_s, OB_s\rangle \Leftrightarrow_{\mathrm{def}} BB_s \subseteq BB\ \&\ OB_s \subseteq OB$ & any action type that A chooses to perform is compatible with $BB_s \cup OB_s$.

According to (3h), an atomic agent who respects a *BO*-system obeys any obligation in the *BO*-system and she/he chooses only action types that are permitted in the *BO*-system. For example, a player of chess respects the *BO*-system of chess and she/he plays chess keeping out of violation of the *BO*-system.

We can update a *BDO*-system ⟨*BB*, *DB*, *OB*⟩ by updating *BB* or *DB* or *OB*. We call the framework that allows this kind of updates *Dynamic BDO-Logic*. A *BDO*-system in *Dynamic BDO-Logic* contains information about its stage. We write a *BDO*-system of *Dynamic BDO-Logic* as follows: $BDO(k) = \langle BB(k), DB(k), OB(k)\rangle$. A play of standard two-man games can be described in *Dynamic BO-Logic* that is a subsystem of *Dynamic BDO-Logic* (Nakayama 2016, 2017a, 2021).

According to Searle, there are two types of rules, namely *regulative* and *constitutive rules* (Searle 1969: Chap. 2.5). Regulative rules regulate a pre-existing activity, an activity whose existence is logically independent of the rules. Regulative rules characteristically take the form of or can be paraphrased as imperatives, e.g., "Officers must wear ties at dinner". Constitutive rules constitute an activity the existence of which is logically dependent on the rules. Constitutive rules can be paraphrased as "*X* counts as *Y* in context *C*". A typical example is an introduction of a term used in a game., e.g., "A checkmate is made when the king is attacked in such a way that no move will leave it unattacked" (p. 34f). Both rules can be expressed in *BO-Logic*. In *BO*-system for officers $\langle BB_{officer}, OB_{officer}\rangle$, the *FO*-translation of sentence "Officers wear ties at dinner" is a member of $OB_{officer}$. Similarly, in *BO*-system of chess $\langle BB_{chess}, OB_{chess}\rangle$, the *FO*-translation of sentence "A checkmate is made if and only if the king is attacked in such a way that no move will leave it unattacked" is a member of BB_{chess}.

2. Ontology for Actions and Agents

Davidson developed an event ontology and considered events as First-Order objects as well as things. Furthermore, he interpreted actions as events that are intentional under some descriptions (Davidson 1980). Nakayama (2017b, 2019) extended this event-based semantics of Davidson and developed an axiomatic theory for *Four-Dimensional Event Ontology* (4EO). This theory is based on *General Extensional Mereology* (GEM) for (four-dimensionally extended) events. 4EO claims that everything is a four-dimensional object (4D-object).

(4a) The *universe* is the maximal 4D-object. This means that any 4D-object is a part of the universe.

(4b) An *event* is a 4D-bject. Thus, an action is also a 4D-object.

(4c) An *agent* is a 4D-object.

Atomic agents can share some parts of their mental states. We describe shared mental states of a group of atomic agents as follows.

(5a) Let group G be the mereological sum of atomic agents $A_1, \ldots, A_n$. Let $BDO(A_k) = \langle BB(A_k), DB(A_k), OB(A_k)\rangle$.

(5b) [Shared belief] φ is a shared belief in $G \Leftrightarrow_{def}$ for all A_k in G, $\varphi \in BB(A_k)$.

(5c) [Shared desire belief] φ is a shared desire belief in $G \Leftrightarrow_{\mathrm{def}}$ for all A_{k} in G, $\varphi \in DB(A_{\mathrm{k}})$.

(5d) [Shared obligation belief] φ is a shared obligation belief in $G \Leftrightarrow_{\mathrm{def}}$ for all A_{k} in G, $\varphi \in OB(A_{\mathrm{k}})$.

(5e) [Shared interpretation] All agents in G share interpretation of language $L \Leftrightarrow_{\mathrm{def}}$ every agent in G interprets all symbols in L in the same way.

(5f) [Shared *BO*-system] *BO*-system $\langle BB, OB \rangle$ is shared in $G \Leftrightarrow_{\mathrm{def}}$ all atomic agents in G share all beliefs in *BB*, all obligation beliefs in *OB*, and interpretation of all symbols in *BB* and *OB*.

(5g) [Game players] If G is a group of players of a game that is defined by a *BO*-system, then this *BO*-system is shared in G and respected by all players in G.

Now, the notion of *extended agent* can be specified as follows (Nakayama 2013).

(6a) [Atomic agent] An *atomic agent* is an agent. Any spatial part of an atomic agent is no agent.

(6b) [Agents and tools] Let *temporal-part* (x, t) denote the temporal part of object x in extended time t. Let A be an agent who uses thing B in t to perform an action. Then, the mereological sum *temporal-part* (A, t) + *temporal-part* (B, t) is an agent.

(6c) [Collective action] For every agent A who is a part of G, if E is a collective action performed by G, then there is an action of A that is a part of E.

(6d) If group G of agents performs a collective action in t, then *temporal-part* (G, t) is an agent.

(6e) If an object satisfies neither (6a) nor (6b) nor (6d), then it is no agent. (Note that this definition of *action* is recursive.)

(6f) [Extended agent] An agent that is not atomic is called an *extended agent*.

The collectivity is created based on the ability of people to share parts of mental states and interpretation of a language. In general, an extended agent is more than the fusion of atomic agents, because it can contain several artifacts as its components (see (6b)). If B_1 is the building of a factory, M_1 is the machine in B_1, and $A_1, \dots, A_n$ are workers in B_1, and t denotes working hours, then *temporal-part* $((A_1 + \dots + A_n) + M_1), t)$ is an extended agent. The workers in B_1 produce goods with M_1 and this production is a collective action (see (6c)). It is a characteristic of our description of collective actions that it takes artifacts as well as humans into consideration.

3. Social Actions and Social Reality

Max Weber thought that social actions of individuals construct the society. Thus, Weber characterized sociology as a science which attempts the interpretive understanding of social action to arrive at a casual explanation of its course and effects (Weber 1922: Sect. 1). This proposal looks persuasive, but it is also true that the society supports social actions. This means that the society and social actions are interconnected. Searle pointed out that some action types and some mental states presuppose some *social institutions*. For example, you can desire to have much money and buy things with money, because there is a monetary system established in the society (Searle 2010: Chap. 6, Sect. 1). This monetary system can be interpreted as *BO*-system $\langle BB_{ms}, OB_{ms}\rangle$ that is shared and respected by almost all members of this society.

Many actions presuppose the existence of the society. For example, if you use a smart phone to play a game, you need a smart phone that is invented and produced in the past. Based on this invention and the spread of smart phones, the action type of *using a smart phone* is created. This type of creation presupposes shared beliefs and shared interpretations of terms for some artifacts. Another type of creation can be found in games. For example, *hitting a home run* is particular action type in a baseball game. This type of creation presupposes shared *BO*-system for a game and shared interpretation of terms in the *BO*-system. Additionally, *playing a team game* presupposes some shared desires among members of a team. These examples show that the existence of many current actions presupposes some current shared *BO*-systems and some collective actions in the past.

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Measurement-Theoretic Remarks on Reducibility of Decision-Theoretic Values of Questions and Answers to Their Information Values (Extended Abstract)

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1 Motivation

The theory of questions and answers is one of the most popular topics in *speech act theory*. According to Cross and Roelofsen [4], *whether-questions* can be classified into at least two categories. The first category is an yes/no question like (1):

(1) Was there a quorum at the meeting?

(1) has the following two direct answers:

(1a) Yes. There was a quorum at the meeting.
(1b) No. There was not a quorum at the meeting.

(1) presupposes that the meeting took place. (1) also has a corrective answer:

(1c) The meeting did not take place.

Although (2) can be read as an yes/no question having two direct answers, it also has a reading on which it presents the following three direct answers:

(2) Does Jones live in Italy, in Spain, or in Germany?

(2a) Jones lives in Italy.
(2b) Jones lives in Spain.
(2c) Jones lives in Germany.

(2) falls under the second category of whether-questions. (2) presupposes that Jones lives in Italy, in Spain, or in Germany. (2) also has a corrective answer:

(2d) Jones does not live in Italy, in Spain, or in Germany.

Whether-questions have a finite number of direct answers, whereas *which-questions* like (3) and (4) may have an indefinite or infinite number of direct answers.

(3) Which Cardinal was elected Pope in 2013?
(4) Who shot J.R.?

Belnap and Steel [1] refer to wether- and which-questions like (3) and (4) as *elementary questions*. Hamblin [7] takes a question to denote, in a world w, the set of all propositions corresponding to a possible answer to the question. A fundamental problem is that Hamblin semantics does not specify what a possible answer is. Groenendijk and Stokhof [6] take a question to denote, in each world, a single proposition corresponding to the true exhaustive answer to the question in that world. What the true *exhaustive answer* to a question in a given world is is much clear than what *all the possible answers* to that question are. Then the meaning of a question can be identified with a set of *mutually exclusive and exhaustive* propositions (i.e., *partition*) of the logical space. In this paper, we would like to argue about the *crossroads* of the *theory of questions and answers*, *decision theory*, and *information theory* in terms of *measurement theory* (cf. Krantz et al. [8]). The aim of this paper is to remark, in terms of such measurement-theoretic concepts as scale types, on the reducibility of the decision-theoretic values of questions to the their information-theoretic values on the basis of Luce [9]'s theorems. The selling point of this paper is not giving a new linguistic (empirical) analysis of questions and answers but giving a new measurement-theoretic (conceptual) analysis of the decision-theoretic and information-theoretic sides of questions and answers.

2 Decision-Theoretic and Information-Theoretic Values of Questions and Answers

According to van Rooij [10, 11], the *relevance* of a question and its answers can be determined in terms of how much it contributes to solving a *decision problem* that can be modeled by a decision space $(\mathbf{W}, \mathscr{F}, P, U)$. When a partition R is given, *decision-theoretic value* $DV_R(B)$ of a proposition B with respect to R is defined as follows:

Definition 1 **($DV_R(B)$).**

$$DV_R(B) := \max_U \sum_{A \in R} P(A|B)U(A \cap B) - \max_U \sum_{A \in R} P(A)U(A).$$

The expected decision-theoretic value $EDV_R(Q)$ of a question (partition) Q with respect to R is defined by $DV_R(B)$:

Definition 2 **($EDV_R(Q)$).**

$$EDV_R(Q) := \sum_{B \in Q} P(B)DV_R(B).$$

On the other hand, the *relevance* of a question and its answers can be analyzed also in terms of information theory. The informational value $IV_R(A)$ of $A \in \mathscr{F}$ with respect to a partition R:

Definition 3 **($IV_R(B)$).**

$$IV_R(B) := H(R) - H_B(R) = \sum_{A \in R} P(A|B) \log P(A|B) - \sum_{A \in R} P(A) \log P(A),$$

where $H_B(R)$ is the entropy of R with respect to the probability function conditionalized on B.

The *expected information-theoretic* value $EIV_R(Q)$ of a question (partition) Q with respect to R that is defined by $IV_R(B)$:

Definition 4 **($EIV_R(Q)$).**

$$EIV_R(Q) := \sum_{B \in Q} P(B) IV_R(B) = \sum_{B \in Q} \sum_{A \in R} P(A \cap B) \log \frac{P(A \cap B)}{P(A)P(B)}.$$

3 Reducibility: Properness, Locality, and Underlying Context

In general, the decision-theoretic values of questions and answers do not agree with their information-theoretic values. Then when the decision-theoretic values of questions and answers can be reduced to their information-theoretic values? We would like to consider this problem. When this problem is considered, such properties of U as properness and locality are often focused. Properness is defined as follows:

Definition 5 **(Properness).** *U is a* proper *iff* $\sum_{A \in R} P(A) \cdot U(P, A) \geq \sum_{A \in R} P(A) \cdot U(P', A)$ *for any P and P'.*

Locality is defined as follows:

Definition 6 **(Locality).** *U is* local *iff U is defined only by $P(A)(P'(A))$ where $A \in R$ but not by $P(P')$.*

Fischer [5] proves the following theorem:

Fact 1 (Logarithmic Utility Function) *If U is differentiable, proper and local utility functions (scoring rules) for probability functions, and R has more than two cells, then $U(P(A)) = \alpha \log P(A) + \gamma$, where $\alpha > 0$.*

From Fact 1, van Rooij [10, p. 395] deduces the following proposition:

Fact 2 (Reducibility) *If U is differentiable, proper and local utility functions (scoring rules) for probability functions, and R has more than two cells, and moreover $\alpha = 1$ and $\gamma = 0$ in $U(P(A)) = \alpha \log P(A) + \gamma$, then both $DV_R(A) = IV_R(A)$ and $EDV_R(Q) = EIV_R(Q)$ hold, that is, $(E)DV_R$ can be reduced to $(E)IV_R$.*

Although deducing itself the logarithmic utility functions from properness and locality is clear, the statuses of these functions and conditions are not clear to us. So we would like to consider these statuses in terms of comparing the logarithmic utility functions with other proper utility functions. Besides the logarithmic utility functions, there are at least two kinds of frequently-used proper utility functions (scoring rules) for probability functions:

1. quadratic: $U(P(A)) := 2P(A) - \sum_{B \in R} P(B)^2$, and
2. spherical: $U(P(A)) := \dfrac{P(A)}{\sqrt{\sum_{B \in R} P(B)^2}}$.

Both the quadratic and spherical utility functions are not local. Among these three types of functions, the logarithmic utility functions only are both proper and local. Which of these three utility functions should be chosen? Bickel [2] criticizes the quadratic and spherical utility functions in the following two points:

1. The quadratic and spherical utility functions often result in extreme ranking differences when compared to the logarithmic utility functions.
2. Because of nonlocality, the quadratic and spherical utility functions allow for the undesirable possibility that one expert receives the highest utility (score) when assigning to the observed proposition a probability lower than the probabilities assigned by other experts.

On the other hand, Selten [12] criticizes the logarithmic utility functions in the following two points:

1. Their resulting utility (score) is too sensitive to small mistakes for small probabilities.
2. An expert's utility (score) is $-\infty$ when a proposition holds that she predicted to be impossible. So the logarithmic utility functions are unbounded and they need to be truncated, but it will be no longer be proper after such a truncation.

According to Carvalho [3, p. 4], "the choice of the most appropriate proper scoring rule is dependent on the desired properties, which in turn is dependent on the *underlying context.*" Properness and locality can be considered to be examples of "desired properties". Because the statuses of the logarithmic utility functions, properness and locality are not clear to us as we said before, we would like to change our viewpoint from the relation between these functions and conditions to the relation between these functions and the "underlying context" to determine when U is a logarithmic function. Then the following problem arises:

Problem 1 (Reducibility and Underlying Context) *What is an underlying context to determine when $(E)DV_R$ can be reduced to $(E)IV_R$, that is, when U is a logarithmic function and so both $DV_R(A) = IV_R(A)$ and $EDV_R(Q) = EIV_R(Q)$ hold?*

4 Luce's Theorems: Psychophysical Laws

Now we try to cope with Problem 1 in terms of such measurement-theoretic concepts as *scale types* based on the class of *admissible transformations*:

Definition 7 (Scale Types). *A scale is a triple* $\langle \mathfrak{U}, \mathfrak{V}, f \rangle$ *where* $\mathfrak{U}$ *is an observed relational structure that is qualitative,* $\mathfrak{V}$ *is a numerical relational structure that is quantitative, and* f *is a homomorphism from* $\mathfrak{U}$ *into* $\mathfrak{V}$*.* A *is the domain of* $\mathfrak{U}$ *and* B *is the domain of* $\mathfrak{V}$*. When the admissible transformations are all the functions* $\varphi : f(A) \to B$*, where* $f(A)$ *is the range of* f*, of the form* $\varphi(x) := \alpha x; \alpha > 0$*.* φ *is called a* similarity transformation*, and a scale with the similarity transformations as its class of admissible transformations is called a* ratio scale*. When the admissible transformations are all the functions* $\varphi : f(A) \to B$ *of the form* $\varphi(x) := \alpha x + \beta; \alpha > 0$*,* φ *is called a* positive affine transformation*, and a corresponding scale is called an* interval scale*.*

Remark 1 (Ratio and Interval Scales) *The indefinite integral of a ratio scale is an interval scale.*

Indeed the concept of (underlying) context is ambiguous. But when $U := \psi(P)$, ψ can be considered to be an *underlying context* to connect P to U and to determine when U is a logarithmic function and so both $DV_R(A) = IV_R(A)$ and $EDV_R(Q) = EIV_R(Q)$ hold. Luce [9] proves the theorems on the types of *psychophysical laws* that connect the *physical scales* to *psychological scales* in terms of *measurement theory*. First, Luce proves the following theorem that connects ratio scales as physical scales to ratio scales as psychological scales:

Fact 3 (From Ratio Scale to Ratio Scale) *Suppose that* $f : A \to \mathbb{R}^+$ *and* $g : A \to \mathbb{R}^+$ *are both ratio scales and that* $g(a) = \psi(f(a))$ *for any* $a \in A$ *and that* ψ *is continuous. Then* $\psi(x) = \alpha x^\beta$*, where* $\alpha > 0$*.*

Second, Luce [9] proves the following theorem that connects ratio scales as physical scales to interval scales as psychological scales:

Fact 4 (From Ratio Scale to Interval Scale) *Suppose that* $f : A \to \mathbb{R}^+$ *is a ratio scale and* $g : A \to \mathbb{R}$ *is an interval scale and that* $g(a) = \psi(f(a))$ *for any* $a \in A$ *and that* ψ *is continuous. Then* $\psi(x) = \alpha x^\beta + \gamma$ *or* $\psi(x) = \alpha \log x + \gamma$*.*

5 Reducibility and Scale Types

Luce proves Fact 4 independently of Fact 3. In addition, he proves Fact 4 as a corollary of Fact 3 on the assumption that ψ is not only continuous but also *differentiable* in such a way that since the *indefinite integral* of a *ratio scale* is an *interval scale*, if f is considered to be a ratio scale and g is an interval scale, then either $\psi(x) = \frac{\alpha}{\beta+1} x^{\beta+1} + \gamma$ if $\beta \neq -1$ or $\psi(x) = \alpha \log x + \gamma$ if $\beta = -1$. Facts 3 and 4 may be originally intended to determine the psychophysical laws that connect the physical scales to psychological scales. But we can regard Luce's theorems as the theorems which have *wider applicability* in the sense that these theorems can make clear connection between *scales in general*. Now we would like to use these theorems in order to furnish a solution to Problem 1:

Proposition 1 (Reducibility and Scale Types). *Suppose that a ratio scale P (in a wide sense) is given, and that an underlying context $\psi(x) := \alpha x^\beta; \alpha > 0$ is given connecting P to a ratio scale (stronger cardinal utility) U^*, and that R has more than two cells. Then if $\beta = -1$ and the integral constant of $\int U^*(P)dP$ equals 0, then such interval scale (weaker cardinal utility) U as $U(P) := \int U^*(P)dP$ is a* logarithmic function *and, when DV_R is defined by U, both $DV_R(A) = IV_R(A)$ and $EDV_R(Q) = EIV_R(Q)$ hold, that is, $(E)DV_R$ can be reduced to $(E)IV_R$, and if $\beta \neq -1$, then U is* no logarithmic function*—it may be quadratic or spherical function—and either $DV_R(A) = IV_R(A)$ or $EDV_R(Q) = EIV_R(Q)$ does not always hold, that is, $(E)DV_R$ cannot be reduced to $(E)IV_R$.*

Remark 2 (Solution to Problem 1) *Proposition 1 states that such conditions as especially the value of β (i.e., $\beta = -1$ or not) concerning the underlying context $\psi(x) := \alpha x^\beta$ connecting a ratio scale P to a ratio scale U^* determines when the* decision-theoretic *value of questions and answers can be reduced to their* information-theoretic *values, which furnishes a solution to Problem 1.*

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A simple logic of the hide and seek game

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We discuss a simple logic to describe one of our favourite games from childhood, hide and seek, and show how a simple addition of an equality constant to describe the winning condition of the seeker makes our logic undecidable. There are certain decidable fragments of first-order logic which behave in a similar fashion with respect to such a language extension, and we add a new modal variant to that class. We discuss the relative expressive power of the proposed logic in comparison to the standard modal counterparts. We prove that the model checking problem for the resulting logic is P-complete. In addition, by exploring the connection with related product logics, we gain more insight towards having a better understanding of the subtleties of the proposed framework. This is joint work with Dazhu Li, Sujata Ghosh and Yaxin Tu.

Graph Games and Logic Design

Johan van Benthem

Amsterdam, Stanford and Tsinghua

Graph games model interesting social scenarios when normal behavior gets disrupted, or (perhaps beneficially) nudged away from its ordinary course. At the same time, these games offer interesting interfaces with old and new logics. In this survey talk, I present some classical results on the sabotage game and its modal logic, then move to a range of new results obtained recently by students, and I end with a general discussion of the logic design/game design interface, including the pressing challenge of bringing in more informational/epistemic aspects.

Johan van Benthem & Fenrong Liu, 2019, Graph Games and Logic Design, Journal of Tsinghua University (Philosophy and Social Sciences), 34:2, 131139.

Inference as Belief Change

Jeremy Seligman

The University of Auckland

The narrative of inference is sequential. You have some information, which you combine and transform in a series of cognitive acts until you arrive at a conclusion. One, two and then three. At each step there is some change in your cognitive state. I will explore the possibility that such changes are changes in belief, and discuss the logic of this kind of belief change.

Completeness of Common sense Term-Sequnce-Deontic-Alethic Logic

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The languages of propositional modal logics has been shown to be highly useful in developing dynamic modal logics that deal with various speech acts. It is also clear, however, that we need more expressive language if we wish to state, for example, a general principle to the effect that if you promise to keep a person safe, you will be committed to keep her safe. Its natural formalization may be something like the following:

$$\forall x \forall y [\mathrm{Promise}(x, y, \mathrm{Safe}(y))] O(x, y, x) \mathrm{Safe}(y),$$

where $[\mathrm{Promise}(x, y, \varphi)]$ means whenever an act of promising to see to it that φ is performed by x addressing y, x will be committed to see to it that φ in the resulting situation and $O(x, y, z)\varphi$ means that it is obligatory for x with respect to y by the name of z to see to it that φ. This talk presents a static base logic *mathsfCTSDAL* (Common sense Term-Sequnce-Deontic-Alethic Logic) that we hope can be extended in to a dynamic language in which we can state things like the one above. We define a logic, *mathsfCTSDAL*, in which we can state, fo example,

$$\forall x \forall y (\mathrm{Parent}(x, y) \wedge \mathrm{Young}(y)) \rightarrow O(x, y, x) \mathrm{Safe}(y),$$

which means that parents are committed to see to it that their young children are safe, and prove its completeness. This presentation is based on joint work with Katsuhiko Sano and Takahiro Sawasaki.

Craig Interpolation for a Sequent Calculus for Combining Intuitionistic and Classical Propositional Logic

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1 Introduction

This paper establishes the Craig interpolation for a multi-succedent sequent calculus for a combination of intuitionistic and classical propositional logic, denoted by $\mathsf{G}(\mathbf{C}+\mathbf{J})$. The calculus was provided in [16] and is based on the semantics offered in [4, 5]. The logic, called $\mathbf{C}+\mathbf{J}$, has two implications: intuitionistic and classical one[1]. They are interpreted in the Kripke semantics as follows (cf. [4, 5]):

$$\begin{array}{lll} w \models_M A \to_{\mathtt{i}} B & \text{iff} & \text{for all } v \in W, (wRv \text{ and } v \models_M A \text{ jointly imply } v \models_M B), \\ w \models_M A \to_{\mathtt{c}} B & \text{iff} & w \models_M A \text{ implies } w \models_M B, \end{array}$$

where M is an intuitionistic Kripke model, w is a possible world in M, and R is a preorder equipped in M. However this semantic treatment breaks one feature of intuitionistic logic called *heredity*, which is defined as: $w \models A$ and wRv jointly imply $v \models A$ for all Kripke models M and all states w and v in M. It is a well-known fact that this feature corresponds to an intuitionistically valid formula $A \to_{\mathtt{i}} (B \to_{\mathtt{i}} A)$. Therefore, the formula is not valid in the Kripke semantics of $\mathbf{C}+\mathbf{J}$. In order to avoid the formula being derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, the right rule for the intuitionistic implication should be restricted as follows:

$$\frac{A, C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow B}{C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow A \to_{\mathtt{i}} B}\ (\Rightarrow \to_{\mathtt{i}}).$$

The resulting calculus is sound and complete and a conservative extension of both an intuitionistic and a classical propositional sequent calculus (see [16]).

It is well-known that classical propositional logic and intuitionistic propositional logic enjoy the Craig interpolation theorem:

> If $A \to B$ is derivable, then there exists a formula C such that both $\Rightarrow A \to C$ and $\Rightarrow C \to B$ are also derivable and that $\mathsf{Prop}(C) \subseteq \mathsf{Prop}(A) \cap \mathsf{Prop}(B)$,

where $\mathsf{Prop}(D)$ denotes the set of all propositional variables in a formula D. The theorem can be shown in terms of a classical sequent calculus $\mathbf{LK}$ by Maehara's method in [9]. In multi-succedent intuitionistic sequent calculus $\mathbf{mLJ}$, the theorem can also be shown, though some modification of the ways is needed, as is noted in [10]. Since $\mathbf{C}+\mathbf{J}$ contains the two kinds of implication, the two types of Craig interpolation theorem can be considered in $\mathsf{G}(\mathbf{C}+\mathbf{J})$.

[1] In addition to $\mathbf{C}+\mathbf{J}$, other attempts to combine intuitionistic and classical logic are displayed in [1, 2, 3, 6, 7, 11, 12, 13, 14].

2 Syntax, Kripke Semantics and Sequent Calculus

2.1 Syntax and Kripke Semantics

This section reviews the syntax and the Kripke semantics of $\mathbf{C}+\mathbf{J}$. The syntax is defined in [16], and the Kripke semantics is based on the ones in [4, 5]. The syntax $\mathcal{L}$ consists of a countably infinite set Prop of propositional variables and the following logical connectives: falsum $\bot$, disjunction $\vee$, conjunction $\wedge$, intuitionistic implication $\to_{\mathtt{i}}$, and classical implication $\to_{\mathtt{c}}$. The set Form of all formulas in our syntax is defined inductively as follows:

$$A ::= p \mid \bot \mid A \vee A \mid A \wedge A \mid A \to_{\mathtt{i}} A \mid A \to_{\mathtt{c}} A,$$

where $p \in \mathsf{Prop}$. We define $\top := \bot \to_{\mathtt{i}} \bot$, $\neg_{\mathtt{c}} A := A \to_{\mathtt{c}} \bot$ and $\neg_{\mathtt{i}} A := A \to_{\mathtt{i}} \bot$.

Let us move to the semantics for the syntax $\mathcal{L}$.

Definition 1. A *model* is a tuple $M = (W, R, V)$ where

- W is a non-empty set of possible worlds,
- R is a preorder on W, i.e., R satisfies reflexivity and transitivity,
- $V : \mathsf{Prop} \to \mathcal{P}(W)$ is a valuation function satisfying the following *heredity* condition: $w \in V(p)$ and wRv jointly imply $v \in V(p)$ for all worlds $w, v \in W$.

Definition 2. Given a model $M = (W, R, V)$, a world $w \in W$ and a formula A, the *satisfaction relation* $w \models_M A$ is inductively defined as follows:

$$\begin{array}{lll}
w \models_M p & \text{iff} & w \in V(p), \\
w \not\models_M \bot, & & \\
w \models_M A \wedge B & \text{iff} & w \models_M A \text{ and } w \models_M B, \\
w \models_M A \vee B & \text{iff} & w \models_M A \text{ or } w \models_M B, \\
w \models_M A \to_{\mathtt{i}} B & \text{iff} & \text{for all } v \in W, (wRv \text{ and } v \models_M A \text{ jointly imply } v \models_M B). \\
w \models_M A \to_{\mathtt{c}} B & \text{iff} & w \models_M A \text{ implies } w \models_M B.
\end{array}$$

Let us say that a formula A is a *semantic consequence* of a set of formulas Γ, represented as $\Gamma \models A$, if $w \models_M C$ for any formula $C \in \Gamma$, then $w \models_M A$ for all models $M = (W, R, V)$ and all worlds $w \in W$. We use $\Gamma \models \Delta$ if $\Gamma \models A$ for some formula $A \in \Delta$. We say that A is *valid* if $\emptyset \models A$ holds. We say a formula A satisfies *heredity* if the following holds: $w \models A$ and wRv jointly imply $v \models A$ for all Kripke models M and all states w and v in M.

Proposition 1. A formula $\neg_{\mathtt{c}} p$ does not satisfy heredity.

Proposition 2. Neither $\neg_{\mathtt{c}} p \to_{\mathtt{i}} (\top \to_{\mathtt{i}} \neg_{\mathtt{c}} p)$ nor $\neg_{\mathtt{c}} p \to_{\mathtt{c}} (\top \to_{\mathtt{i}} \neg_{\mathtt{c}} p)$ is valid.

Proposition 2 implies that an intuitionistic tautology $A \to_{\mathtt{i}} (B \to_{\mathtt{i}} A)$, which is known for the correspondence to heredity in intuitionistic logic, is no longer valid.

2.2 Multi-succedent sequent calculus $\mathsf{G}(\mathbf{C}+\mathbf{J})$

This section reviews the sequent calculus $\mathsf{G}(\mathbf{C}+\mathbf{J})$ provided in [16]. In what follows, we use the ordinary notion of multi-succedent sequent. A *sequent* is a pair of finite multisets denoted by $\Gamma \Rightarrow \Delta$, which is read as "if all formulas in Γ hold then some formulas in Δ hold." Table 1 provides our multi-succedent sequent calculus $\mathsf{G}(\mathbf{C}+\mathbf{J})$, where the notion of derivability is defined as an existence of a

Table 1: Sequent Calculus $\mathsf{G}(\mathbf{C}+\mathbf{J})$

Axioms

$$\frac{}{A \Rightarrow A}\ (Id) \quad \frac{}{\bot \Rightarrow}\ (\bot)$$

Structural Rules

$$\frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A}\ (\Rightarrow w) \quad \frac{\Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta}\ (w \Rightarrow) \quad \frac{\Gamma \Rightarrow \Delta, A, A}{\Gamma \Rightarrow \Delta, A}\ (\Rightarrow c) \quad \frac{A, A, \Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta}\ (c \Rightarrow)$$

$$\frac{\Gamma \Rightarrow \Delta, A \quad A, \Pi \Rightarrow \Sigma}{\Gamma, \Pi \Rightarrow \Delta, \Sigma}\ (Cut)$$

Propositional Logical Rules

$$\frac{A, C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow B}{C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow A \to_{\mathtt{i}} B}\ (\Rightarrow \to_{\mathtt{i}}) \quad \frac{\Gamma_1 \Rightarrow \Delta_1, A \quad B, \Gamma_2 \Rightarrow \Delta_2}{A \to_i B, \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\ (\to_i \Rightarrow)$$

$$\frac{A, \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \to_{\mathtt{c}} B}\ (\Rightarrow \to_{\mathtt{c}}) \quad \frac{\Gamma_1 \Rightarrow \Delta_1, A \quad B, \Gamma_2 \Rightarrow \Delta_2}{A \to_{\mathtt{c}} B, \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2}\ (\to_{\mathtt{c}} \Rightarrow)$$

$$\frac{\Gamma \Rightarrow \Delta, A \quad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \wedge B}\ (\Rightarrow \wedge) \quad \frac{A, \Gamma \Rightarrow \Delta}{A \wedge B, \Gamma \Rightarrow \Delta}\ (\wedge \Rightarrow_1) \quad \frac{B, \Gamma \Rightarrow \Delta}{A \wedge B, \Gamma \Rightarrow \Delta}\ (\wedge \Rightarrow_2)$$

$$\frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, A \vee B}\ (\Rightarrow \vee_1) \quad \frac{\Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \vee B}\ (\Rightarrow \vee_2) \quad \frac{A, \Gamma \Rightarrow \Delta \quad B, \Gamma \Rightarrow \Delta}{A \vee B, \Gamma \Rightarrow \Delta}\ (\vee \Rightarrow)$$

finite tree, which is called a *derivation*, generated by inference rules of Table 1 from initial sequents (Id) and $(\bot)$ of Table 1.

Our basic strategy of constructing $\mathsf{G}(\mathbf{C}+\mathbf{J})$ is to add classical implication to the propositional fragment of multi-succedent sequent calculus $\mathbf{mLJ}$ of intuitionistic propositional logic, proposed by Maehara [8]. However, if the ordinary left and right rules of classical implication were added, the soundness of the resulting calculus would fail, because a formula $\neg_{\mathtt{c}} p \to_{\mathtt{c}} (\top \to_{\mathtt{i}} \neg_{\mathtt{c}} p)$, which is not valid by Proposition 2, would be derivable. This is the reason why the original right rule

$$\frac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A \to_{\mathtt{i}} B}$$

of intuitionistic implication of $\mathbf{mLJ}$ is restricted to the right rule given in Table 1. Based on the abbreviation defined in Section 2.1, the following rules for negations are obtained respectively:

$$\frac{A, C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow}{C_1 \to_{\mathtt{i}} D_1, \ldots, C_m \to_{\mathtt{i}} D_m, p_1, \ldots, p_n \Rightarrow \neg_{\mathtt{i}} A}\ (\Rightarrow \neg_{\mathtt{i}}) \quad \frac{\Gamma \Rightarrow \Delta, A}{\neg_{\mathtt{i}} A, \Gamma \Rightarrow \Delta}\ (\neg_i \Rightarrow)$$

$$\frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg_{\mathtt{c}} A, \Delta}\ (\Rightarrow \neg_{\mathtt{c}}) \quad \frac{\Gamma \Rightarrow \Delta, A}{\neg_{\mathtt{c}} A, \Gamma \Rightarrow \Delta}\ (\neg_{\mathtt{c}} \Rightarrow).$$

Proposition 3. For any $\Gamma \cup \Delta \subseteq \mathsf{Form}$, $\Gamma \Rightarrow \Delta$ is derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$ iff $\Gamma \models \Delta$ holds.

Proposition 4. If $\Gamma \Rightarrow \Delta$ is derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, then $\Gamma \Rightarrow \Delta$ is derivable in $\mathsf{G}^{-}(\mathbf{C}+\mathbf{J})$, where $\mathsf{G}^{-}(\mathbf{C}+\mathbf{J})$ is the calculus obtained by removing the rule (Cut) from $\mathsf{G}(\mathbf{C}+\mathbf{J})$.

By Proposition 4, the subformula property is obtained, which ensures the calculus is a conservative extension of both intuitionistic and classical propositional logic.

3 Craig Interpolation

In this section, we establishes two types of Craig interpolation theorem for $\mathsf{G}(\mathbf{C}+\mathbf{J})$, based on Maehara's partition argument in [9]. This argument is originally for classical sequent calculus $\mathbf{LK}$, and is dependent on the fact that the cut elimination holds in the calculus. Since cut elimination holds also in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, as is guaranteed by Proposition 4, this method can be employed. In the following part of this section, $\mathsf{Prop}(D)$ denotes the set of all propositional variables in a formula D. And if Γ is a finite multiset of formulas, we define $\mathsf{Prop}(\Gamma) = \bigcup\{\mathsf{Prop}(D) \mid D \in \Gamma\}$. Especially, we have $\mathsf{Prop}(\bot) = \emptyset$. We call $\langle(\Gamma_1 : \Delta_1); (\Gamma_2 : \Delta_2)\rangle$ a *partition* of a sequent $\Gamma \Rightarrow \Delta$, if Γ is Γ_1, Γ_2 and Δ is Δ_1, Δ_2. Let us say that C is an *interpolant* of $\langle(\Gamma_1 : \Delta_1); (\Gamma_2 : \Delta_2)\rangle$ if $\Gamma_1 \Rightarrow \Delta_1, C$ and $C, \Gamma_2 \Rightarrow \Delta_2$ are derivable and $\mathsf{Prop}(C) \subseteq \mathsf{Prop}(\Gamma_1, \Delta_1) \cap \mathsf{Prop}(\Gamma_2, \Delta_2)$.

Although the main idea of giving $\mathsf{G}(\mathbf{C}+\mathbf{J})$ is adding classical implication to intuitionistic logic, our proof is similar to that in classical logic. For establishing the Craig interpolation theorem for $\mathbf{mLJ}$, we cannot employ the notion of partition of the form $\langle(\Gamma_1 : \Delta_1); (\Gamma_2 : \Delta_2)\rangle$. This is because we cannot find an interpolant for $\langle(\emptyset : A); (A : \emptyset)\rangle$ as noted in [10]. Therefore, in order to show the theorem for $\mathbf{mLJ}$, the form of a partition should be restricted to $\langle(\Gamma_1 : \emptyset); (\Gamma_2 : \Delta)\rangle$. However, this restriction makes it possible to show neither of the two types of theorem in $\mathsf{G}(\mathbf{C}+\mathbf{J})$. Considering this situation, it seems difficult to establish the theorem for $\mathsf{G}(\mathbf{C}+\mathbf{J})$. However, the classical negation (or implication) enables us to use partitions of the form $\langle(\Gamma_1 : \Delta_1); (\Gamma_2 : \Delta_2)\rangle$ without any restriction to calculate an interpolant by Maehara method. This fact about the way of showing Craig interpolation theorem implies that $\mathbf{C}+\mathbf{J}$ can be regarded as the logic obtained by adding the special (intuitionistic) implication to classical logic[2].

Lemma 1. Suppose that $\Gamma \Rightarrow \Delta$ is derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$. Then for any partition $\langle(\Gamma_1 : \Delta_1); (\Gamma_2 : \Delta_2)\rangle$ of the sequent, there exits an interpolant C in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, i.e., such that both $\Gamma_1 \Rightarrow \Delta_1, C$ and $C, \Gamma_2 \Rightarrow \Delta_2$ are also derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, and $\mathsf{Prop}(C) \subseteq \mathsf{Prop}(\Gamma_1, \Delta_1) \cap \mathsf{Prop}(\Gamma_2, \Delta_2)$.

With Lemma 1, which is the core of the proof, we can easily show the following two types of Craig interpolation theorem.

Theorem 1. (Intuitionistic Craig Interpolation Theorem of $\mathsf{G}(\mathbf{C}+\mathbf{J})$). If $\Rightarrow A \to_{\mathtt{i}} B$ is derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, then there exists a formula C such that $\Rightarrow A \to_{\mathtt{i}} C$ and $\Rightarrow C \to_{\mathtt{i}} B$ are also derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$ and that $\mathsf{Prop}(C) \subseteq \mathsf{Prop}(A) \cap \mathsf{Prop}(B)$.

Theorem 2. (Classical Craig Interpolation Theorem of $\mathsf{G}(\mathbf{C}+\mathbf{J})$). If $\Rightarrow A \to_{\mathtt{c}} B$ is derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$, then there exists a formula C such that $\Rightarrow A \to_{\mathtt{c}} C$ and $\Rightarrow C \to_{\mathtt{c}} B$ are also derivable in $\mathsf{G}(\mathbf{C}+\mathbf{J})$ and that $\mathsf{Prop}(C) \subseteq \mathsf{Prop}(A) \cap \mathsf{Prop}(B)$.

4 Further Direction

In [15], the first-order expansion $\mathsf{G}(\mathbf{FOC}+\mathbf{J})$ of $\mathsf{G}(\mathbf{C}+\mathbf{J})$ can be given by adding classical universal quantifier to first-order multi-succedent intuitionistic sequent calculus $\mathbf{mLJ}$, although the similar restriction on the right rule for the intuitionistic universal quantifier is needed. Whether Craig interpolation holds in this expansion is an open question, which deserves being inquired.

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Simple Model and the Deduction System for Dynamic Epistemic Quantum Logic

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Quantum logic (QL) has been studied to handle strange propositions of quantum physics. In particular, logic based on *orthomodular lattices*, namely, *orthomodular logic* (OML), has been studied since 1936, proposed by Birkhoff and Von Neumann [10]. An orthomodular lattice is related to the closed subspaces of a Hilbert space, which is a state space of a particle in quantum physics. Instead of these lattices, the *Kripke model* (possible world model) of OML can be used, which is called the *orthomodular-model* (OM-model) [11] [12]. Intuitively, each possible world of an OM-model expresses a one-dimensional subspace of a Hilbert space, corresponding to a *quantum state*.

In quantum mechanics, due to the *uncertainty principle*, exact values cannot be simultaneously obtained for a specific set of physical quantities (for example, momentum and position along an axis). This statistical property is the nature of the states of the object and exist independently of an experimenter's knowledge. OML handles the most basic part of this strange nature of states.

To treat an agent's knowledge in quantum mechanics, some studies combine *epistemic logic* (EL) with QL. EL is a field of modal logic that treats the proposition of an agent's knowledge. In the Kripke model of EL, the *indistinguishability of states* is used to express knowledges. That is, if a formula ϕ is true at all states that are indistinguishable from the current state for agent i, then agent i knows that ϕ is true. Furthermore, *dynamic EL* (DEL) has been studied to handle the transitions of knowledge [15]. In general, *public announcement logic* (PAL) is treated as the most basic and simple logic in DEL. Basic PAL includes only two types of modal symbols: the symbols for knowledge K_i of individual agents and the symbol [ ] for public announcements. $[\phi]\psi$ can be read as " After a public announcement ϕ, ψ is true. "

Ref [8] and [9] can be cited as one of the studies of logic that deal with the concept of knowledge with quantum physics. In these studies, the models which incorporate specific *quantum information* concepts were used. Ref [2] and [3] can be cited as the studies of knowledge with more general concepts of quantum physics. In these studies, similar to EL, knowledge was expressed using the indistinguishability of states.

To discuss the general *change* of knowledges due to the procurement of informations, other concepts have to be introduced and the field of *dynamic epistemic QL* (DEQL) has to be developed. In [4], *quantum test frame* is introduced as a part of the study of the *dynamic logic of test* (DLT). DLT is a logic for dealing with general changes in knowledge due to information obtained by testing. Quantum test frame is based on the frame for DLT and the frame for *dynamic QL* (DQL) [5] [6] [7]. DQL uses modal symbols for several types of transitions of quantum states, such as *unitary evolutions* and *projections*. An important aspect of quantum physics is the change of state due to measurement. In quantum physics, when a physical quantity is observed, the state is projected to an eigenstate of the physical quantity. That is, the state of the particle itself changes depending on the obtained information. In (classical) EL, if $x(\phi)y$, then $x = y$; where x and y denote states and (ϕ) is the relation for information ϕ. Reflecting the nature of quantum physics, in quantum test frame, this property is not true [5] [6] [7].

As mentioned above, the transition of knowledge in quantum mechanics has been analyzed in some directions. However, some problems remain.

1. These models in previous studies are little complicated because these models introduce almost every modal element related to quantum mechanics. Such a model is also needed, but a somewhat simple and abstracted model that leaves only the important notions is also useful to analyze specific feature of knowledge in quantum mechanics.

2. As the models and symbols are complicated, constructing a *deduction system* for this types of logic is somewhat complicated task because we have to deal with the mutual consistency of many conditions. Actually, deduction systems for DEQL have not been analyzed much.

Therefore, in this study, as a basis for solving these problems, we construct new logic and models for the transition of knowledge in quantum mechanics that is simpler than previous studies, while retaining the essence of these studies. Furthermore, we construct a deduction system that holds soundness and completeness for those new models. Because of these purposes, herein, we mainly focus on mathematical and logical analysis, rather than quantum mechanical analysis.

We construct *dynamic epistemic orthomodular logic* (DEOML) by combining the frames and models of OML and PAL, and we simply use a combination of logical symbols for OML and PAL. The meaning of $[\phi]$ in DEOML is different from that in PAL. In DEOML(and in quantum test frames), $[\phi]$ denotes the action that the agent obtains the information ϕ by observing a state of the particle. However, they are the same in terms of "obtaining the information that ϕ is true." Therefore, in fact, the logical nature for this symbol are almost the same in each logic. Moreover, due to the simplicity of DEOML, this similarity is used to prove useful theorem (which is described in last paragraph) similar to PAL, which is difficult to established in the models in previous studies.

OML is adopted instead of DQL for the foundation of logic because of the following advantages.

1. Although OML is not a modal logic, OM-models *implicitly* include the concept of the modality of projection as binary relations that satisfies some important conditions [17]. Therefore, OML can handle the concept of projection while being a simpler model than DQL, which include the notion of of projections explicitly.

2. OML does not include the other dynamic concepts of quantum mechanics, such as unitary evolutions. However, the most important strange properties of the agent's knowledge that appear in quantum mechanics are related to projective observations. Therefore, the important properties can be analyzed as long as the concept of projection is included in the logic.

3. Different from DQL, deduction systems for OML are well argued in previous studies [13] [14] [16] [18] [19], and we can use them directly to construct a deduction system for DEOML.

We construct a *sequent calculus* type deduction system for DEOML and prove the soundness and completeness theorem with respect to new models.

Sequent calculus is suitable for this study because it is compatible with OML and modal logics [13] [18] [19]. Hilbert-style systems for OML have also been studied [14] [16]. However, they contain unique symbols for creating the Hilbert-style system, which are not suitable for combination with other (modal) symbols.

In this new logic, two types of formulae are used: a *quantum formula* (q-formula), and a *general formula* (g-formula).

q-formula $A ::= p \mid \perp \mid\sim A \mid A\wedge A$

g-formula $\phi ::= A \mid \neg\phi \mid \phi\wedge\phi \mid \mathsf{K}\phi \mid [A]\phi$

The q-formulae are included in g-formulae. The q-formulae are correspond to the propositions in OML. That is, q-formulae are used to express the propositions of quantum mechanics. g-formulae are used to express modal notions including knowledge and change of informations. We use the definition that only q-formulae can be placed in the modal symbol [ ] because we deal with the situation where the agent gets information about the particles in an experiment. By using this condition, the same concept of projections [ ] defined in advanced OM-model [17] can be used.

In this study, similar to [1] [4], we focus on the situations where only one agent is present. The main reason for this restriction is that models for QL which are currently configured are not very suitable for dealing with *product* Hilbert spaces, which represent state spaces of multiple particles and agents. Therefore, a study of logic that includes more than one agent or more than one particle in binary relational model is somehow different from this study.

It is shown that even with these restricted definition, important parts of knowledge in quantum mechanics still can be expressed. For example, $\mathsf{K}p \to [A]\mathsf{K}p$ is valid in PAL but not always valid in models of DEOML. Intuitively, this is because an announcement may change an agent's knowledge but not change the environment in PAL. In contrast, as mentioned earlier, in quantum mechanics, when we obtain information from the environment (particles), the state of the environment may change because of projections.

The main contributions of this study are as follows.

A novel model for DEQL that can analyze the transitions of knowledge is constructed, and it is simpler than the models in previous studies. The method of configuration of the model is also completely different from the previous studies.

Using the new model, we construct a new logic DEOML.

Some similarity and differences between PAL and DEOML from the mathematical logic perspective are analyzed. That is, following formulae are valid in DEOML.

$$[A]B \leftrightarrow \sim A \sqcup (A \wedge B))$$

$$[A](\phi \wedge \psi) \leftrightarrow ([A]\phi \wedge [A]\psi)$$

$$[A]\neg\phi \leftrightarrow (\neg \sim A \rightarrow \neg[A]\phi)$$

$$[A]\mathsf{K}\phi \leftrightarrow (\neg \sim A \rightarrow \mathsf{K}[A]\phi)$$

Deduction system for DEOML, which is sound and complete with respect to these new models are established. This results of deduction system for DEQL is completely new.

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On the Degrees of Ignorance: via Epistemic Logic and μ-Calculus

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Epistemic Logic normally discourses on knowledge, belief, and related concepts. We here study ignorance instead. With the help of the μ-calculus, we analyze the degrees of ignorance in which an agent doesn't know whether or not a given proposition is true. Building on the study by Stalnaker [14], we argue that logics "closer" to S4.2 allow greater degrees of ignorance, compared to logics "closer" to S5.

We consider Epistemic Logic with the modal operators K (for knowledge) and B (for belief). We will focus on the case where there is only one agent, following Hintikka [7], Lenzen [8], and Stalnaker [14]. We suppose that K satisfies (at least) S4 and B satisfies KD45. By belief, we mean strong belief, and suppose that the agent's beliefs has no contradiction and they have introspection about their own beliefs:

$$K\varphi \to B\varphi,$$
$$B\varphi \to KB\varphi, \text{ and}$$
$$\neg B\varphi \to K\neg B\varphi.$$

Lenzen [8] showed that the interaction axioms above imply that $B\varphi \leftrightarrow \hat{K}K\varphi$, that is, belief can be defined by knowledge. In practice, we consider B as a defined modality. Lenzen's proof also implies that K satisfies S4.2.

The concept of ignorance was also studied by van der Hoek and Lomuscio [6]. They defined a modal operator for ignorance by

$$I\varphi :\leftrightarrow \neg K\varphi \wedge \neg K\neg\varphi.$$

They also define a logic for ignorance Ig and prove that it is sound and complete over the class of all frames. Fine [5] studied the n^{th}-order ignorance $I^n\varphi$, where $I^{n+1}\varphi :\leftrightarrow I(I^n\varphi)$ and $I^0\varphi :\leftrightarrow \varphi$. In particular, it is shown that $I^2\varphi$ is equivalent to the so-called Rumsfeld ignorance "the unknown unknown", $I\varphi \wedge \neg KI\varphi$. Fine also showed that, for any φ, $\neg KI^2\varphi$ is valid on any frame of S4.2. Therefore, the knowledge of second-order ignorance is unobtainable. Note that the ignorance modality I is a particular case of the contingency modality ∇ (see Montgomery and Routley [10]). ∇ is defined by

$$\nabla\varphi :\leftrightarrow \Diamond\varphi \wedge \Diamond\neg\varphi.$$

We build on Stalnaker's [14] analysis and consider the logics S4.2, S4.3, S4.3.2, S4.4, and S5 for knowledge. They are defined using the axioms in Table 1 with the necessitation rule. Aucher [2] characterized these logics by axioms relating knowledge and belief/conditional belief similar to Lenzen's characterization of S4.2.

Axiom Name	Axiom	Frame conditions
K	$K(\varphi \to \psi) \to (K\varphi \to K\psi)$	(no condition)
D	$K\varphi \to \hat{K}\varphi$	Serial
T	$K\varphi \to \varphi$	Reflexive
4	$K\varphi \to KK\varphi$	Transitive
5	$\hat{K}\varphi \to K\hat{K}\varphi$	Euclidean
.2	$\hat{K}K\varphi \to K\hat{K}\varphi$	Convergent
.3	$K(K\varphi \to \psi) \vee K(K\psi \to \varphi)$	Weakly Connected
.3.2	$(\hat{K}\varphi \wedge \hat{K}K\psi) \to K(\hat{K}\varphi \vee \psi)$	Semi-Euclidean
.4	$(\varphi \wedge \hat{K}K\varphi) \to K\varphi$	(no particular name)

Table 1: Modal axioms for K.

The (modal) μ-calculus is obtained by adding to modal logic the fixed-point operators μ and ν, for least and greatest fixed-points. The μ-formulas are generated by the grammar

$$\varphi := P \mid \neg P \mid X \mid \varphi \wedge \varphi \mid K\varphi \mid \mu X.\varphi \mid \nu X.\varphi.$$

We denote the dual operators of K and B by $\hat{K}$ and $\hat{B}$. For reasons that will become clear later, we consider only alternation-free formulas, that is, formulas with no nested alternation of μ and ν operators. More rigorously, a μ-formula is alternation-free if it has no subformula of the form $\mu X.\varphi$ (or $\nu X.\varphi$) such that φ has a subformula $\nu Y.\psi$ (or $\mu Y.\psi$) with a free occurrence of X in ψ.

The relational semantics for the μ-calculus is defined as follows. Given a model M and a μ-formula φ, we will define $\|\varphi\|^M$ to be the set of worlds w where φ holds. Propositional operators and modal operators are treated as usual. For fixed-point operators, letting $\Gamma_\varphi(X) = \|\varphi(X)\|^M$, we have

$$\|\mu X.\varphi(X)\|^M \text{ is the least fixed point of } \Gamma_\varphi, \text{ and}$$
$$\|\nu X.\varphi(X)\|^M \text{ is the greatest fixed point of } \Gamma_\varphi.$$

For an example of the use of fixed-point operators, suppose we have modality E for "everyone knows". Then a formula φ is common knowledge iff the following formula holds:

$$\nu X.(\varphi \wedge EX).$$

That is, φ is common knowledge iff it is true, everybody knows that φ is true, everybody knows that "everybody knows that φ is true", and so on.

The operators μ and ν induce a (syntactical) hierarchy of the μ-formulas, measuring the entanglement of least and greatest fixed-point operators. Bradfield [4] showed that, in general, the hierarchy is strict: for all natural number

n there is a formula with alternation depth $n+1$ which is not equivalent to any formula of alteration depth n. But this strictness may fail in a restricted class of models. In fact, Alberucci and Facchini [1] showed that on a frame satisfying $\mathsf{S4}$, the hierarchy collapses to its alternation-free fragment: every μ-formula is equivalent to an alternation-free μ-formula. This justifies our restriction to alternation-free formulas in the definition of our μ-calculus. They also showed that the hierarchy collapses to modal logic on frames of $\mathsf{S5}$: every μ-formula is equivalent to a modal formula. Also note that we can define a *weak* alternation hierarchy on the alternation-free fragment. The authors have shown the strictness of the weak alternation hierarchy on recursive frames [11].

In [12], the authors show that the alternation hierarchy collapses to its alternation-free fragment over frames of

$$\mathsf{S4.2} \text{ and } \mathsf{S4.3};$$

and collapses to modal logic over frames of

$$\mathsf{S4.3.2}, \mathsf{S4.4} \text{ and } \mathsf{KD45}.$$

Therefore there must be an (alternation-free) formula φ which is not equivalent to any modal formula over $\mathsf{S4.2}$ and $\mathsf{S4.3}$; but *is* equivalent to a modal formula over $\mathsf{S4.3.2}$, $\mathsf{S4.4}$ and $\mathsf{KD45}$. While this abstract uses only relational semantics, the collapses of the alternation hierarchy can be transferred to topological semantics by a result of Baltag *et al.* [3].

We analyze a formula which is not equivalent to any modal formula over $\mathsf{S4.2}$ and $\mathsf{S4.3}$. Let φ be any μ-formula, and define

$$\alpha_\varphi(X) := \hat{K}(\varphi \wedge X) \wedge \hat{K}(\neg\varphi \wedge X).$$

We study $\alpha_\varphi^\infty := \nu X.\alpha_\varphi$ and its approximants $\alpha_\varphi^1 := \alpha_\varphi(T)$, $\alpha_\varphi^{n+1} := \alpha_\varphi(\alpha_\varphi^n)$; they will be used to measure the agent's degree of ignorance with respect to φ. Each α_φ^i will represent a degree of ignorance. Over $\mathsf{S4.2}$, any degree implies the weaker degrees but the converse may not hold. That is, if $i, j \in \mathbb{N} \cup \{\infty\}$ and $i < j$, then α_φ^j implies α_φ^i; and the converse doesn't hold as $\alpha_\varphi^i \wedge \neg\alpha_\varphi^j$ is satisfiable. Therefore we have, in general, infinitely many degrees of ignorance. Our first degree of ignorance α_φ^1 is equivalent to $I\varphi$, and all the α_φ^i can be thought of as generalizations of $I\varphi$.

Van der Hoek and Lomuscio [6] state that the ignorance modality I is not intended to capture degrees of ignorance, while our α_φ^i's are intended to do so. Furthermore, $K\alpha_\varphi^i$ is satisfiable for any $i \in \mathbb{N} \cup \{\infty\}$. Therefore the α_φ^i are different from second-order ignorance $I^2\varphi$, and not obtainable by iterations of I.

If we change our settings, we may have finitely many degrees of ignorance. Consider $\mathsf{S4.4}$, the logic of knowledge as true belief. We can show here that $\alpha_\varphi^1 \wedge \neg\alpha_\varphi^2$ is equivalent to the agent having a false belief and that α_φ^2 is equivalent to α_φ^∞. That is, we have only two non-equivalent degrees of ignorance: α_φ^1, where the agent's belief is false; and α_φ^2, where the agent believes neither φ nor $\neg\varphi$.

The same analysis holds for S4.3.2, which has the same two degrees. This logic is used, for example, in [13].

In S5, the standard logic for multi-agent epistemic logic, we have only one degree of ignorance. In this setting, α^1_φ is equivalent to α^∞_φ. We also have that belief is equivalent to knowledge, $B\varphi \leftrightarrow K\varphi$, so the agent has no wrong beliefs, and being ignorant of φ also means that they believe neither φ nor $\neg\varphi$.

Now consider the interpretation of $\alpha^i_\varphi \wedge \neg\alpha^{i+1}_\varphi$ over S4.2 and S4.3. S4.2 is the logic of knowledge according to Lenzen [8] and Stalnaker [14]. S4.3 is Lehrer and Paxon's undefeated justified true belief [9]. Here, $\alpha^1_\varphi \wedge \neg\alpha^2_\varphi$ is equivalent to the agent's belief being false and the agent knowing whether φ holds in every world other than the real world. Likewise, $\alpha^2_\varphi \wedge \neg\alpha^3_\varphi$ holds exactly when the agent has a true belief but considers it possible that their belief is false. Symbolically,

$$\alpha^2_\varphi \wedge \neg\alpha^3_\varphi \equiv [\varphi \wedge B\varphi \wedge \hat{K}(\neg\varphi \wedge B\varphi)] \vee [\neg\varphi \wedge B\neg\varphi \wedge \hat{K}(\varphi \wedge B\neg\varphi)].$$

This analysis can be extended to other $\alpha^i_\varphi \wedge \neg\alpha^{i+1}_\varphi$ to show that each degree of ignorance expresses a higher level of the agent's self-doubt.

Still in S4.2 and S4.3, note that, for $n \in \mathbb{N}$, α^n_φ implies the agent has a belief (which may be true or false) and the agent not having a belief implies α^∞_φ. In other words, having no belief implies a high degree of ignorance, but a high degree of ignorance does not deny the agent having a belief.

From the point of view of our degrees of ignorance, S4.3 and S4.3.2 are very different: S4.3 has infinitely many degrees of ignorance; while S4.3.2 has only two degrees. This contrasts with Stalnaker's critics of S4.3 and S4.3.2, which argues that in both logics false belief can deny knowledge: in S4.3 a false belief can deny some knowledge the agent may be justified in having; and in S4.3.2 a false belief denies all non-trivial knowledge.

At last, we can do a similar analysis to belief, using

$$\delta_\varphi(X) := \hat{B}(\varphi \wedge X) \wedge \hat{B}(\neg\varphi \wedge X),$$

a belief variant of $\alpha_\varphi(X)$. Define $\delta^1_\varphi := \delta_\varphi(\top)$, $\delta^{n+1}_\varphi := \delta_\varphi(\delta^n_\varphi)$ and $\delta^\infty = \nu X.\delta(X)$. Then, for $i \in \mathbb{N} \cup \{\infty\}$, δ^1_φ is equivalent to δ^i_φ over KD45, similar to the case where knowledge satisfies S5. Therefore we can only define one degree of disbelief by our approach.

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The Creation and Change of Social Networks

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Recently, epistemic-social phenomena have received more attention from the logic community, analyzing peer pressure, studying informational cascades, inspecting priority-based peer influence, modeling diffusion and prediction, and examining reflective social influence. In this presentation, I will contribute to this line of work and focus in particular on the logical features of social group creation. I pay attention to the mechanisms which indicate when agents can form a team based on the correspondence in their set of features (behavior, opinions, etc.). Our basic approach uses a semi-metric on the set of agents, which is used to construct a network topology. This structure is then extended with epistemic features to represent the agents' epistemic states, allowing us to explore group-creation alternatives where what matters is not only the agent's differences but also what they know about them. The logical settings in this work make use of the techniques of dynamic epistemic logic to represent group-creation actions, to define new languages in order to describe their effects, and to provide sound and complete axiom systems. This talk is based on joint work with Fernando Velazquez Quesada.

Learning what Others Know

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I present recent work on modelling scenarios in which agents are given (or gain) access to all the relevant information possessed by some other agents (including information of a non-propositional nature, such as data, passwords etc). Modelling such scenarios requires us to extend the framework of epistemic logics to one in which we abstract away from specific announcements. In order to do this, I introduce a general framework for such informational events, that subsumes actions such as sharing all you know with a group or individual, giving one access to some folder or database, exchanging all relevant information within a closed subgroup, hacking a database without the owners knowledge, etc. We formalize their effect, i.e. we express the state of affairs in which one agent (or group) has epistemic superiority over another agent/group, using comparative epistemic assertions (the extend to groups the individual comparative formulas considered in [5]). Another ingredient is a new modal operator for common distributed knowledge, that combines features of both common knowledge and distributed knowledge, and characterizes situations in which common knowledge can be gained in a larger group of agents (formed of a number of subgroups) by communication only within each of the subgroups. This is joint work with Sonja Smets [1], though I position it in the context of other related work [2-8].

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