Proceedings of
SOCREAL 2019

5th International Workshop
On Philosophy and Logic of Social Reality
15-17 November 2019
Hokkaido University, Sapporo, Japan

(Ed. ) Tomoyuki Yamada
About SOCREAL 2019

Since the last years of the 20th century, a number of attempts have been made in order to model various aspects of social interaction among agents including individual agents, organizations, and individuals representing organizations. The aim of SOCREAL Workshop is to bring together researchers working on diverse aspects of such interaction in logic, philosophy, ethics, computer science, cognitive science and related fields in order to share issues, ideas, techniques, and results.

The earlier editions of SOCREAL Workshop was held in March 2007, March 2010, October 2013, and October 2016. Building upon the success of these editions, its fifth edition was held from 15 November till 17 November 2019 under the auspices of Philosophy and Ethics Laboratory at Faculty of Humanities and Human Sciences, Hokkaido University, CAEP (Center for Applied Ethics and Philosophy) at Faculty of Humanities and Human Sciences, Hokkaido University, and LOG-UCI (An interdisciplinary study of the logical dynamics of the interaction between utterances and social contexts), a research project funded by JSPS (JSPS KAKENHI Grant Number JP 17H02258).

SOCREAL 2019 consisted of keynote lectures by invited speakers and presentations of submitted papers. Researchers from various fields, including logic, philosophy, ethics, computer science, cognitive science had been invited to submit an extended abstract (up to two thousand words) by 30 August 2019. We had received 16 abstracts and each of them had been peer-reviewed by the program committee of the workshop. Here you will find abstracts or presentation slides of 8 accepted papers and 4 keynote lectures.
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What is and what might have been

Jeremy Seligman
The University of Auckland, New Zealand

A central theme of Arthur Prior's 1956 Locke Lectures on "Time and Modality" is the trouble raised for logic by the contingency of existence. His tentative solution was to abandon bivalence with the curious and poorly-understood System Q. I present a simple alternative that has been overlooked by the mainstream development of modal predicate logic, and which provides an easy way of combing actualism (the view that all that exists actually exists) with contingentism (the view that there might have been things other than there are). Timothy Williamson's argument for the necessity of existence is then re-examined. For those familiar with my work, this relates directly to "common sense" modal predicate logic, but focusses more on the philosophical context.
A Sequent Calculus for K-restricted
Common Sense Modal Predicate Logic

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November 15–17, 2019

Abstract

In recent years, Common sense Modal Predicate Calculus (CMPC) has been pro-
posed by J. van Benthem in [4, pp. 120–121] and further developed by J. Seligman in [1, 3, 2]. It allows us to ‘take $\exists$ to mean just “exists” while denying the Constant Domain thesis’ [1, p. 8]. This is done in terms of talking about only things in each world in which they exist. From a proof-theoretical view, the Hilbert-style system for CMPC given by Seligman is a system for modal predicate logic $S5$ which has the following axiom $K_{in}$ instead of axiom $K$:

$$\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$$

provided that all free variables in $\varphi$ are free variables in $\psi$.

It is quite interesting because it might make a clean sweep of all philosophical dis-
cussions on possible world semantics between actualists and possibilists. How-
ever, neither van Benthem nor Seligman have developed K-restricted CMPC and
expansions of the logic with some well known axioms. Moreover, proof-theoretic
studies for such logics have not been done yet.

In this talk, I shall propose a sequent calculus for K-restricted CMPC. The main
mathematical contributions of this talk are the completeness result (Theorem 1)
and cut elimination theorem (Theorem 2) for the calculus. If time allows I shall
also introduce sequent calculi for K-restricted CMPC with T axiom and D-like
axioms. In what follows, I will outline the contents of this talk.

The language $L$ of K-restricted Common sense Modal Predicate Calculus $CK$
consists of a countably infinite set $\text{Var} = \{x, y, \ldots\}$ of variables, a countably
infinite set $\text{Pred} = \{P, Q, \ldots\}$ of predicate symbols each of which has a fixed finite

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1The Constant Domain thesis is a thesis that ‘[e]very possible world has exactly the same objects as every other possible world.’ [1, p. 5]
arity, and logical symbols, $\bot, \supset, \Box, \forall$. The set Form of formulas of $L$ is defined recursively as follows:

$$\text{Form} \ni \phi := P x_1 \ldots x_n \mid \bot \mid (\phi \supset \phi) \mid \forall x \phi \mid \Box \phi$$

where $P$ is a predicate symbol with arity $n$ and $x, x_1, \ldots, x_n$ are variables. The other connectives are defined as usual. We also define the sets $\text{FV}(\phi)$ and $\text{FV}(\Gamma)$ of free variables in a formula $\phi$ and a set $\Gamma$ of formulas, respectively, as usual.

Semantics for CK is given as follows. A frame is a tuple $(W, R, D)$, where $W$ is a nonempty set, $R$ is a binary relation on $W$; $D$ is a $W$-indexed family $\{D_w\}_{w \in W}$ of nonempty sets. Thus $R$ does not need to satisfy the inclusion requirement: if $wRv$ then $D_w \subseteq D_v$. A model is a tuple $(F, V)$, where $F$ is a frame and $V$ is a valuation that maps each world $w$ and each predicate $P$ to a subset $V_w(P)$ of $D_w$. An assignment $\alpha$ is a partial function from variables to entities and $\alpha(x|d)$ stands for the same assignment as $\alpha$ except for assigning $d$ to $x$. In addition to these notions, we follow [1, p. 15] and say that a formula $\phi$ is an $\alpha_w$-formula if $\alpha(x) \in D_w$ for any variable $x \in \text{FV}(\phi)$. Then, similarly as in [1, pp. 15–16], the satisfaction relation and validity are defined as follows.

**Definition 1** (Satisfaction relation). Let $M$ be a model, $\alpha$ be an assignment, and $w$ be a world in $W$. The satisfaction relation $M, \alpha, w \models \phi$ between $M, \alpha, w$ and an $\alpha_w$-formula $\phi$ is defined as follows:

- $M, \alpha, w \not\models \bot$ if $\alpha(x_i), \ldots, \alpha(x_n)) \in V_w(P)$
- $M, \alpha, w \models \psi \supset \gamma$ if $M, \alpha, w \models \psi$ implies $M, \alpha, w \models \gamma$
- $M, \alpha, w \models \forall x \psi$ if $M, \alpha(x|d), w \models \psi$ for any $d \in D_w$
- $M, \alpha, w \models \Box \psi$ if $M, \alpha, v \models \psi$

for any $v$ such that $wRv$ and $\psi$ is an $\alpha_v$-formula

**Definition 2** (Validity). Let $\Gamma \cup \{ \phi \}$ be a set of formulas. We say that $\phi$ is valid in a frame if for any model $M$ based on the frame, assignment $\alpha$ and world $w$ such that $\phi$ is an $\alpha_w$-formula, $M, \alpha, w \models \phi$. We also say that $\phi$ is valid in a class of frames if $\phi$ is valid in all frames in the class.

The following propositions that Seligman proves in [1, pp. 16–17] are noteworthy.

**Proposition 3** (Converse Barcan formula). A formula $\Box \forall x \phi \supset \forall x \Box \phi$ is valid in the class of all frames.

**Proof.** Fix any model $M$, assignment $\alpha$, world $w$ such that $\Box \forall x \phi \supset \forall x \Box \phi$ is an $\alpha_w$-formula. Suppose $M, \alpha, w \models \Box \forall x \phi$ and fix any element $d \in D_w$, any world $v$ such that $wRv$ and $\phi$ is an $\alpha(x|d)_v$-formula. We show $M, \alpha(x|d), v \models \phi$. Since

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2Strictly speaking, he considers the dual formulas of those in Proposition 3.4.
Proposition 4. A formula \( \forall x \square \varphi \supset \square \forall x \varphi \) is not valid in the class \( \mathcal{F} \) of all frames \( F = (W, R, D) \) such that \( R \) is an equivalence relation.

Proof. Consider a model \( M = (W, R, D, V) \), where \( W = \{0, 1\} \); \( R = W \times W \); \( D_0 = \{a\} \) and \( D_1 = \{b\} \); \( V_P = \{a\} \) and \( V_1(P) = \emptyset \) for some predicate symbol \( P \) with arity 1, and \( V_1(Q) = \emptyset \) for the other predicate symbols \( Q \) with arity \( n \). Then, we can establish \( M, \alpha, 0 \models \forall x \square \varphi \) but \( M, \alpha, 0 \not\models \Box \forall x \varphi \). Therefore, \( \forall x \square \varphi \supset \square \forall x \varphi \) is not valid in \( \mathcal{F} \).

Given finite multisets \( \Gamma, \Delta \) of formulas, we call an expression \( \Gamma \Rightarrow \Delta \) a sequent. Then a sequent calculus \( G(\mathbb{CK}) \) for \( \mathbb{CK} \) is given in Table 1. The rule \( \Box \text{inv} \) in it plays roles of axiom \( K_{\text{inv}} \) and the necessitation rule in the Hilbert-style system for CMPC given by Seligman. The notion of a derivation in \( G(\mathbb{CK}) \) is defined as usual.

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\( \dagger \): \( y \) does not occur in \( \Gamma, \Delta, \forall x \varphi \). \( \ddagger \): \( \text{FV}(\Gamma) \subseteq \text{FV}(\varphi) \).
Theorem 1 (Completeness). Let $\Gamma \cup \{ \varphi \}$ be a set of formulas. If $\Gamma \Rightarrow \varphi$ is valid in the class of all frames, then $\Gamma \Rightarrow \varphi$ is derivable in $G(CK)$.

Theorem 2 (Cut elimination). Let $\Gamma, \Delta$ be finite multisets of formulas. If $\Gamma \Rightarrow \Delta$ is derivable in $G(CK)$, then it is also derivable in $G(CK)$ without any application of Cut.

References


Mereological group ontology analyses groups as wholes composed of physical parts. Such mereological accounts have enjoyed renewed popularity in recent years (Sheehy 2003, 2006a, 2006b; Ritchie 2013, 2015, 2018; Hawley 2017; Strohmaier 2018), but they have not yet addressed a crucial challenge. How can such accounts respond to proposed instances of groups without members or any other parts? The literature suggests that the Supreme Court can persist when all judges serving on it resign simultaneously (Epstein 2015) and that corporations do not require any physical parts (Smith 2003).

If these supposed examples of empty groups held up to scrutiny, it would undermine the mereological approach to group ontology. Groups cannot be wholes without having parts. In the present paper, I show how mereological accounts can face this challenge and dispel the force of these counterexamples.

To specify neo-Aristotelian mereology, I use Fine’s hylomorphic approach as laid out in his 1999 paper Things and their Parts (see also Uzquiano 2018). In this text he introduces rigid and variable embodiments. A rigid embodiment has a constituent structure that can be represented as “<a, b, c.../R>”, where a, b, c... are objects, R is a relation and the “/” denotes the primitive relation of rigid embodiment. The objects as well as the relations are timeless parts of the whole. According to Fine, a ham sandwich can be analysed as a rigid embodiment of three timeless parts, two pieces of bread and a ham piece. The relation R would be the ham being between the bread pieces.

Variable embodiments are represented as “f=/F/”, where F is a principle and there are a series of manifestations f. F picks out the objects as manifestations of the embodiment and can be thought of as a function from world-times to things (Fine 1999: 69). The water in the Thames can be analysed as such a variable embodiment where a principle, presumably related to the riverbed, picks out various quantities of water as manifestations. These manifestations are a part of the variable embodiment at the time of manifestations and the embodiment exists whenever a manifestation of it exists.

Fine analyses a car using both resources. It is a variable embodiment picking out a rigid embodiment of a motor, a chassis and other parts at a world-time. The combination of variable and rigid embodiments allows the car to undergo changes in parts, while also capturing that there is a structuring relation to the whole. At each point in time during which the car exists it is manifested by a rigid embodiment of parts, but over time these can be different embodiments. The wheels might be replaced, which entails a change of rigid embodiments, while the car persists.

Following Fine, one can analyse groups the same way as cars (see also Uzquiano 2018). A reading group is a variable embodiment /G/ manifested by rigid embodiments, that is the manifestation of the group at a world-time is a whole with parts standing in a particular relation to each other. If Rory, Paris, and Doyle are the members of the reading group today, there is a rigid embodiment <Rory, Paris, Doyle/R>. Should at a later point Marty join the reading group, then another rigid embodiment <Rory, Paris, Doyle, Marty/R> would manifest the group at that later world-time. The relation could also change, for example, because Paris appoints herself successfully leader of the reading group. Again, that would lead to a different embodiment <Rory, Paris, Doyle, Marty/R’> manifesting the group at
that point. In virtue of being a variable embodiment the group can persist through all these changes of rigid embodiments that manifest it. The theory, however, requires physical manifestations of groups at each point of their existence.

I focus in my discussion on the case of the Supreme Court. It has been a prime example in group ontology (see Uzquiano 2004) and Brian Epstein has suggested it as a candidate for a group which exists at some world-time without members. In his book the Ant Trap, he raises the question whether the Supreme Court existed, “with no members, when the Constitution was ratified, or when the Judiciary Act of 1789 was passed?” (Epstein 2015: 158) In an illustrative example, Epstein also assumes that “once [the Supreme Court] has come to exist, it continues to exist in perpetuity” (Epstein 2015: 159). Accordingly, the Supreme Court will also persist without any members.

This proposal of an empty group is motivated within Epstein’s approach to social ontology. According to Epstein, facts about the Supreme Court are not exclusively grounded in facts about its members and some of them are not even partially grounded in facts about the members. The Supreme Court has certain powers, such as revoking the decisions of lower court, independently of its members and one might propose that the Supreme Court has these powers even when it does not have any members. But the Supreme Court can have such powers only while it exists.

As can be seen, Epstein raises multiple questions, which can receive different responses. For the present purposes, the decisive questions are (1) whether the Supreme Court can exist at any world-time without having members; (2) whether the Supreme Court can exist at any world-time without having parts; and (3), if the response to either of these questions is affirmative, whether this is compatible with a mereological account of groups.

One simple solution I will neglect here is to argue that the Supreme Court has other material parts than its members, e.g. the Supreme Court Building. While this would address the difficulty in the specific case of the Supreme Court, it would be easy to adapt the supposed counterexamples to evade this response. It would be too much to require that all candidate groups have non-member material parts.

Instead, I propose, the case of the Supreme Court is best dealt with by comparing it with the case of the US President. The US Presidency was also created prior to the first President being sworn in and the role of the Presidency would also continue if the President unexpectedly died and before anyone else replaced them. Nonetheless, in neither case would one say that there exists a US President although there is no person who is the President. Since the US presidency does not confer immortality, there is no President persisting throughout the history of the United States.

What would persist, I suggest, is a whole that allows for someone to be sworn in and become the US President under appropriate conditions. The federal government is a mereological composite that typically manifests as a rigid embodiment including a President. Likewise, prior to any Supreme Court justice being confirmed, what exists is a whole which allows for a group, the actual Supreme Court, to fill a specific role. The Judiciary Act of 1789 established the federal judiciary and thereby changed the federal government to enable groups to serve as the Supreme Court.

The proposal of groups without members rests on a confusion between type and token. The type Supreme Court can exist in the absence of any justices serving on the court, because it can exist in the absence of a token group. The type Supreme Court has certain powers lacking a material token, in the same way the US President has pardoning power even when there is no token President. It is a power conferred to tokens in virtue of being to a certain type. As soon as we remind ourselves of the type-token distinction, there are no special mysteries here.
What makes the confusion between types and tokens so tempting is that there can always only be one realising token of the type Supreme Court. But that is also the case for the Presidency, where only one person can be the US President at a world-time. Nonetheless, no one would suppose that the US Presidents can persist independently of a physical object and there exists a separate term for the type.

Having accounted for the intuitions in the case of the Supreme Court, I also address how this solution can be generalised to other groups, including limited liability corporations. As a consequence of my proposal, one needs to distinguish a specific type for each corporation from its tokens. There is a Microsoft type in addition to a Microsoft token. Such a multiplication of types might appear counter-intuitive, but I propose it is not unusual within the field of social ontology. We can set up kinds such as the US Presidency without much ado (cf. Epstein 2015).

I will also address the problem that some ordinary discourse seems to presuppose the existence of a group token despite the lack of physical parts. One might say of a corporation lacking any physical embodiment that it still owns abstract assets such as patents. My account is committed to not take these utterances at face value. I will discuss how such property ascriptions can be re-interpreted and propose that we understand them as falling under a subjunctive conditional; that is if the type were to be instantiated, the token would have these properties.

By dispelling examples of groups seemingly lacking members, my paper resolves one of the few remaining challenges to mereological accounts of groups. Consequently, group mereology exerts great appeal and can serve as the foundation of future research, empirical as well as philosophical.
Some Earlier Works

Earlier works on the kind of regularity in question include:


Why this is a problem?

In Grossi and Jones (2013, p.416), Jones and Sergot (1996) are said to represent count-as conditional as $\varphi_1 \Rightarrow_c \varphi_2$.

They proposed the following principle as one of the “minimal core of the logical principles for the logic of count-as” (Grossi et al. 2013, pp. 416-417. Cf. Jones and Sergot, 1996, pp. 436):

$$((\varphi_1 \Rightarrow_c \varphi_2) \land (\varphi_2 \Rightarrow_c \varphi_3)) \Rightarrow (\varphi_1 \Rightarrow_c \varphi_3)$$

As Jones and Sergot (1996, p. 430) understand c as an institution, it is natural to think of c as fixed.

If c is understood just as an arbitrary context, however, we have to admit the possibility of a context being part of two or more institutions.

Formal approaches to count-as conditionals

Several attempts to capture the logic of count-as conditionals have been made in the deontic logic literature recently.

Grossi and Jones (2013) gives a succinct overview of the following works:

1. Jones et al. (1996),
2. Gelati et al. (2002, 2004),
4. Lorini et al. (2008, 2009),
5. Governatori et al. (2008),
7. Lindahl et al. (2006, 2008a, b).

A problem

Count-as conditionals are introduced by John Searle (1969) as “constitutive rules” of the following form.

$$X \text{ counts as } Y \text{ in context } C.$$

I'm wondering whether the recent discussions pay enough attention to the distinction between concrete particular contexts in which entities or processes of type X count as Y and the common type C shared by such contexts.

The purpose of this paper is to show how this problem can be avoided by modeling contexts and actions done in them in channel theory of Barwise and Seligman (1997).

Iteration

Consider the following quotation from Searle (1995).

Making certain noises counts as uttering an English sentence, uttering a certain sort of English sentence in certain circumstance counts as entering into a contract, entering into certain sorts of contracts counts as getting married (Searle, 1995, p. 83).

Consider a particular context $c_1$ in which a person a gets married.
Count-as Conditionals

Here we can assume that \( C_1 \) is:

the context in which \( a \)'s entering into a certain sort of contract counts as getting married.

But if so, it can also be

the context in which \( a \)'s uttering a certain sort of English sentence counts as entering into a certain sort of contract

and similarly.

the context in which \( a \)'s making certain noises counts as uttering an English sentence.

Context Types

Compare that with the following:

Performing such and such speech acts (the \( X \) term) in front of a presiding official (the \( C \) term) now counts as getting married (the \( Y \) term). Saying those very same words in a different context, while making love, for example, will not constitute getting married (Searle 1995, p. 82).

Here the \( C \) term seems to refer to a repeatable condition “in front of a presiding official”.

Two hierarchical structures

Now let us compare the following two hierarchical structures.

\[ C_1 \subseteq X_1 \subseteq X_2 \subseteq X_3 \]

\[ X_2 \text{ counts-as } Y_2 \text{ in } C_2 \]

\[ X_1 \text{ counts-as } Y_1 \text{ in } C_1 \]

Suppose \( X_1 = \{ x : x \text{ is of type } C_1 \} \), etc. Then we have

\( c_1 \) is of type \( C_1 \), of type \( C_2 \) and of type \( C_3 \).

But we can also say:

\( C_1, C_2 \) and \( C_3 \) are distinct from each other.

Conditions on contexts

Channel theory enables us to talk not only about particular contexts such as \( C_1 \) but also about types of contexts such as \( C_1, C_2 \) and \( C_3 \).

If we are to be able to say under what conditions \( X \) counts as \( Y \) we need to be able to say, at least partly, what these types are.

This is one of the things we need to do in order to develop a logical analysis of social institutions in general and speech acts in particular.

Context Tokens

Let \( \Sigma_1 \) be the set of all the contexts in which making of certain noises counts as uttering a particular English sentence \( S \).

Let \( \Sigma_2 \) be the set of all the contexts in which uttering of that particular English sentence \( S \) counts as entering into a certain sort of contract.

Let \( \Sigma_3 \) be the set of all the contexts in which entering into that sort of contract counts as getting married.

Now we can say:

\[ c_1 \in \Sigma_1 \subseteq \Sigma_2 \subseteq \Sigma_3 \]

A hierarchical structure

“We can impose status-functions on entities that have already had status-functions imposed on them. In such cases the \( X \) term at a higher level can be a \( Y \) term from an earlier level” (Searle 1995, p. 80).

\[ X_3 \text{ counts-as } Y_3 \text{ in } C_3 \]

\[ X_2 \text{ counts-as } Y_2 \text{ in } C_2 \]

\[ X_1 \text{ counts-as } Y_1 \text{ in } C_1 \]

\( c_1 \) is of type \( C_1 \), of type \( C_2 \) and of type \( C_3 \).

\( C_1, C_2 \) and \( C_3 \) are distinct from each other.

Channel theory enables us to talk not only about particular contexts such as \( c_1 \) but also about types of contexts such as \( C_1, C_2 \) and \( C_3 \).

What channel theory enables us to do

Channel theory enables us to talk not only about particular contexts such as \( c_1 \) but also about types of contexts such as \( C_1, C_2 \) and \( C_3 \).

A failed illocutionary act

A private: Clean this room.

A sergeant: You don't have the authority to give me a command.

Normally, privates would not say things like this to a sergeant. By contrast, the following looks normal.

A sergeant: Clean this room.

A private: Yes, sir.
Judith’s flashlight (Barwise and Seligman, 1997, p. 23)

In doing things in everyday life, we rely on various regularities that hold normally.

For example, by turning the switch of her flashlight on, Judith lights its bulb.

1. The switch being on entails that the bulb is lit.

What will happen, however, if the battery is dead?

Weakening ? (Barwise & Seligman, p. 23)

By applying the inference rule called weakening, we could derive the following:

2. The switch being on and the battery being dead entails that the bulb is lit.

Since this conclusion is unacceptable, we might wish to revise (1) and say:

3. The switch being on and the battery being live entails that the bulb is lit.

What will happen, however, if the bulb is gone?

Classification (Barwise & Seligman, p. 69)

Definition. A classification $A = \langle \text{tok}(A), \text{typ}(A), \models_A \rangle$ consists of
1. a set, $\text{tok}(A)$ of objects to be classified, called the tokens of $A$,
2. a set, $\text{typ}(A)$, of objects used to classify the tokens, called the types of $A$, and
3. a binary relation, $\models_A$, between tokens of $A$ and types of $A$.

If $a \models_A \alpha$, then $a$ is said to be of type $\alpha$ in $A$.

A classification is depicted by means of a diagram as follows:

$$
\begin{align*}
\text{typ}(A) & \models_A \beta \\
\text{tok}(A) & \models_A \alpha
\end{align*}
$$

Sequents, constraints, the complete theory (Barwise & Seligman, p. 29)

By a sequent we just mean a pair $(\Gamma, \Delta)$ of sets of types.

Definition. Let $A$ be a classification and let $(\Gamma, \Delta)$ be a sequent of $A$.

- A token $a$ of $A$ satisfies $(\Gamma, \Delta)$ provided that if $a$ is of type $\alpha$ for every $\alpha \in \Gamma$ then $a$ is of type $\beta$ for some $\beta \in \Delta$.
- We say that $\Gamma$ entails $\Delta$ in $A$, written $\Gamma \vdash_A \Delta$, if every token $a$ of $A$ satisfies $(\Gamma, \Delta)$.
- If $\Gamma \vdash_A \Delta$ then the pair $(\Gamma, \Delta)$ is called a constraint supported by the classification $A$.

Information Channels (Barwise & Seligman, pp. 34–35)

We say that $f = (f', f'')$ is a contravariant pair from $A$ to $B$, and write $f : A \rightarrow B$, if $f' : \text{typ}(A) \rightarrow \text{typ}(B)$ and $f'' : \text{tok}(A) \rightarrow \text{tok}(B)$.

We think of an infomorphism $f = (f', f'')$ as an infomorphism from $A$ to $B$ if it is a contravariant pair form $A$ to $B$.

Definition. An information channel consists of an indexed family $C = \{ t : A \rightarrow C \}$ of infomorphisms with a common codomain $C$ called the core of the channel.
An example.

\[ \{ f^{\text{Switch}}_{\text{ON}} \} \vdash \text{Flaslight} \{ f^{\text{Switch}}_{\text{LIT}} \} \]

Bulb

\( f^{\text{Switch}}_{\text{LIT}} \)

Switch

\( f^{\text{Switch}}_{\text{LIT}} \Rightarrow \text{Switch} \text{ON} \)

Flashlight

A refinement (Barwise & Seligman, pp. 43–44)

Even if \( \{ f^{\text{Switch}}_{\text{ON}} \} \vdash \{ f^{\text{Switch}}_{\text{LIT}} \} \) holds, \( \{ f^{\text{Switch}}_{\text{ON}} \} \not\vdash \{ f^{\text{Switch}}_{\text{LIT}} \} \) might not hold.

Local logic (Barwise & Seligman, p. 40)

Definition. A local logic \( \mathcal{L} = (A, \vdash, N) \) consists of:
- A classification \( A \),
- A set \( \mathcal{L} \) of sequents (satisfying certain structural rules) involving the types of \( A \), called the constraints of \( \mathcal{L} \), and
- A subset \( N \) of the set of all the tokens of \( A \), called the normal tokens of \( \mathcal{L} \), which satisfy all the constraints of \( \mathcal{L} \).

\( \mathcal{L} \) is sound if every token is normal; it is complete if every sequent that holds of all normal tokens is in the consequence relation \( \vdash \).

The Outline of a Dynamic Theory of Action (Barwise & Seligman, pp. 50-65)

Acts of using flashlights in channel theory

Flashlight using actions are modeled as connections that connects initial states and final states of such actions by constructing an information channel \( A_d = (f_{\text{act}} : C_{\text{act}} = C_{\text{act}} : f_{\text{in}} : C_{\text{in}} = C_{\text{act}}) \) such that \( f_{\text{act}} \) classifies flashlight using action tokens, and \( f_{\text{in}} \) and \( f_{\text{in}} \) classify their initial states and final states respectively. Thus two copies of the earlier classification Flashlight can be used as \( f_{\text{act}} \) and \( f_{\text{in}} \).

Then, the local logic on \( f_{\text{act}} \) can be defined.

Non-normal action tokens in the flashlight example

Even if \( \{ f^{\text{Switch}}_{\text{OFF}} \} \vdash \{ f^{\text{Switch}}_{\text{LIT}} \} \) holds, \( \{ f^{\text{Switch}}_{\text{OFF}} \} \not\vdash \{ f^{\text{Switch}}_{\text{LIT}} \} \) might not hold.
The problem

Actions in channel theory (Barwise & Seligman 1997)
Logical dynamics of speech acts
Acts of commanding in channel theory

Logical dynamics of speech acts

Acts of commanding in channel theory

The Language of MDL+ III and DMDL+ III

\[ O_{i,j,k}^{\varphi} \]

- It is obligatory for \( i \) with respect to \( j \) by the name of \( k \) to see to it that \( \varphi \).
- The agent who owes the obligation (obligor)
- The agent to whom the obligation is owed (obligee)

\[ \text{[command}_{i,j}^{\varphi}] \varphi \]
Whenever an agent \( i \) commands an agent \( j \) to see to it that \( \varphi \).

\[ \text{[promise}_{i,j,k}^{\varphi} ] \varphi \]
Whenever an agent \( i \) promises an agent \( j \) that \( k \) will see to it that \( \varphi \).

More formally

Take a countably infinite set \( \text{Prop} \) of proposition letters and a finite set \( I \) of agents, with \( p \) ranging over \( \text{Prop} \) and \( i, j, k \) over \( I \). The languages of MDL+ III and DMDL+ III are given respectively by:

\[ \varphi ::= \top \mid p \mid \neg \varphi \mid (\varphi \land \psi) \mid [\varphi]_I \mid O_{i,j,k}^{\varphi} \]

An \( L_{MDL+ III} \)-model

By an \( L_{MDL+ III} \)-model, we mean a quadruple \( \mathcal{M} = \langle W^\mathcal{M}, A^\mathcal{M}, \{D_{i,j,k}^{\mathcal{M}}\} \mid i,j,k \in I \rangle \) where:

- \( W^\mathcal{M} \) is a non-empty set (heuristically, of ‘possible worlds’),
- \( A^\mathcal{M} \subseteq W^\mathcal{M} \times W^\mathcal{M} \)
- \( \{D_{i,j,k}^{\mathcal{M}}\} \mid i,j,k \in I \subseteq A^\mathcal{M} \)
- \( V^\mathcal{M} \) is a function that assigns a subset \( V^\mathcal{M}(p) \) of \( W^\mathcal{M} \) to each proposition letter \( p \in \text{Prop} \).
Your boss’s act of commanding

A command and a promise can lead to a dilemma

Acts of commanding in channel theory

Deontic state classification


The CUGO Principle

If $\varphi$ is a formula of MDL III and is free of modal operators of the form $O_{ij}$, the following formula is valid:

$$[\text{Com}(ij)\varphi]O_{ij}\varphi$$

The PUGO Principle

If $\varphi$ is a formula of MDL III and is free of modal operators of the form $O_{ij}$, the following formula is valid:

$$[\text{Prom}(ij)\varphi]O_{ij}\varphi$$

How to do that

- For the sake of simplicity, ignore acts of promising and alethic modality. Thus, we will work with a fragment of $\mathcal{L}_{\text{DMDL-III}}$ which lacks the promise modalities and the alethic modality. Call it the command fragment of $\mathcal{L}_{\text{DMDL-III}}$.
- We will work with the whole class of $\mathcal{L}_{\text{DMDL-III}}$-models but with a set of $\mathcal{L}_{\text{DMDL-III}}$-models obtained by updating an arbitrary chosen one.

Deontic state classification 1/2

Deontic state classification 2/2
Count-as Conditionals

Then an information channel
\[ D^O = (\alpha, D^O_\delta = D^O_{\delta_0} \upharpoonright x, \bar{D}^O_{\delta_0} = D^O_{\delta_0} \upharpoonright x) \]
can be defined as follows:

- Action tokens of \( D^O_{\delta_0} \) can be modeled as connections of the form
  \( \langle a, b \rangle \) such that \( a \) is a token of \( D^O_{\delta_0} \) and \( b \) is a token of \( D^O_{\delta_0} \).
- For some sequence \( \sigma \) of action types and an action type \( \text{com}_{D^O_{\delta_0}} \),
  \( a = (\text{Act}_i(\sigma), b) \) and \( a = (\text{Act}_i(\sigma), b) \).
- We can define the set \( \text{inf}(D^O_{\delta_0}) \) of tokens of \( D^O_{\delta_0} \).

We will omit the superscript "*" henceforth.

Constraints of \( \mathcal{L}_{\text{Act}} \)

Constraints in \( \vdash_{\mathcal{D}_{\text{Act}}} \) can be derived from the valid formulas of \( \text{DMDL}^{\text{III}} \). For example:

\[
\vdash_{\text{DMDL}^{\text{III}}} \text{Com}_{\mathcal{L}(\nu)}(\varphi \land \psi) \rightarrow \text{Com}_{\mathcal{L}(\nu)}(\varphi).
\]

we have

\[
\begin{align*}
\{ f_{\mathcal{D}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi \land \psi)) \} & \vdash_{\mathcal{D}_{\text{Act}}} \{ f_{\mathcal{D}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi)) \} \\
\{ f_{\mathcal{D}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi \land \psi)) \} & \vdash_{\mathcal{D}_{\text{Act}}} \{ f_{\mathcal{D}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi)) \} \\
\end{align*}
\]

Local logic on \( \text{Act} \)

Now we can define a local logic \( \mathcal{L}_{\text{Act}} = (\mathcal{L}_{\text{Act}}, \vdash_{\mathcal{L}_{\text{Act}}}, \mathcal{N}_{\mathcal{L}_{\text{Act}}}) \) on \( \mathcal{L}_{\text{Act}} \).

More generally, for any \( \varphi \) such that \( \vdash_{\text{DMDL}^{\text{III}}} \varphi \), we have

\[
\emptyset \vdash_{\mathcal{L}_{\text{Act}}} \{ f_{\mathcal{L}_{\text{Act}}}(\varphi) \}.
\]

More interestingly, we have

\[
\{ f_{\mathcal{L}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi)), \text{Com}_{\mathcal{L}(\nu)}(\varphi) \} \vdash_{\mathcal{L}_{\text{Act}}} \{ f_{\mathcal{L}_{\text{Act}}}(\varphi) \}.
\]

How about CUGO Principle?

The CUGO Principle

If \( \varphi \) is a formula of \( \text{MDL}^{\text{III}} \) and is free of modal operators of the form \( \text{Com}_{\mathcal{L}(\nu)} \), the following formula is valid:

\[
\text{Com}_{\mathcal{L}(\nu)}(\varphi) = \text{Com}_{\mathcal{L}(\nu)}(\varphi).
\]

If we wish to have a sound local logic, we have to accept

\[
\emptyset \vdash_{\mathcal{L}_{\text{Act}}} \{ f_{\mathcal{L}_{\text{Act}}}(\text{Com}_{\mathcal{L}(\nu)}(\varphi)) \}.
\]

The problem of how we could characterize the class of formulas \( \varphi \) such that \( \text{Com}_{\mathcal{L}(\nu)}(\varphi) \) is valid is still open.
If we only include constraints derived from valid formulas of DMDL^{+} III, both $D_{DA}$ and $D_{DR}$ will be sound.

How then is $D_{DA}$ different from $D_{AR}$?

In the flashlight example, the background condition that the battery is live is the condition that must be satisfied in order for an act of pushing the switch into the on position to be a way of turning the flashlight on.

Similarly, the condition that the agent has the suitable authority to issue such and such a command is the condition that must be satisfied in order for her act of saying so and so counts as an act of commanding.

The language of DMDL^{+} III has to be substantially extended in other respect as well, if we are to capture the kind of regularities included in normal situations. We have to be able to talk about acts of saying so and so (locutionary acts).

An act of type $\text{Say}_{ij}(CTR)$ counts as an act of type $\text{Com}_{ij}(p)$ in a context of type $\text{Auth}_{ij}(y)^{+}$ for some $h$.

The problem
Actions in channel theory (Barwise & Seligman 1997) Logical dynamics of speech acts
Acts of commanding in channel theory

An enriched classification

This is a channel theoretic analogue of count-as conditional.

Context types

Authority

The problem
Actions in channel theory (Barwise & Seligman 1997) Logical dynamics of speech acts
Acts of commanding in channel theory

The language of DMDL^{+} III has to be substantially extended if we are to talk about such background conditions. This is partly done in Yamada (2015).

Let $\text{org}_{i}$ indexed by a finite set of organizations $H$ be a function that assigns a possibly empty subset of the set of command types $\text{com}_{ij}(i,j)$ to each pair of agents $(i,j) \in I \times I$ for each world $w$. Thus, $\text{org}_{i}(j, w)$ is a set of command types an organization $h$ authorizes $i$ to give $j$ in $w$. Then we define

$\mathcal{M}, w \models \text{DMDL}^{+} \text{III.}\ 	ext{Auth}_{ij}(y)^{+} \iff \text{com}_{ij}(y)^{+} \in \text{org}_{i}(j, w)$.

Let $\text{org}_{i}$ indexed by a finite set of organizations $H$ be a function that assigns a possibly empty subset of the set of command types $\text{com}_{ij}(i,j)$ to each pair of agents $(i,j) \in I \times I$ for each world $w$. Thus, $\text{org}_{i}(j, w)$ is a set of command types an organization $h$ authorizes $i$ to give $j$ in $w$. Then we define

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$\mathcal{M}, w \models \text{DMDL}^{+} \text{III.}\ 	ext{Auth}_{ij}(y)^{+} \iff \text{com}_{ij}(y)^{+} \in \text{org}_{i}(j, w)$.

A failed illocutionary act again

A private: Clean this room.
A sergeant: You don’t have the authority to give me a command.

Normally, privates would not say things like this to a sergeant. By contrast, the following looks normal.

A sergeant: Clean this room.
A privates: Yes, sir.
Count-as conditionals and CUGO Principle

A sergeant a said “Clean this room”, addressing a private b and pointing to the room r.

\[
\Downarrow \text{[count-as]} \quad \{ \text{Say}_{a,b}(CTR) \vdash \text{Com}_{a,b}(p) \}
\]

a commanded b see to it that r is clean.

\[
\Downarrow \text{[CUGO]} \quad \{ \text{Com}_{a,b}(p) \vdash \text{Sus}_{a,b}(f_C(b,a,b)\rho) \}
\]

b is obligated to see to it that r is clean with respect to a by the name of a.

## Acknowledgement

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- Earlier versions of this talk was presented in AWPL 2018, held in Tsinghua University, Beijing in October, 2018, and in CLMPST 2019, held in Czech Technical University in Prague in August, 2019. I am grateful to the participants of these meetings for their helpful comments and discussions.
The aim of this paper is to distinguish between two types of constitutive rules that I will call “definitional” vs. “essential” constitutive rules, to make sense of the distinction, and further to try to show its implications for an account of social reality. Very roughly, constitutive rules are rules that constitute social reality and play a role in the determination of what social practices are. What I want to argue is that we have to distinguish two ways in which constitutive rules can make sense of social practices: some constitutive rules are there to give meaning to activities within those practices and to define those activities; others operate on a deeper level and underlie, in an essential way, those social practices themselves. I will call the first type of constitutive rules “definitional” and the second type “essential”, and I try to give a possible explanation of this distinction.

How I will proceed is the following: I start by introducing the distinction between the two types of constitutive rules through speech act theories and games via which I came to this distinction, by reference to two conflated ways of characterizing constitutive rules in speech act theories: the Searlian characterization (Searle J., 1969) and the Williamsonian one (Williamson T., 2000). According to both authors, speech acts, as well as games, are governed by constitutive rules, but whereas a constitutive rule in the Williamsonian sense is a rule that is essential to an act, such that it necessarily governs every performance of the act (Williamson T., 2000: 239), a constitutive rule in the Searlian sense is a rule that is tautological in character, such that it can be seen, now as a rule, now as an analytic truth based on the meaning of the activity term in question. (Searle J., 1969: 34). Therefrom, I introduce the distinction between “definitional” vs. “essential” constitutive rules. Definitional rules correspond to the Searlian sense of ‘constitutive’, and essential rules correspond to the Williamsonian sense. The difference lies in the fact that if a constitutive rule is definitional, we do not engage in the act of which the rule is definitional if we do not act in accordance with the rule, but if a constitutive rule is essential, obeying it is not a necessary condition for performing the act which is constituted by that rule. I argue that whereas competitive games are governed by definitional rules, speech acts are governed by essential rules.

I then suggest a possible way to trace this distinction in an institutional framework, by introducing a parallel distinction between intra-institutional concepts and trans-institutional concepts (Miller D., 1981). The former are concepts that are entirely defined or that exist only in virtue of a rule within a certain institution, and the latter are somewhat-floating concepts used in different institutions. (Miller D., 1981) I then suggest that there is a parallel between intra- vs. trans-institutional concepts and the definitional vs. essential constitutive rules, which can help us find an explanation, in the institutional framework, of the distinction between the two types of constitutive rules: essential rules are those in the formulation of which a trans-institutional concept is used and which give the point and significance of the practice of which they are constitutive; definitional rules are those constitutive rules which do not involve any trans-institutional concept, and involve at least one intra-institutional concept.
In a second part I try to situate these distinctions in two different frameworks of accounting for social practices: First, an essentialist framework though the work of A. Reinach, and then a conventionalist framework through the work of A. Marmor. According to Reinach (Reinach A., 1983), social and legal entities form a specific ontological category of temporal objects which have their own independent being and are governed by what he calls “essential laws”. I aim to situate the distinction between the two types of constitutive rules by reference to two characteristics of Reinachien essential laws: their immediate intelligibility and their non-forgettability. I will then compare this “essentialist” account with Marmor’s account of social conventions (Marmor A., 2009), according to which social practices are results of [constitutive] conventions, and he distinguishes between two types of conventions in these domains: surface conventions and deep conventions. I again aim to situate the distinction between the two types of constitutive rules with respect to Marmorian surface and deep conventions. I conclude that in whichever way we want to defend the emergence of social and legal institutions, we had better be disposed with the distinction between definitional and essential rules.

Main References

Extended abstract of:

*A classification of discursive references to settle what is modified by talking and why it is so.*

Dra. Maribel Narváez Mora (Universitat de Girona)

I have claimed (Narváez, 2018) that in order to build a dynamic model of sense/knowledge transformation, we need to reshape our notion of discursive reference. In this presentation, I will introduce a classification of discursive references or modes of aboutness. My aim is to offer a scalable tool to settle what can be modify by talking in communicative interactions, and why it is so.

Over forty years ago, the Canadian philosopher Ian Hacking explained how language came to matter to philosophy while the notion of knowledge was being transformed (Hacking, 1975, p. 186). Platonic ideas as perfect objects, or thoughts as a type of Cartesian substance, left room for statements and propositions. When statements were considered a suitable way of representing and expressing knowledge, their meanings –propositions- became the contents of justified true beliefs. In the analytical tradition, to speak about this transformation, the phrase linguistic turn, first popularised by Richard Rorty, is of common use.1

However, the pioneering discussions that strengthened the relationship between language and knowledge took place in Vienna during the late nineteenth and early twentieth centuries, and they had to do directly with meaning (Bedeutung) (Mulligan, 2012, p. 109 et seq.). The debate between those who, like Brentano, maintained the meaning of a word was the object named and those like Husserl, who rejected such a position, allowed Mauthner to state, ‘Philosophy is theory of knowledge and the theory of knowledge is critic of language’ (Mauthner, 1901). To be sure, this was the thematization that involved Frege, Wittgenstein2 and Russell in their respective works, establishing the relationships between word-meaning-object.3

As intuitive as these relationships may seem,4 the truth is that the issue remains problematic despite the long and sophisticated discussion about them. The readjustments

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1 What Richard Rorty called linguistic philosophy was ‘the view that philosophical problems are problems that may be solved (or dissolved) either by reforming language or by understanding more about the language we presently use’ (Rorty, 1967, p. 3). Although it was Rorty who made popular the *linguistic turn* label, nowadays, the discussion about its history and scope is well alive. See Koopman (2007), Wagner (2010) and Hacker (2013). The latter paper includes an illustrative graphic about the linguistic turn history located as a gif image at the following: http://www.oxfordhandbooks.com/view/10.1093/oxfordhb/9780199238842.001.0001/oxfordhb-9780199238842-oxfordhb_9780199238842_graphic_018-full.gif.

2 When Wittgenstein moves away from the criticism of language from Mauthner’s view, saying it explicitly in the Tractatus (4.0031), and accepts the criticism of language in the sense of Russell, he takes sides with a type of purification to distinguish the linguistic appearances of the logical form of the proposition. A presentation of the similarities and differences between Mauthner and Wittgenstein can be found in Santibáñez (2007).

3 The most common way of introducing the relationships (of correction, adequacy or truth) between symbols (words), thoughts and references (meanings) and referents (objects) is the semiotic triangle of Ogden and Richards (1923).

4 Probably, the relationship between word-meaning-object is familiar to us from the work of Ferdinand de Saussure, *Cours de Linguistique Générale* (1916). However, the famous significant-signifier pair is not at all equivalent to the pair word-meaning. In Saussure’s linguistic treatment, words are an inseparable union of significant-signifier. The significant was conceived as an acoustic image of the word and the signifier as the concept that the sign expresses. This is important because the semantic relations that are contemplated are made arbitrary when associated with an acoustic image or signify a certain concept or meaning but not when it comes to the association of a meaning to a word. Put in a different way, to be a word is not to be an acoustic or graphic image.
between conceptions of language and epistemological positions seem to have no end. Under certain conditions, we can say that the meaning of words determines what we are talking about, but what we talk about seems to determine the meaning of the words.

In philosophical treatments of language to refer to something is to relate linguistic representations with what they represent. This relationship is influenced by the model of the proper name in which a word refers to an individual entity, and under this influence, the idea that words are names that represent and bring into the discourse more or less complex entities (objects, properties, situations, states of affairs, processes, events, classes and so forth) keeps exercising a strong influence. The sentences in which (and by which) something about those entities is predicated are then considered their descriptions. Of course, many other functions of language are admitted in addition to the descriptive one, but independently of the speech act enacted, the truth is that the very same way of referring is presupposed. Regardless of the purposes for and the ways of using language, the semantic value (Bedeutung) of the terms and phrases that refer is the entity brought into the discourse, that is, its referent. In this model, naming, referring and representing are closely related semantic relationships. The (logical) name is used to represent a referent, so a part of the problem in the philosophy of language has to do with how to disentangle this relationship.

In this presentation borrowing to some degree the structure of Horwick’s argumentation (Horwick, 1990, 1998) about a minimalist conception of truth – as within the realm of deflationists theories – “to refer to” becomes a transparent element but not a redundant one. The role of the binary predicate ‘… refers to…’ is expressive and inferential. Asserting that a statement – sentence, or speech – refers to something is to take it as a statement – sentence or speech. To carry out this project, as I said, a classification for discursive references will be advanced. This classification covers discursive references or modes of aboutness, not types of referents seen as a function of whatsoever modes of existence. As such, it has to be a useful and scalable tool to talk about what we talk about in any communicative interaction.

The main criterion used here to discriminate discursive references is given by the predicate ‘…is expressible’ and its negation ‘…is not expressible’. Being expressible is a constitutive feature of some discursive references. The relationship stated in an assertion between a discursive reference and the predicate ‘…is expressible’ is definitional and not attributive, and here, internal is preferred. By opposition, the same happens with non-

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5 A full development of those debates can be seen in Reimer and Michaeldenson (2016).
6 (Donnellan, 1966, 1970)
7 From a lexical etiology perspective, to have a name is not a condition that can be referred to. The detection of patterns, their recurrence and the interactions with them can give a name in an efficient manner to manage information.
8 The identity of the semantic content requires referents being the semantic value of referring expressions. However, note that the phrase ‘semantic value’ coined by Miller (1998, 2007, pp. 7, 9, 340) to clarify the Fregean notion Bedeutung is used to justify that the same semantic content can be asserted, ordered and asked. So far in the text, we have seen that Bedeutung is understood in some cases as meaning, in others as reference and eventually as semantic value hesitating from being a concept to being an object.
9 A paradigmatic case is the treatment of fictions in the philosophy of language; see Garcia-Carpintero and Martí (2014). The title of this collection of works is empty representations trying to highlight a problem. Because proper names in the logical sense serve ‘[…] to pick out an object, to bring it into our talk or thought, to call our attention to it for further representational purposes such as saying something about it, asking about it or giving directions concerning it’ how can it be explained as representational and referential functions in front of non-existent objects?
10 See also McDonald 2011 for a minimalist approach of properties and facts.
11 There is no impediment assuming that our criterion is the presence or absence of a property: the property of being expressible. In any case, the relationship between predicates and properties functions in parallel to the relationship between references and referent.
expressible discursive references. In order to keep this explication in the domain of discourse and contrary to what happened within the framework of the linguistic turn, we will not move from language to epistemological and ontological spheres. In any case, the main implications will turn to be epistemological and ontological.

When referring is already considered a (semantic) relation establishing a connection between language and reality (Robertson, 2012, p. 189; Martí, 2012, p. 106), it becomes impossible to avoid the usual recalcitrant dualities classifying references – referents actually – using some fundamental ontological or epistemological criteria. As soon as referents – extra-linguistic realities to which we refer to – come into play, we stop classifying discursive references, and start to classify reality according to several modes of existence, hence, we take an ontological perspective. Then, as far as those referents cannot ontologically be material, physical, empirical, factual or whatever the sustained ontological naturalized commitment advocates for, they are postulated, by exclusion, to be immaterial, spiritual, abstract or ideal provided that those types of reality are accepted; otherwise, they are eliminated as inexistent or reduced to existent ones. I submit that the other way around is possible: if what we refer to can be said (narrated, explained, etc.), then it is expressible; otherwise, we are talking about language independent realities – of a variety of types and scales to be discussed. Because the predicate used here to classify discursive references points to the verbal ability of expressing by language, the assumptions made about expressibility are shared among the speakers of the linguistic community. Consequently, our semantic interpretation of reference – namely our accepted semantic substitute for it – will be ‘what we talk (it talks) about’ and what we talk about is expressible or non-expressible.

When what we talk (it talks) about is fully expressible: a norm, an idea, a concept, a story, our way of representing it is linguistic and our way of experiencing it is cognitive and emotional, which is where its motivational power emerges. This power can be understood as exhibiting top-down causation (Ellis, 2016). When what we talk (it talks) about is non-expressible: a cell, a hurricane, a bike, our way of representing it can be linguistic, using criteria which encapsulates some natural or conventional description of that extension (entity, pattern, process). Our way of experiencing it, besides being, to some extent, cognitive and emotional, is physical too when actualised at certain scale. These references have an impact, affecting our bodies physically in a biological manner or aided by technology.

Only to the extent to which a discursive reference is non-expressible does it make sense to treat it as extra-linguistic and to assume ontological and epistemic naturalised commitments. However, to the extent to which a discursive reference is expressible, its

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12 What an ontological perspective is before the linguistic turn has nothing to do with the Quinean demand of ontological commitment.
13 Following Terrence W. Deacon’s (2013) proposal in *Incomplete Nature*, expressible references are ententional, and the central feature of ententional phenomena is to produce a limitation or constraint, being absences ‘are intrinsically incomplete in the sense of being in relationship to, constituted by, or organized to achieve something non-intrinsic’ (p. 549).
14 The concept of semantic interpretation here follows the Wittgensteinian one (1953 PI §201; 1958: 63: 1967: PG §229, 41e; 1974 Z.§9) and is something that is given in signs, substitutes one expression for another or adds a new symbol to an old one.
15 When Wittgenstein points out the problem of ostensive definitions is recognizing that *having a name* is already a move into a language game.
16 To deal with problems of existence and actualisation, the concept of scale, introduced later on in the presentation, will play a key role.
17 According to Price’s terminology, it can be said that objective naturalism has a place in a subjective naturalistic project.
naturalisation will have a more limited scope.\textsuperscript{18} There is an internal relationship between sharing a language and converging on whether what is talked about is or is not expressible. Any question as to why a reference is expressible points to a constitutive feature of it: norms, principles, ideas and concepts are expressible; otherwise, they would not be norms, principles, ideas or concepts. Although we represent and name norms, principles, ideas and concepts, only non-expressible references can be described in spite of the traditional considerations of philosophy of language.

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\textsuperscript{18} It is worth mentioning Terrence W. Deacon’s work again because the naturalised understanding of ententional phenomena is something difficult to achieve because his research shows us an emergentist explanation of mind and meaning. Here, we do not deal with the nature of a concept, a norm or a story precisely because the only feature of expressible references we are interested in is their expressible character.
The Dynamics of Group Knowledge and Belief

SOCREAL 2019, Hokkaido University, 16 November 2019

Thomas Ågotnes
University of Bergen, Norway
Southwest University, China

Group knowledge

The students know that there is no lecture in Bergen this week

During the lecture last week I told the students that there is no lecture in Bergen this week

The police know my speed was too high

I know that if you got at least 70 points you pass the exam, you know that you got 85 points. Together we know that you pass the exam.
Group knowledge

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Group knowledge, dynamics and ability

(this talk) C all of you:

• Group knowledge is all about dynamics
Group knowledge, dynamics and ability

(this talk) C_all of you:

- Group knowledge is all about dynamics
- Group ability is fundamental in reasoning about group knowledge
- Until recently common knowledge has received most attention in the dynamic epistemic logic literature
- I will focus a little more on distributed knowledge

Plan

- Background: multi-agent epistemic/doxastic logic
- Group knowledge
- Group belief
  - Generalised
  - Adding dynamics
- Group ability and group knowledge
  - General ability
  - Ability through informative updates
  - Maximal ability

We assume given

a finite set \( \mathcal{N} = \{1, \ldots, n\} \) of agents

a countably infinite set of primitive propositions

let \( \mathcal{G} \mathcal{R} = \wp(\mathcal{N}) \setminus \emptyset \) (the set of non-empty groups)
Models

A model is a tuple $M = (W, \sim_1, \ldots, \sim_n, V)$:

- $W$ is a set of states
- $\sim_i$ is an accessibility relation
  - Assumed to be an equivalence relation (S5) when we model knowledge
  - Assumed have weaker properties when we talk about belief, e.g., transitive, euclidian and serial (KD45)
- $V$ is a valuation function, assigning primitive propositions to each state

Epistemic/doxastic logic

Language $\mathcal{EL}$:

$$\phi ::= p \mid K_i \phi \mid \neg \phi \mid \phi_1 \land \phi_2$$

Interpretation:

- $M, s \models p$ iff $p \in V(s)$
- $M, s \models K_i \phi$ iff for all $t$ s.t. $s \sim_i t$, $(M, t) \models \phi$
- $M, s \models \neg \phi$ iff $M, s \not\models \phi$
- $M, s \models \phi \land \psi$ iff $M, s \models \phi$ and $M, s \models \psi$

For belief we often write $B_i$ instead of $K_i$

Group Knowledge

General Knowledge ("everybody-knows")

$$\sim^G_i = \bigcup_{G \in \mathcal{G}} \sim_i$$

$M, s \models E_G \phi$ iff for all $t$ s.t. $s \sim^G_i t$, $(M, t) \models \phi$

Already expressible: $E_G \phi \equiv \bigwedge_{i \in G} K_i \phi$

Common Knowledge

$$\sim^C = (\bigcup_{G \in \mathcal{G}} \sim_i)^*$$

$M, s \models C_G \phi$ iff for all $t$ s.t. $s \sim^C t$, $(M, t) \models \phi$
Distributed Knowledge

\[ \neg D_\phi \models D_G, \forall i \in \phi \]

\[ M, s \models D_G \iff \text{for all } t \text{ s.t. } s \models D_G t, (M, t) \models \phi \]

\[ \neg D_{(1,2)} \models D_G, \forall i \in \phi \]

\[ M, s \models D_G \iff \text{for all } t \text{ s.t. } s \models D_G t, (M, t) \models \phi \]

\[ \text{“... the knowledge of } \phi \text{ is distributed among its members, so that by pooling their knowledge together the members of the group can deduce } \phi \text{”} \]

\[ \text{Fagin et al., 1995} \]

\[ \text{“... it should be possible for the members of the group to establish } \phi \text{ through communication”} \]

\[ \text{van der Hoek et al., 1999} \]

\[ \text{“... the knowledge that would result of the agents could somehow “combine” their knowledge”} \]

\[ \text{Roelofsen, 2006} \]
Distributed Knowledge

“...the knowledge of $\phi$ is distributed among its members, so that by pooling their knowledge together the members of the group can deduce $\phi$”

\[ D_{1,2}(p \land \neg K_1 p) \]

What Distributed Knowledge Actually Is

- Common interpretations of distributed knowledge:
  - Knowledge the group could obtain if they had unlimited means of communication
  - “A group has distributed knowledge of a fact $\phi$ if the knowledge of $\phi$ is distributed among its members, so that by pooling their knowledge together the members of the group can deduce $\phi$...”

A group has distributed knowledge of a fact $\phi$ if after “pooling their knowledge together” the members of the group know that $\phi$ was true before they did that.
Group Knowledge

\[ \models C_G \phi \rightarrow E_G \phi \]
\[ \models E_G \phi \rightarrow K_i \phi \quad (i \in G) \]
\[ \models K_i \phi \rightarrow S_G \phi \]
\[ \models S_G \phi \rightarrow D_G \phi \]

Group Belief

\[ \models C_G \phi \rightarrow E_G \phi \]
\[ \models E_G \phi \rightarrow B_i \phi \quad (i \in G) \]
\[ \models B_i \phi \rightarrow S_G \phi \]
\[ \models S_G \phi \rightarrow D_G \phi \]
Group Belief

Generalised Distributed Belief

\[ \sim_D = \bigcap_{i \in G} \sim_i \]

- The group considers a state
- possible iff all the agents in the group considers it possible
- impossible iff at least one member of the group considers it impossible
- For S5 agents this makes sense
- If an S5 agent considers a state impossible, then it is impossible
- .. and this is common knowledge
Distributed belief for non-S5 agents

- For non-S5 agents, in particular agents without T/reflexivity (e.g., KD45):
  - If one agent considers a state impossible, that agent might in fact be wrong
  - Ruling out a state based on the evidence of a single agent is then a very credulous group attitude
  - Curious asymmetry between the evidence need for possibility vs. impossibility
    - impossibility: every agent is a veto voter, possibility: unanimity

Generalised Distributed Belief

- The group considers a state
  - possible iff at least k agents in the group considers it possible
  - impossible iff not at least k agents in the group considers it impossible

\[ M, s \models D_G^{\geq k} \phi \iff \forall (s, t) \in \sim_i^k M, t \models \phi \]

\[ \sim_G^{\geq k} = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_{i \in H} \sim_i \]

Generalised distributed belief: the extremes

- k = |G|: the group considers a state
  - impossible iff at least one member of the group considers it impossible
  - possible iff all the agents in the group considers it possible

\[ \sim_G^{\geq |G|} = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_{i \in H} \sim_i \]

Generalised distributed belief: the extremes

- k = |G|: the group considers a state
  - impossible iff at least one member of the group considers it impossible
  - possible iff all the agents in the group considers it possible

\[ \sim_G^{\geq |G|} = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_{i \in H} \sim_i \]
Generalised distributed belief: the extremes

\[ \sim^k_G = \bigcup_{H \subseteq G, |H| \geq k} \bigcap_i \sim_i \]

- \( k = |G| \): the group considers a state impossible if at least one member of the group considers it impossible
- \( k = 1 \): the group considers a state impossible if all agents in the group considers it impossible
- possible if at least one agent in the group considers it possible

Generalised distributed belief: conclusions

- Between distributed and general belief
- Intuitively two entirely different concepts
- Difference between them can be explained quantitatively rather than qualitatively
- Specific instances of the same concept, corresponding to which voting threshold is used
- There is a scale of intermediate concepts between them

Adding dynamics

Public Announcement Logic with Distributed Knowledge

\[ \mathcal{PAD}: \phi ::= p | K_i \phi | D_G \phi | [\phi]_i \phi | \sim_i \phi | \phi_1 \land \phi_2 \]

PC
- All instances of propositional tautologies
- K
- K \( \phi \rightarrow \psi \rightarrow K_a \phi \rightarrow K_a \psi \rightarrow T \)
- K \( \phi \rightarrow \psi \rightarrow D_{A} \phi \rightarrow D_{A} \psi \rightarrow T \)
- D \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)
- D \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)
- K \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)
- D \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)
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- K \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)
- D \( \psi \rightarrow \psi \rightarrow \psi \rightarrow \psi \rightarrow T \)

Sound and complete: by reduction to \( \mathcal{ELD} \).
Public Announcement Logic with Common Knowledge

$\mathcal{PAC}$: $\phi ::= p \mid K_i \phi | C_G \phi | [\psi] \phi | \neg \phi | \phi_1 \land \phi_2$

PC All instances of tautologies

$T_K$ $K_n \phi \rightarrow \phi$

$K_K$ $K_n(\phi \rightarrow \psi) \rightarrow K_n \phi \rightarrow K_n \psi$

$5K$ $\neg K_n \phi \rightarrow K_n \neg K_n \phi$

$R_{\Gamma p}$ $[p] \varphi \leftrightarrow ([p] \varphi \land \chi) \rightarrow K_n \varphi$

$R_{\Gamma K}$ $K_n \psi \leftrightarrow [\psi] \varphi \land \chi \rightarrow K_n \psi$

$R_{\Gamma p}$ $\chi \land \psi \rightarrow [\psi] \varphi \land \chi \rightarrow \chi \rightarrow [\psi] C_A \psi$

$N_{\Gamma}$ $\varphi \land \psi \rightarrow K_n \varphi$

$N_{\Gamma}$ $\varphi \land \psi \rightarrow [\psi] \varphi \land \chi \land \varphi \rightarrow E_{A \chi} \Rightarrow \chi \rightarrow [\psi] C_A \psi$

Sound and complete.

Public Announcement Logic with Common and Distributed Knowledge

$\mathcal{PACD}$: $\phi ::= p \mid K_i \phi | C_G \phi | D_G \phi | [\psi] \phi | \neg \phi | \phi_1 \land \phi_2$

PC All instances of tautologies

$T_K$ $K_n \phi \rightarrow \phi$

$K_K$ $K_n(\phi \rightarrow \psi) \rightarrow K_n \phi \rightarrow K_n \psi$

$D_K$ $C_A(\phi \rightarrow \psi) \rightarrow C_A \phi \rightarrow C_A \psi$

$5K$ $\neg K_n \phi \rightarrow K_n \neg K_n \phi$

$R_{\Gamma p}$ $[p] \varphi \leftrightarrow ([p] \varphi \land \chi) \rightarrow D_{A \chi} \psi$

$R_{\Gamma K}$ $K_n \psi \leftrightarrow ([p] \varphi \land \chi) \rightarrow K_n \psi$

$R_{\Gamma p}$ $\chi \land \psi \rightarrow [\psi] \varphi \land \chi \land \varphi \rightarrow E_{A \chi} \Rightarrow \chi \rightarrow [\psi] C_A \psi$

Sound and complete.

A note on completeness proofs for epistemic logic with distributed knowledge

Several claims about completeness for ELD can be found in the literature (Fagin et al. 1992, van der Hoek and Meyer 1992, Halpern and Moses 1992, Agathoklis et al. 1995, van der Hoek and Meyer 1997, Gerbrand 1999). Most of them either only allow distributed knowledge operators for the grand coalition; and/or do not provide detailed proofs.

Sound and complete.
Public Announcement Logic with Common and Distributed Knowledge: expressivity

![Diagram]

S5D ≡ PAD → S5CD
S5 ≡ PAC → PACD

Open problem: relax S5 assumptions

Some complexity results

<table>
<thead>
<tr>
<th>Logic</th>
<th>Result</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{EL} )</td>
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<td>Halpern and Moses 1992</td>
</tr>
<tr>
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<td>EXPTIME-complete</td>
<td>Fisher and Ladner 1977</td>
</tr>
<tr>
<td>( \mathcal{ELD} ) (D only for grand coal.)</td>
<td>PSPACE-complete</td>
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</tr>
<tr>
<td>( \mathcal{ELCD} ) (C, D only for grand coal.)</td>
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<td>Halpern and Moses 1992</td>
</tr>
<tr>
<td>( \mathcal{ELCD} ) (no restrictions)</td>
<td>EXPTIME-complete</td>
<td>Wang and Ågotnes 2013</td>
</tr>
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<td>( \mathcal{PA} )</td>
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<td>Lutz 2006</td>
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</tbody>
</table>

Coalitional Ability Logics

- Logics with coalition operators. Typical notation:
  \[ \langle C \rangle \phi \quad \langle \langle C \rangle \rangle \phi \quad [C] \phi \]
  - where \( C \) is a coalition (set of agents, possibly empty)
  - Intuitive meaning: \( C \) has the ability to make \( \phi \) true

Coalitional ability: examples

\[ \langle \{ Thomas, Meiyan \} \rangle \langle \text{students}\_\text{happy} \rangle \]
\[ \langle \{ alibaba, tencent \} \rangle \langle \neg \text{applepay}\_\text{successful} \rangle \]
Coalition Logic
\[ \phi ::= p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \{C\}\phi \quad C \subseteq N \]

Alternative: neighbourhood semantics.
\[ M, s \models \{C\} \iff \phi^M \in E_s(C) \]

Coalition Logic
\[ \phi ::= p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \{C\}\phi \quad C \subseteq N \]

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Alternative: neighbourhood semantics.
\[ M, s \models \{C\} \iff \phi^M \in E_s(C) \]

Epistemic Coalition Logic
\[ \phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \{G\}\phi \mid K_i \phi \mid C_G \phi \mid D_G \phi \quad G \in \mathcal{G} \]

\[ M, s \models \{Ann\} jail \]

Representation theorem
\[ M, s \models \{C\} \iff \phi^M \in E_s(C) \]
Epistemic Coalition Logic

\[ \phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \{G\} \phi \mid K_i \phi \mid C_{G'} \phi \mid D_{G'} \phi \quad G' \in \mathcal{G} \quad G \subseteq \mathcal{N} \]

\( K_i \phi \rightarrow \{\{i\}\} K_j \phi \): i can communicate her knowledge of \( \phi \) to j

\( C_{G'} \rightarrow \{G\} \psi \): common knowledge in \( G \) of \( \phi \) is sufficient for \( G \) to ensure that \( \psi \)

\( \{G\} \psi \rightarrow D_{G'} \phi \): distributed knowledge in \( G \) of \( \phi \) is necessary for \( G \) to ensure that \( \psi \)
Epistemic Coalition Logic

\[ \phi ::= p \mid \neg \phi \mid \phi \wedge \phi \mid \langle G \rangle \phi \mid K_i \phi \mid C_G \phi \mid D_{G'} \phi \]

**Open problem:** completeness of ECL with the distributed knowledge axiom

### Epistemic Coalition Logic: adding interaction axioms

<table>
<thead>
<tr>
<th>Property</th>
<th>Axiom</th>
<th>Completeness?</th>
</tr>
</thead>
<tbody>
<tr>
<td>s ∼_G t ⇒ E(s)(i) = E(t)(i)</td>
<td>( \langle i \rangle \phi \rightarrow K_i [\phi] \phi )</td>
<td>Yes</td>
</tr>
<tr>
<td>s ∼_G t ⇒ E(s)(G) = E(t)(G)</td>
<td>( \langle G \rangle \phi \rightarrow C_{G'}[G] \phi )</td>
<td>Yes</td>
</tr>
<tr>
<td>s ∼_G t ⇒ E(s)(G) = E(t)(G)</td>
<td>( \langle G \rangle \phi \rightarrow D_{G'}[G] \phi )</td>
<td>?</td>
</tr>
</tbody>
</table>

**Sound and complete**  
(all combinations of operators: \( CLC, CLD, CLL, CLLC \))

### Epistemic ATL: knowing vs. knowing how

\[ \phi ::= p \mid \neg \phi \mid \phi \wedge \phi \mid \langle G \rangle \phi \mid K_i \phi \mid C_G \phi \mid D_{G'} \phi \]

**Open problem:** complete axiomatisation of ATL with group knowledge

### ATL with group knowledge

\[ \phi ::= p \mid \neg \phi \mid \phi \wedge \phi \mid \langle A \rangle \phi \mid \{ A \} \phi \mid \{ A \} \phi U \phi \mid C_{A'} \phi \mid E_{A'} \phi \mid D_A \phi \]

- Plain ATL completely axiomatised
- A lot of work on epistemic extensions, but no completeness proof yet
- Completeness claim with common knowledge only

**Open problem:** complete axiomatisation of ATL with group knowledge

### Epistemic ATL: knowing that vs. knowing how (knowledge of ability de dicto vs. de re)

\[ C_G \langle G \rangle \gamma : in every G-reachable state G has a strategy that will ensure \gamma \]

??: \( G \) has a strategy that in every \( G \)-reachable state will ensure \( \gamma \)

### Some complexity results

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<td>( \mathcal{ELCD} + )</td>
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**Logic Result Reference**
- \( CLC \): EXP\( T \)\( E \) -complete
- \( CLD \): EXP\( T \)\( E \) -complete
- \( PLL \): PSPACE-complete
- \( PLLCD \): EXP\( T \)\( E \) -complete
- \( PLLCD+ \): EXP\( T \)\( E \) -complete
- \( PLLCD++ \): EXP\( T \)\( E \) -complete
- \( PLLCD+++ \): EXP\( T \)\( E \) -complete
- \( PLLCD++++ \): EXP\( T \)\( E \) -complete

---

\[ K_1 \phi \rightarrow \phi \]
\[ DT_1 \phi \rightarrow \phi \]
\[ D3 \phi \rightarrow D3D_1 \phi \]
\[ D5 \phi \rightarrow D5D_1 \phi \]
\[ C_G \phi \rightarrow EC_1 \phi \]
\[ 4 \phi \rightarrow K_i \phi \]
\[ 5 \phi \rightarrow K_i \phi \]

---

\[ \phi ::= p \mid \neg \phi \mid \phi \wedge \phi \mid \langle G \rangle \phi \mid K_i \phi \mid C_G \phi \mid D_{G'} \phi \]

**Open problem:** completeness of ECL with the distributed knowledge axiom
Epistemic ATL: knowing that vs. knowing how
(knowledge of ability de dicto vs. de re)

\[ C_G \{G\} \gamma: \text{in every } G\text{-reachable state } G \text{ has a strategy that will ensure } \gamma \]

\[ ??: \ G \text{ has a strategy that in every } G\text{-reachable state will ensure } \gamma \]

\text{not expressible in Epistemic ATL}

Group knowing how: who knows that the group strategy is winning?

- Common knowledge in the group: requires the least amount of coordination
- General knowledge in the group
- Distributed knowledge in the group: if they communicate they can identify a winning strategy
- A single agent (e.g., the leader)
- A subgroup (e.g., the executive committee)
- A disjoint group (e.g., a consulting company)
- ...
Group knowing how: who knows that the group strategy is winning?

- Common knowledge in the group requires the least amount of coordination
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- ...

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

Constructive Knowledge

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \{A\} \varphi \mid \{A\} \Box \varphi \mid \{A\} \varphi U \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi. \]

\[ M, q \models C_G \varphi \iff M, [q],_{\neg C_G} \models \varphi \]

M, Q \models \{G\} \varphi \iff G has a joint strategy that will ensure that \( \gamma \) is true in all states in Q

Constructive Knowledge

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \{A\} \varphi \mid \{A\} \Box \varphi \mid \{A\} \varphi U \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi. \]

\[ M, q \models C_H \varphi \iff M, [q],_{\neg C_H} \models \varphi \]

M, Q \models \{H\} \varphi \iff G has a strategy that will ensure that \( \gamma \) is true, starting in any state \( H\)-reachable from q

Open problems: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators
Constructive Knowledge

\[ \varphi ::= p \mid \lnot \varphi \mid \varphi \land \varphi \mid \langle A \rangle \Box \varphi \mid \langle A \rangle \Box \lnot \varphi \mid \langle A \rangle \Box U \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi \mid C_A \varphi \mid E_A \varphi \mid D_A \varphi. \]

\[ M,q \models C_G \varphi \iff M,[q]_{\neg G} \models \varphi \]

\[ M,q \models C_H \langle G \rangle \varphi \iff G \text{ has a strategy that will ensure that } \gamma \text{ is true in all states in } Q \]

\[ M,q \models C_H \langle G \rangle \varphi \iff G \text{ has a joint strategy that will ensure that } \gamma \text{ is true, starting in any state } H\text{-reachable from } q \]

I.e., \( H \) knows how \( G \) can achieve \( \gamma \)

**Open problems**: complete axiomatisation of (even fragments of) ATL with constructive knowledge operators

---

**Ability**

through publicly observed informational actions

What if we interpret group ability modalities directly on epistemic models, in terms of possible public announcements?

"Group \( G \) can make a joint announcement such that, no matter what the other agents announce, \( \phi \) will be true"

Coalition Announcement Logic (GAL)

\[ \psi ::= p \mid K \varphi \mid \lnot \varphi \mid \varphi \land \varphi \mid \langle \psi \rangle \langle \psi \rangle \mid \langle G \rangle \varphi. \]

Ågotnes and van Ditmarsch, AAMAS 2008
What if we interpret group ability modalities directly on epistemic models, in terms of possible public announcements?

"Group $G$ can make an $(G)\phi$ announcement after which $\phi$ is true"

"Group $G$ can make a joint announcement such that, no matter what the other agents announce, $\phi$ will be true"

**Group Announcement Logic (GAL)**

$\phi ::= p | K_i \models \neg \phi | \phi_1 \land \phi_2 | (\{i\})\phi$

Agotnes et al., JAL 2010

**Coalition Announcement Logic (GAL)**

$\phi ::= p | K_i \models \neg \phi | \phi_1 \land \phi_2 | \{i\} \phi_1 | \{G\} \phi$

Agotnes and van Ditmarsch, AAMAS 2008

Related: Arbitrary Public Announcement Logic

$\phi ::= p | K_i \models \neg \phi | \phi_1 \land \phi_2 | \{i\} \phi_1 | \{G\} \phi$

Balbiani et al., TARK 2007

---

**GAL: example (Russian Cards)**

$(Ann)\{Bill\}(one \land two \land three)$

$(\{Ann,Bill\})(one \land two \land three)$

---

**GAL: expressing knowing-how**

Knowledge of ability, de dicto

$\forall s \sim u \exists \psi \models t \models (K_u \psi)\phi$

Knowledge of ability, de re

$\exists \psi \forall s \sim u \models t \models (K_u \psi)\phi$

$s \models K_u(a)\phi$

$s \models (a)K_u\phi$

$\exists \psi \models (K_u \psi)K_u\phi$
GAL: expressing knowing-how

Knowledge of ability, de dicto

∀s ∼ a t ∃ψ t |= ⟨K_aψ⟩φ

s |= K_a(aφ)

Knowledge of ability, de re

∃ψ s ∼ a t t |= ⟨K_aψ⟩φ

s |= (a)K_aφ

Depends on
(1) the fact that actions are announcements
(2) the S5 properties

GAL: expressing knowing-how

∃ψ ∃s ∼ a t t |= ⟨K_aψ⟩φ

s |= (a)K_aφ

∀ψ ∀s s ∼ a t t |= (K_aψ)φ

s |= (a)K_aφ

∀ψ: i ∈ G ∀(s, t) ∈ (∩_{ψ ∈ G} ψ_i), t |= (Λ_{ψ ∈ G} K_iψ_i)φ

s |= (G)D_{ψ_i}φ

∀ψ: i ∈ G ∀(s, t) ∈ (∩_{ψ ∈ G} ψ_i), t |= (Λ_{ψ ∈ G} K_iψ_i)φ

s |= (G)E_{ψ_i}φ
GAL: expressing knowing-how

\[ \forall s \in \phi \rightarrow (K_a \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

Open problem: express common knowledge de re

GAL: expressing knowing-how

\[ \forall s \in \phi \rightarrow (K_a \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

\[ \exists \psi : \psi(t) \in (\neg \psi \rightarrow \phi) \]

GAL: infinitary axiomatisation

Propositional tautologies

\[ K_a(\phi \rightarrow \psi) \rightarrow K_a \phi \rightarrow K_a \psi \]

\[ (\phi \rightarrow \psi \rightarrow \phi \rightarrow \neg \psi) \]

\[ (\phi \rightarrow \psi \rightarrow \phi \rightarrow \neg \psi) \]

\[ (\phi \rightarrow \psi \rightarrow \phi \rightarrow \neg \psi) \]

Sound and complete.

GAL-D: ability and distributed knowledge

\[ \phi := p \mid K_a \phi \mid \neg \phi \mid \phi \land \phi_2 \mid (\phi_1 \phi_2) \mid (G) \phi \mid DC \phi \]

Sound and complete.

Open problem: finitary axiomatisation (same for APAL)
Detour: Distributed Knowledge and the Principle of Full Communication

**Full communication:** $M, s \models D_G \phi \Rightarrow KS_G(M, s) \vdash \phi$

$KS_G(M, s) = \{\varphi \in \mathcal{L}_G : M, s \models \forall_{i \in G} K_s \varphi\}$

Does not always hold.

Roelofsen gives a complete characterisation of the models in which it does.

---

van Benthem: what about public communication?

GAL-D: ability and distributed knowledge

$\varphi ::= p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \langle G \rangle \varphi \mid D_G \varphi$

$D_G \varphi \rightarrow \langle G \rangle E_G \varphi$
GAL-D: ability and distributed knowledge

\[ \varphi := p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid (\varphi_1)\varphi_2 \mid (G)\varphi \mid D_G\varphi \]

\[ D_G\varphi \to (G)E_G\varphi \]

\[ M, t \models D_{(1,2)}(p \land \neg K_1 p) \]

E_{(1,2)}(p \land \neg K_1 p) not S5-consistent

---

GAL-D: ability and distributed knowledge

\[ \varphi := p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid (\varphi_1)\varphi_2 \mid (G)\varphi \mid D_G\varphi \]

\[ D_G\varphi \to (G)E_G\varphi \]

\[ M, t \models D_{(1,2)}(p \land \neg K_1 p) \]

E_{(1,2)}(p \land \neg K_1 p) not S5-consistent

---

Group announcement logic with distributed knowledge

\[ \varphi := p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid (\varphi_1)\varphi_2 \mid (G)\varphi \mid D_G\varphi \]

\( (40) \) Propositional tautologies

\[ (A11) \neg p \leftrightarrow (p \rightarrow p) \]

\[ (A12) \neg \neg \varphi \leftrightarrow (\varphi \rightarrow \neg \neg \varphi) \]

\[ (A13) K_i \varphi \leftrightarrow \varphi \]

\[ (A14) K_i \varphi \leftrightarrow K_i \neg \neg \varphi \]

\[ (A15) D_G\varphi \leftrightarrow D_G \neg \neg \varphi \]

\[ (A16) D_G\varphi \leftrightarrow G\varphi \]

\[ (A17) D_G\varphi \leftrightarrow D_{G\varphi} \]

\[ (A18) D_G\varphi \leftrightarrow D_G \neg \neg \varphi \]

\[ (A19) D_G\varphi \leftrightarrow G\varphi \]

\[ (A20) D_G\varphi \leftrightarrow D_{G\varphi} \]

Sound and complete:

\[ \forall \varphi \varphi_1 \vdash \eta(\varphi_1) \varphi \Rightarrow \eta(\varphi) \]

---

Group announcement logic with distributed knowledge

\[ \varphi := p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid (\varphi_1)\varphi_2 \mid (G)\varphi \mid D_G\varphi \]

\( (40) \) Propositional tautologies

\[ (A11) \neg p \leftrightarrow (p \rightarrow p) \]

\[ (A12) \neg \neg \varphi \leftrightarrow (\varphi \rightarrow \neg \neg \varphi) \]

\[ (A13) K_i \varphi \leftrightarrow \varphi \]

\[ (A14) K_i \varphi \leftrightarrow K_i \neg \neg \varphi \]

\[ (A15) D_G\varphi \leftrightarrow D_G \neg \neg \varphi \]

\[ (A16) D_G\varphi \leftrightarrow G\varphi \]

\[ (A17) D_G\varphi \leftrightarrow D_{G\varphi} \]

\[ (A18) D_G\varphi \leftrightarrow D_G \neg \neg \varphi \]

\[ (A19) D_G\varphi \leftrightarrow G\varphi \]

\[ (A20) D_G\varphi \leftrightarrow D_{G\varphi} \]

Sound and complete:

\[ \forall \varphi \varphi_1 \vdash \eta(\varphi_1) \varphi \Rightarrow \eta(\varphi) \]
Open problem: can CAL express everything APAL can express?

Open problem: can GAL express everything CAL can express?

Sound and complete.

Relative expressivity of logics of quantified announcement

Open problem: can GAL express everything CAL can express?

Open problem: can APAL express everything CAL can express?

Complexity

<table>
<thead>
<tr>
<th>Logic</th>
<th>Result</th>
<th>Reference</th>
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<tbody>
<tr>
<td>EL</td>
<td>PSPACE-complete</td>
<td>[Halpern and Moses 1992]</td>
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<td>CLD</td>
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<td>[Halpern and Moses 1992]</td>
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<td>[Ågotnes et al., JAL 2010]</td>
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<td>PSPACE-complete</td>
<td>[Fisher and Ladner 1977]</td>
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<tr>
<td>PAP</td>
<td>EXPTIME-complete</td>
<td>[Lutz 2006]</td>
</tr>
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<td>PAPC</td>
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<td>[Ågotnes et al., JAL 2010]</td>
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<td>[Ågotnes and Alechina 2016]</td>
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<tr>
<td>CAL</td>
<td>undecidable</td>
<td>[Ågotnes, van Ditmarsch and French, 2016]</td>
</tr>
</tbody>
</table>
Resolving distributed knowledge

- Logics with distributed knowledge do not reason about what happens when the group actually share their information.

- In this work we introduce a new modality, saying that a formula is true after the group have shared their information - after their distributed knowledge has been resolved.

"Communication core" (van Benthem, 2011)

Similar to "tell all you know" logic, Baltag ESSLLI 2010
What do other agents know about the fact that a group G resolve their knowledge?

- We assume that it is common knowledge that G resolve their knowledge

\[ M = (S, \sim_1, \ldots, \sim_n, V) \] (S5 model)

For a group of agents G, the (global) G-resolved update of M is the model \( M|_G \) where \( M|_G = (S', \sim_1', \ldots, \sim_n', V') \) and

- \( S' = S \)
- \( \sim_i' = \bigcap_{j \in G} \sim_j \) if \( i \in G \)
  otherwise
- \( V' = V \)

Resolving Distributed Knowledge: Logic

\[
\begin{align*}
RD \cdot \phi := & p \mid \neg \phi \mid \neg \phi \land \phi \mid K_\phi \mid D_\phi \mid C_\phi \mid R_\phi \\
RCD \cdot \phi := & p \mid \neg \phi \mid \neg \phi \land \phi \mid K_\phi \mid D_\phi \mid C_\phi \mid R_\phi \\
RCD \cdot \neg \phi := & p \mid \neg \phi \land \phi \mid K_\phi \mid D_\phi \mid C_\phi \mid R_\phi
\end{align*}
\]

\[ M, s \models R_G \phi \iff M|_G, s \models \phi \]

Resolution: from distributed to common knowledge

\[ \phi := p \mid \neg \phi \mid \phi \land \phi \mid K_\phi \mid D_\phi \mid C_\phi \mid R_\phi \]

\[ D_G \phi \rightarrow R_G C_G \phi \]
Resolution: from distributed to common knowledge

\[ \phi := p | \neg \phi | \phi \land \phi | K_i \phi | D_G \phi | C_G \phi | R_G \phi \]

\[ \models D_G \phi \rightarrow R_G \phi \]

\[ \models D_G R_G \phi \leftrightarrow R_G C_G \phi \]

Resolving distributed knowledge: expressivity

\[ \mathcal{ECD} = \mathcal{LCD} = \mathcal{RD} = \mathcal{RCD} = \mathcal{PAD} = \mathcal{ACD} \]

(a) \(|\mathcal{M}| = 1\)

(b) \(|\mathcal{M}| = 2\)

(c) \(|\mathcal{M}| \geq 3\)

AXIOMATISATION: RD

\[ \mathcal{RD} \cdot \phi := p | \neg \phi | \phi \land \phi | K_i \phi | D_G \phi | R_G \phi \]

\[ \mathcal{RCD} \cdot \phi := p | \neg \phi | \phi \land \phi | K_i \phi | D_G \phi | C_G \phi | R_G \phi \]

AXIOMATISATION: RD

\[ (S5) \text{ classical proof system for multi-agent epistemic logic} \]

\[ (CK) \text{ axioms and rules for common knowledge} \]

\[ (DK) \text{ characterization axioms for distributed knowledge} \]

\[ (N_R) \text{ from } \phi \text{ infer } R_G \phi \]

\[ (RR) \text{ reduction axioms for resolution} \]

\[ (RR_c) \text{ from } \phi \rightarrow (E_G \phi \land R_{G_1} \cdots \land R_{G_n} \psi) \text{ infer } \phi \rightarrow R_{G_1} \cdots R_{G_n} C_G \psi \]

CK:

\[ (K_D) \quad C_G (\phi \rightarrow \psi) \rightarrow (C_G \phi \rightarrow C_G \psi) \]

\[ (T_D) \quad C_G \phi \rightarrow \phi \]

\[ (C1) \quad C_G \phi \rightarrow E_G C_G \phi \]

\[ (C2) \quad C_G (\phi \rightarrow E_G \phi) \rightarrow (\phi \rightarrow C_G \phi) \]

\[ (N_C) \quad \text{ from } \phi \text{ infer } C_G \phi. \]

Theorem: sound and complete.

Resolution: some open issues

- Other assumptions about what other agents know about the resolution event
  - E.g., local updates
- Syntax vs. semantics and full communication
- Belief
  - Expressive power:
    - compare to languages with relativised common knowledge
- Computational complexity
Discussion

What Distributed Knowledge Actually Is

• Common interpretations of distributed knowledge:
  • Knowledge the group could obtain if they had unlimited means of communication
  • “A group has distributed knowledge of a fact phi if the knowledge of phi is distributed among its members, so that by pooling their knowledge together the members of the group can deduce phi ...

A group has distributed knowledge of a fact phi if after “pooling their knowledge together” the members of the group know that phi was true before they did that

Interesting (and pretty much unexplored!) connection

$D_{G\varphi}$: after resolving $G$ has common knowledge that $\varphi$ was true before that

Relativised common knowledge

van Benthem et al., 2006

Interesting (and pretty much unexplored!) connection

$D_{G\varphi}$: after resolving $G$ has common knowledge that $\varphi$ was true before that

Relativised common knowledge

van Benthem et al., 2006

Interesting (and pretty much unexplored!) connection

$C_{G\varphi}$: after $\psi$ is announced $G$ has common knowledge that $\varphi$ was true before that

Interesting (and pretty much unexplored!) connection

$D_{G\varphi}$: after resolving $G$ has common knowledge that $\varphi$ was true before that

Relativised common knowledge

van Benthem et al., 2006
Interesting (and pretty much unexplored!) connection

\[ D_{C \varphi} \text{: after resolving } G \text{ has common knowledge that } \varphi \text{ was true before that} \]

\[ C_{G \varphi} \text{: after } \psi \text{ is announced } G \text{ has common knowledge that } \varphi \text{ was true before that} \]

Issues with distributed knowledge and the literature

- What it is ("pooling")
- Distributed belief is not belief, under many common assumptions about what belief is
- Unsound axiomatisations
- Allowing the empty coalition (universal modality)
- "Not invariant under bisimulation"

Some related things I didn’t talk about

- Deeper philosophical accounts of group belief - both reductionist and non-reductionist
- Formalisations: see Gaudou et al., 2015
- Group knowledge in plausibility models (Baltag and Smets)
- Belief merge (Baltag and Smets, MALLO 2009)
- Christoff et al., 2019 (under review) on priority merge (with resolution!)

The road ahead: group knowledge in social networks

- Parikh and Pacuit (2004): first steps towards analysing the information that can be shared by a group of agents restricted to a communication network
- Seligman et al. (TARK 2013): epistemics of network events
- On group formation in social networks
  - Smets and Velázquez-Quesada, LORI 2017: social selection
  - Xiong and Ågotnes, JdLL 2019: on the logic of balance in social networks
- Pedersen, Smets and Ågotnes, LORI 2019: on the formation of echo chambers

Summary

- Group belief is most often not actually belief
- There is a range of notions of group belief corresponding to different aggregation rules, the extremes being general and distributed belief
- We developed techniques for dealing with distributed knowledge in completeness proofs, used for PAL, CL, GAL, CAL, resolving...
- Epistemic coalition logic: general reasoning about group knowledge and group ability
  - Group ability and constructive knowledge: separating who knows how
  - Group and coalition announcement logics: ability through announcement
- Resolving distributed knowledge
  - Captures exactly the relationship between distributed and common knowledge
  - Between group knowledge, dynamics and ability
A Measurement-Theoretic Modification of Harvey’s Aggregation Theorem

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Contents of This Talk

1. Measurement-Theoretic Considerations of Harsanyi’s Aggregation Theorem
2. Harvey’s Aggregation Theorem and Cardinal Utility
3. Our Two Criticisms on Harvey’s Aggregation Theorem from Measurement-Theoretic Point of View
4. Our Aggregation Theorems
5. Summary

Harsanyi’s Aggregation Theorem (1)

- Harsanyi (1955) attempts to develop expected utility theory of von Neumann and Morgenstern (1944) to provide a formalization of (weighted) utilitarianism.
- Weymark (1991) refers to this result as Harsanyi’s Aggregation Theorem.
Harsanyi’s Aggregation Theorem (2)


Measurement-Theoretic Concepts

Here we would like to define such measurement-theoretic concepts as

- scale types,
- representation and uniqueness theorems, and
- measurement types

on which the argument of this talk is based:

Scale Types (1)

First, we classify scale types in terms of the class of admissible transformations $\varphi$.

- A scale is $\langle \mathcal{U}, \mathcal{V}, f \rangle$ or $f$, where $\mathcal{U}$ is an observed relational structure that is qualitative, $\mathcal{V}$ is a numerical relational structure that is quantitative, and $f$ is a homomorphism from $\mathcal{U}$ into $\mathcal{V}$.
- $A$ is the domain of $\mathcal{U}$ and $B$ is the domain of $\mathcal{V}$.
- When the admissible transformations are all the functions $\varphi : f(A) \rightarrow B$ of the form $\varphi(x) := \alpha x + \beta; \alpha > 0$, $\varphi$ is called a positive affine transformation, and a corresponding scale is called an interval scale.
- $\mathcal{U}$ is called a similarity transformation, and a scale with the similarity transformations as its class of admissible transformations is called a ratio scale.
- Length is an example of a ratio scale.

Scale Types (2)

- When the admissible transformations are all the functions $\varphi : f(A) \rightarrow B$ of the form $\varphi(x) := \alpha x + \beta; \alpha > 0$, $\varphi$ is called a positive affine transformation, and a corresponding scale is called an interval scale.
- Temperature on the Fahrenheit scale and temperature on the Celsius scale are examples of interval scales.
The representation problem: Given a quantitative (numerical) scale types \( \alpha \succ \beta \) and \( \succ \) satisfy the Strong Pareto condition. Furthermore, suppose that the individual and social binary preference relations
\begin{align*}
\succeq_i (i = 1, \ldots, n) \text{ and } \succeq \text{ on the set of lotteries satisfy von Neumann-Morgenstern axioms, and also suppose that } \succeq_i \text{ and } \succeq \text{ satisfy the Strong Pareto condition. Furthermore, suppose that } \succeq_i \text{ and } \succeq \text{ are represented by individual and social expected utility functions } U_i (i = 1, \ldots, n) \text{ and } U \text{ respectively. Then, there are real numbers } \alpha_i (>0)(i=1,\ldots, n) \text{ and } \beta \text{ such that }
U(p) = \sum_{i=1}^{n} \alpha_i U_i(p) + \beta,
\end{align*}
for any lottery \( p \).
The next corollary directly follows from this theorem:

**Corollary (Weighted Utilitarianism on Set of Lotteries)**

Lotteries are socially ranked according to a weighted utilitarian rule:

$$U(p) \geq U(q) \text{ iff } \sum_{i=1}^{n} \alpha_i U_i(p) \geq \sum_{i=1}^{n} \alpha_i U_i(q),$$

for any lotteries $p,q$.

---

**Sen’s Criticism**

- There are at least two well-known criticisms on Harsanyi’s Aggregation Theorem.
- The first criticism is by Sen (1976):
  - Von Neumann-Morgenstern axioms on individual and social binary preference relations in Lemma (Representation) are for ordinal measurement and, therefore, any monotone increasing (even non-affine) transform of an expected utility function is a satisfactory representation of individual and social binary preference relations.
- However, (weighted) utilitarianism requires a theory of cardinal utility, and so Harsanyi is not justified in giving his theorems utilitarian interpretations.

---

**Probability Agreement Theorem**

- The second criticism is based on the following probability agreement theorem that is provided by Broome (1991):

**Theorem (Probability Agreement Theorem)**

Suppose that individual and social binary preference relations $\succeq_i (i = 1, \ldots, n)$ and $\succeq$ on the set of lotteries satisfy von Neumann-Morgenstern axioms. Then, $\succeq_i$ and $\succeq$ cannot satisfy the strong Pareto condition unless every individual agrees about the probability of every elementary event.

- In fact, under many circumstances, the members of a society have different beliefs (probabilities) of elementary events.
Harvey’s Aggregation Theorem (1)

In order to escape these two criticisms, we can resort to Harvey’s Aggregation Theorem (1999) that has \textit{quaternary preference relations} as primitive that can be represented by \textit{utility differences}, and is concerned only with quaternary preference relations on the set of \textit{outcomes} but is not concerned with binary preference relations on the set of lotteries in Harsanyi’s Aggregation Theorem.


---

History of Cardinal Utility (1)

- Lange (1934) is the first to connect formally the ranking of \textit{utility differences} with \textit{positive affine transformations} of utility functions.
- However, he does not use the expression “cardinal utility”.
- Alt (1936) is considered to be the first to prove the representation theorem for quaternary preference relations that can be represented by utility differences, and the uniqueness theorem on the uniqueness of the utility functions up to positive affine transformations.
- However, he also dose not connect utility differences with the expression “cardinal utility”.
- Samuelson (1938) is the first to connect utility differences in which utility functions are unique up to positive affine transformations “cardinal utility”, though he takes a negative position toward cardinal utility.

---

History of Cardinal Utility (2)

- Lange, O.: The determinateness of the utility function. Review of Economic Studies 1, 218–224 (1934)
Harvey’s Aggregation Theorem (2)

Harvey (1999, p. 69) defines difference-worth conditions as follows:

We will use conditions on a quaternary preference relation \( \succeq \) as any set of conditions that are satisfied iff there exists a worth function \( w \) such that

\[
(a, b) \succeq (c, d) \text{ iff } w(a) - w(b) \geq w(c) - w(d)
\]

for any outcome \( a, b, c, d \), and we will refer to any such conditions as a set of difference-worth conditions.

Harvey’s Aggregation Theorem (3)

Then Harvey’s Aggregation Theorem can be stated in the following way:

**Theorem (Harvey’s Aggregation Theorem)**

Suppose that individual and social quaternary preference relations \( \succeq_i (i = 1, \ldots, n) \) and \( \succeq \) on the set of outcomes satisfy a certain set of difference-worth conditions. Then, \( \succeq_i \) and \( \succeq \) satisfy the strong Pareto condition iff there are real numbers \( \alpha_i (> 0)(i = 1, \ldots, n) \) and \( \beta \) such that

\[
w(a) = \sum_{i=1}^{n} \alpha_i w_i(a) + \beta,
\]

for any outcome \( a \).

Representation and Uniqueness Lemmas

The next corollary directly follows from this theorem:

**Corollary (Weighted Utilitarianism on Set of Outcomes)**

Outcomes are socially ranked according to a weighted utilitarian rule.

**Lemma (Representation)**

Suppose that \( \succeq_i (i = 1, \ldots, n) \) and \( \succeq \) on the set of outcomes satisfy a certain set of difference-worth conditions. Then, there exist individual and social worth functions \( w_i(i = 1, \ldots, n) \) and \( w \) such that

\[
(a, b) \succeq_i (c, d) \text{ iff } w_i(a) - w_i(b) \geq w(c) - w(d),
\]

\[
(a, b) \succeq (c, d) \text{ iff } w(a) - w(b) \geq w(c) - w(d),
\]

for any outcome \( a, b, c, d \).

**Lemma (Uniqueness)**

Suppose that \( \succeq_i (i = 1, \ldots, n) \) and \( \succeq \) on the set of outcomes satisfy the conditions for the representation above. Then, \( w_i(i = 1, \ldots, n) \) and \( w \) are unique up to a positive affine transformation, that is, \( w_i \) and \( w \) are interval scales.

Because any set of difference-worth conditions is for algebraic-difference measurement that is a kind of cardinal measurement, this theorem can escape the first criticism.
Hammond’s Position

- When Hammond (1982) attempts to salvage utilitarianism in the way that the (strong) Pareto condition can apply only to outcomes.
- Harvey takes the same position as Hammond that enables this theorem to escape the second criticism.


First Criticism

- Now we inspect Harvey’s Aggregation Theorem from a measurement-theoretic point of view.
- We offer two criticisms on Harvey’s Aggregation Theorem: The first criticism is as follows:
- As Roberts (1979, p.139) says, the only set of necessary and sufficient difference-worth conditions is due to Scott (1964) and requires the assumption that the set of outcomes is finite.
- So when there is no domain-size limitation, the set of necessary and sufficient difference-worth conditions is still unknown.

Second Criticism (1)

- The second criticism is as follows:
- The most essential task of aggregation theorem from a measurement-theoretic point of view is to prove the existence of individual and social worth functions that represent individual and social quaternary preference relations which satisfy not only difference-worth conditions but also the strong Pareto condition.
- However, Harvey’s Aggregation Theorem is not of such a form.
A Measurement-Theoretic Modification of Harvey's Aggregation Theorem

Our Two Criticisms on Harvey's Aggregation Theorem from Measurement-Theoretic Point of View

Second Criticism (2)

- For, in Lemma (Representation), individual and social quaternary preference relations satisfy only difference-worth conditions.
- So the existence of individual and social worth functions that represent individual and social quaternary preference relations which satisfy both difference-worth conditions and the strong Pareto condition is not guaranteed.

Second Criticism (3)

- Harvey (1999, p.72) comments on the feature of his own theorem:
  
  *I view the result in Harsanyi (1955) and the result presented here as uniqueness results rather than as existence results. ... an expected-utility function or a worth function is unique up to a positive affine function.*

- Then, does what Harvey says keep to the point?
- What should be proved is the uniqueness of individual and social worth functions that represent individual and social quaternary preference relations which satisfy not only difference-worth conditions but also the strong Pareto condition.

Second Criticism (4)

- However, in Lemma (Uniqueness), individual and social quaternary preference relations satisfy also only difference-worth conditions.
- So the uniqueness of individual and social worth functions that represent individual and social quaternary preference relations which satisfy both difference-worth conditions and the strong Pareto condition is not guaranteed either.
- After all, Harvey's Aggregation Theorem can give any answer neither to the representation problem nor to the uniqueness problem.

Contents of This Talk

1. Measurement-Theoretic Considerations of Harsanyi's Aggregation Theorem
2. Harvey's Aggregation Theorem and Cardinal Utility
3. Our Two Criticisms on Harvey's Aggregation Theorem from Measurement-Theoretic Point of View
4. Our Aggregation Theorems
5. Summary
The aim of this talk is that we show new aggregation theorems, which escape these two criticisms, inspired by Harvey's Aggregation Theorem. Our aggregation representation and uniqueness theorems can be stated in the following way:

**Theorem (Aggregation Representation Theorem)**

Suppose that individual and social quaternary preference relations \( \succsim^i (i = 1, \ldots, n) \) and \( \succsim \) on the set of outcomes satisfy Weak Order, Order Reversal, Weak Monotonicity, Solvability and Archimedean condition in Krantz et al. (1971), and also suppose that \( \succsim_i \) and \( \succsim \) satisfy the strong Pareto condition. Then, there exist individual and social utility functions \( u_i (i = 1, \ldots, n) \) and \( u \) such that NOT ONLY

\[
\begin{cases}
(a, b) \succsim^i (c, d) \iff u_i(a) - u_i(b) \geq u_i(c) - u_i(d), \\
(a, b) \succsim (c, d) \iff u(a) - u(b) \geq u(c) - u(d),
\end{cases}
\]

for any outcome \( a, b, c, d \) BUT ALSO there are real numbers \( \alpha_i (\alpha_i > 0) \) \((i = 1, \ldots, n)\) and \( \beta \) such that

\[
u(a) = \sum_{i=1}^{n} \alpha_i u_i(a) + \beta,
\]

for any outcome \( a \).

---

**Weighted Utilitarianism on Set of Outcomes**

The next corollary directly follows from this theorem.

**Corollary (Weighted Utilitarianism on Set of Outcomes)**

Outcomes are socially ranked according to a weighted utilitarian rule.

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**Theorem (Aggregation Uniqueness Theorem)**

Suppose that \( \succsim^i (i = 1, \ldots, n) \) and \( \succsim \) on the set of outcomes satisfy the conditions for the representation above. Then, \( u_i (i = 1, \ldots, n) \) and \( u \) are unique up to a positive affine transformation, that is, \( u_i \) and \( u \) are interval scales.

---


One of key techniques for proving this theorem is a version of Moment Theorem in abstract linear spaces in Domotor (1979).

Escapes from Criticisms

- Because our aggregation theorems do not include any set of necessary and sufficient difference-worth (algebraic difference) conditions but include only some sufficient conditions, it escapes the first criticism.
- Because our aggregation representation theorem guarantees the existence of individual and social utility functions that represent individual and social quaternary preference relations which satisfy not only difference-worth (algebraic difference) conditions but also the strong Pareto condition, and our aggregation uniqueness theorem guarantees the uniqueness of such functions, they escape the second criticism.

Possible Criticism (1)

- Finally, we would like to discuss the following possible criticism, which is similar to the first criticism on Harsanyi’s Aggregation Theorem by Sen, to our aggregation representation and uniqueness theorems.
- We can prove the following propositions similar to Lemma (Representation) and Lemma (Uniqueness) of Harsanyi’s Aggregation Theorem:

Proposition (Representation)
Suppose that individual and social quaternary preference relations $\succsim_i$ ($i = 1, \ldots, n$) and $\succsim$ on the set of outcomes satisfy Weak Order, Order Reversal, Weak Monotonicity, Solvability and Archimedean condition in Krantz et al. (1971). Then, there exist individual and social utility functions $u_i$ ($i = 1, \ldots, n$) and $u$ such that

\[
\begin{gathered}
(a, b) \succeq_i (c, d) \iff \frac{u_i(a)}{u_i(b)} \geq \frac{u_i(c)}{u_i(d)}, \\
(a, b) \succeq (c, d) \iff \frac{u(a)}{u(b)} \geq \frac{u(c)}{u(d)},
\end{gathered}
\]

for any outcome $a, b, c, d$.

Proposition (Uniqueness)
Suppose that $\succsim_i$ ($i = 1, \ldots, n$) and $\succsim$ on the set of outcomes satisfy the conditions for the representation above. Then, $u_i$ ($i = 1, \ldots, n$) and $u$ are unique up to a transformation of functions of the form $\alpha x^\beta; \alpha, \beta > 0$, that is, $u_i$ and $u$ are log-interval scales.

Possible Criticism (2)

- These propositions imply that Weak Order, Order Reversal, Weak Monotonicity, Solvability and Archimedean condition in Krantz et al. (1971) can satisfy not only (1) "algebraic-difference measurement" but also (2) "algebraic-quotient measurement".
- So our aggregation theorems cannot justify weighted utilitarianism.
- How can we escape this criticism?
A Measurement-Theoretic Modification of Harvey's Aggregation Theorem

Our Aggregation Theorems

Escape from Criticism (1)

- Von Neumann-Morgenstern axioms on individual and social binary preference relations in Lemma (Representation) are considered, as we have argued earlier, to be for ordinal measurement according to the first criticism by Sen.
- In this criticism, the fact that any monotone increasing (even non-affine) transform of an expected utility function is a satisfactory representation of individual and social binary preference relations is used to prove that von Neumann-Morgenstern axioms on individual binary preference relations in Lemma (Representation) are not for cardinal measurement but for ordinal measurement.
- In Lemma (Uniqueness), von Neumann-Morgenstern axioms together with $U_i$ and $U$ being expected utility functions imply the cardinality of $U_i$ and $U$.
- So von Neumann-Morgenstern axioms only do not justify the cardinality of $U_i$ and $U$.

Escape from Criticism (2)

- On the other hand, our axioms on individual and social quaternary preference relations can be for utility-difference measurement (algebraic-difference measurement) that is regarded historically as the definition of cardinal utility.
- Our aggregation representation theorem can justify the existence of the cardinal utilities $u_i$ and $u$ implying weighted utilitarianism.
- Propositions (Representation) and (Uniqueness) are not about cardinal utility but about algebraic-quotient measurement.
- Propositions (Representation) can justify the existence of the non-cardinal utilities $u_i$ and $u$ implying non-weighted utilitarianism.
- Then these propositions do not relate to the cardinality of $u_i$ and $u$.
- So we do not have to take these propositions into consideration.

Summary

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Summary

- Harsanyi’s Aggregation Theorem is an attempt to justify (weighted) utilitarianism.
- However, there are at least two well-known criticisms on Harsanyi’s Aggregation Theorem.
- In order to escape these two criticisms, we can resort to Harvey’s Aggregation Theorem.
- In this talk, we have offered two criticisms on this theorem from a measurement-theoretic point of view.
- Then, we have proposed new aggregation theorems, which escape these two criticisms on Harvey’s Aggregation Theorem.
- Moreover, we have shown that these theorems can escape a possible criticism that seems to plausible.
- In this sense, these theorems can justify weighted utilitarianism in a strict way.
Thank You for Your Attention!
Single-peakedness of preferences via deliberation: A formal study
(An extended abstract)

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1 Introduction

There are two important aspects of any democratic decision: aggregation of preferences and deliberation about preferences. They are essential and complementary components of any decision making process. While the well-studied process of aggregation focuses on accumulating individual preferences without discussing their origin [3], deliberation can be seen as a conversation through which individuals justify their preferences, a process that might lead to changes in their opinions as they get influenced by one another. Till now, there has been a lot of work on the ‘aggregation’ aspect (e.g., [12, 13, 6]). However, some recent work has focussed on the deliberation aspect as well [8, 9, 10, 15].

Sometimes, deliberation does not lead to unanimity in preferences, but the discussion can lead to some ‘preference uniformity’ (see how deliberation can help in bypassing social choice theory’s impossibility results in [5]), which might facilitate their eventual aggregation. In addition, the combination of both processes provides a more realistic model for decision making scenarios. In light of this status quo, our focus is on the formal study of achieving such preference uniformities, e.g., single-peaked, single-caved, single-crossing, value-restricted, best-restricted, worst-restricted, medium-restricted, or group-separable profiles. In this short abstract we provide our preliminary ideas towards achieving single-peakedness of preference profiles via deliberation.

In what follows, we define two preference upgrade operators based on [8, 9] and provide a preliminary discussion on how single-peaked preference profiles can be achieved through such operations. We will delve into the details of the logical language in the main paper.
2 Basic concepts

The focus of this work is public deliberation, so let \( Ag \) be a finite non-empty set of agents with \(|Ag| = n \geq 2\) (if \( n = 1 \), there is no scope for joint discussion). Below we present the most important definitions of this framework.

**Definition 1 (PR frame).** A preference and reliability (PR) frame \( F \) is a tuple \( \langle W, \{\leq_i, \preceq_i\}_{i \in Ag} \rangle \) where

- \( W \) is a finite non-empty set of worlds;
- \( \leq_i \subseteq (W \times W) \) is a total preorder (a total, reflexive and transitive relation), agent \( i \)'s preference relation over worlds in \( W \) (\( u \leq_i v \) is read as “world \( v \) is at least as preferable as world \( u \) for agent \( i \)”);
- \( \preceq_i \subseteq (Ag \times Ag) \) is a total order (a total, reflexive, transitive and antisymmetric relation), agent \( i \)'s reliability relation over agents in \( Ag \) (\( j \preceq_i j' \) is read as “agent \( j' \) is at least as reliable as agent \( j \) for agent \( i \)”).

Some further useful definitions are given below.

**Definition 2.** Let \( F = \langle W, \{\leq_i, \preceq_i\}_{i \in Ag} \rangle \) be a PR frame.

- \( u <_i v \) (“\( u \) is less preferred than \( v \) for agent \( i \)”) iff \( \leq_i u \) and \( v \not\leq_i u \).
- \( u \simeq_i v \) (“\( u \) and \( v \) are equally preferred for agent \( i \)”) iff \( \leq_i u \) and \( v \leq_i u \).
- \( j <_i j' \) (“\( j \) is less reliable than \( j' \) for agent \( i \)”) iff \( \preceq_i j <_i j' \) and \( j' \not\preceq_i j \).
- \( \text{mr}(i) = j \) (\( j \) is agent \( i \)'s most reliable agent) iff \( \preceq_i j <_i j \) for every \( j' \in Ag \).
- \( \text{Max}_{\leq_i}(U) \), the set containing agent \( i \)'s most preferred worlds among those in \( U \subseteq W \), is formally defined as \( \{v \in U \mid u \leq_i v \text{ for every } u \in U\} \).

3 Preference dynamics: lexicographic upgrade

Intuitively, a public announcement of the agents’ individual preferences might induce an agent \( i \) to adjust her own preferences according to what has been announced and the reliability she assigns to the set of agents.\footnote{Note that we do not study the formal representation of such announcement, but rather the representation of its effects.} Thus, agent \( i \)'s preference ordering after such announcement, \( \leq'_i \), can be defined in terms of the just announced preferences (the agents' preferences before the announcement, \( \leq_1, \ldots, \leq_n \)) and how much \( i \) relied on each agent (\( i \)'s reliability before the announcement, \( \preceq_i \)): \( \leq'_i := f(\leq_1, \ldots, \leq_n, \preceq_i) \) for some function \( f \). Below, we define a general upgrade operation based on agent reliabilities from [8].
Definition 3 (General lexicographic upgrade). A lexicographic list $\mathcal{R}$ over $W$ is a finite non-empty list whose elements are indices of preference orderings over $W$, with $|\mathcal{R}|$ the list’s length and $\mathcal{R}[k]$ its $k$th element ($1 \leq k \leq |\mathcal{R}|$). Intuitively, $\mathcal{R}$ is a priority list of preference orderings, with $\approx_{\mathcal{R}[1]}$ the one with the highest priority. Given $\mathcal{R}$, the preference ordering $\leq_{\mathcal{R}} \subseteq (W \times W)$ is defined as

$$u \leq_{\mathcal{R}} v \iff (u \leq_{\mathcal{R}[|\mathcal{R}|]} v \wedge \bigwedge_{k=1}^{|\mathcal{R}|-1} u \approx_{\mathcal{R}[k]} v) \lor$$

$$\bigvee_{k=1}^{|\mathcal{R}|-1} (u \leq_{\mathcal{R}[k]} v \wedge \bigwedge_{l=1}^{k-1} u \approx_{\mathcal{R}[l]} v)$$

Thus, $u \leq_{\mathcal{R}} v$ holds if this agrees with the least prioritised ordering ($\leq_{\mathcal{R}[|\mathcal{R}|]}$) and for the rest of them $u$ and $v$ are equally preferred (part 1), or if there is an ordering $\leq_{\mathcal{R}[k]}$ with a strict preference for $v$ over $u$ and all orderings with higher priority see $u$ and $v$ as equally preferred (part 2).

Proposition 1. Let $\mathcal{R}$ be a lexicographic list over $W$. If every ordering $\mathcal{R}[k]$ ($1 \leq k \leq |\mathcal{R}|$) is reflexive (transitive, total, respectively), then so is $\leq_{\mathcal{R}}$.

As a consequence of this proposition, the general lexicographic upgrade preserves total preorders (and thus our class of semantic models) when every preference ordering in $\mathcal{R}$ satisfies the requirements.

Even though the general lexicographic upgrade covers many natural upgrades [8], there are also ‘reasonable’ policies that fall outside its scope. Sometimes we are not interested in considering the complete order among the choices of the most reliable agent, but only her most preferred choices. To model such upgrades, as mentioned in [9] we provide the following preference upgrade definition.

Definition 4 (General layered upgrade). A layered list $\mathcal{S}$ over $W$ is a finite (possibly empty) list of pairwise disjoint subsets of $W$ together with a default preference ordering over $W$. The list’s length is denoted by $|\mathcal{S}|$, its $k$th element is denoted by $\mathcal{S}[k]$ (with $1 \leq k \leq |\mathcal{S}|$), and $\approx_{\mathcal{S}}$ is its default preference ordering. Intuitively, $\mathcal{S}$ defines layers of elements of $W$ in the new preference ordering $\leq_{\mathcal{S}}$, with $\mathcal{S}[1]$ the set of worlds that will be in the topmost layer and $\approx_{\mathcal{S}}$ the preference ordering that will be applied to each individual set and to those worlds not in $\bigcup_{k=1}^{|\mathcal{S}|} \mathcal{S}[k]$. Formally, given $\mathcal{S}$, the ordering $\leq_{\mathcal{S}} \subseteq (W \times W)$ is defined as

$$u \leq_{\mathcal{S}} v \iff_{\text{def}} \left( u \leq_{\approx_{\mathcal{S}}} v \wedge \bigcup_{k=1}^{|\mathcal{S}|} \mathcal{S}[k] = \emptyset \lor \bigvee_{k=1}^{|\mathcal{S}|} \{u, v\} \subseteq \mathcal{S}[k] \right)$$

$$\lor \left( \bigwedge_{k=1}^{|\mathcal{S}|} \bigwedge_{l=1}^{k-1} u \approx_{\mathcal{S}[l]} v \wedge \bigvee_{k=1}^{|\mathcal{S}|} \{v \in \mathcal{S}[k] \wedge u \notin \bigcup_{l=1}^{k} \mathcal{S}[l]\} \right)$$
Thus, $u \leq_S v$ holds if this agrees with the default ordering $\leq_{\text{def}}$ and either neither $u$ nor $v$ are in any of the specified sets in $S$ or else both are in the same set (part 1), or if there is a set $S[k]$ in which $v$ appears and $u$ appears neither in the same set (a case already covered in part 1) nor in one with higher priority (part 2).

**Proposition 2.** Let $S$ be a layered list over $W$. If $\leq_{\text{def}}$ is reflexive (transitive, total, respectively), then so is $\leq_S$.

**Definition 5.** Let $M = \langle W, \{\leq_i, \preceq_i\}_{i \in A_g}, V \rangle$ be a PR model.

- Let $S$ be a layered list whose default ordering is reflexive, transitive and total; let $j \in A_g$ be an agent. The PR model $\text{gy}_j^S(M) = \langle W, \{\leq'_i, \preceq'_i\}_{i \in A_g}, V \rangle$ is such that, for every agent $i \in A_g$, $\leq'_i := \leq_S$ if $i = j$, and $\leq'_i := \leq_i$ otherwise.

- Let $S$ be a list of $|A_g|$ layered lists whose default ordering are reflexive, transitive and total, with $S_i$ its $i$th element. The PR model $\text{gy}_S(M) = \langle W, \{\leq'_i, \preceq'_i\}_{i \in A_g}, V \rangle$ is such that, for every agent $i \in A_g$, $\leq'_i := \leq_S$.

We have proposed different preference upgrade operators based on agent reliabilities. Now, the question is under what conditions these upgrade operators may lead to single-peakedness of agent preferences.

## 4 Deliberating towards single-peakedness

On the one hand we have the general lexicographic upgrade operation which considers a particular list to define the upgraded preferences. On the other hand we have this layered upgrade operation which is based on arbitrary subsets of choices and providing an order between them. There is a whole territory of possible upgrade operators in between these possibilities that is uncharted as of now. We would like to focus on charting the territory with a special emphasis on single-peakedness. We now assume the preference orderings to be asymmetric in addition to being total and transitive. Each agent is endowed with such a preference relation over the worlds.

**Definition 6.** A preference profile is single-peaked if there exists a world $w_i$ for each agent $i$ and a linear order $L$ such that $w_i \preceq L w''$ or $w'' \preceq L w_i$ imply $w' < w''$.

Ballester and Haeringer [2] showed that the following two conditions characterize single-peakedness.

- For any subset of worlds the set of worlds considered as the worst by all agents cannot contains more than 2 elements (known as the worst-restricted condition in the literature).
There cannot be four worlds \( w_1, w_2, w_3, w_4 \) and two agents \( i, j \) such that \( w_1 <_i w_2 <_i w_3, \ w_3 <_j w_2 <_j w_1, \) and \( w_4 <_i w_2, w_4 <_j w_2. \) In other words, two agents cannot disagree on the relative ranking of two alternatives with respect to a third alternative but agree on the (relative) ranking of a fourth one.

Our task is to investigate that under what conditions the given deliberation processes can achieve these properties. The first one should be easy to get: Since the orderings are asymmetric, the lexicographic upgrade policy will be identified with the drastic upgrade policy \[ \] which would lead to unanimity or oscillation. If unanimity is reached, we have single-peakedness trivially. In case of oscillation, we need to make sure that whichever be the agents included in oscillation for each agent, the least preferred world can only vary between (at most) two of the given worlds. For the layered upgrade ordering we will have a more interesting property of ensuring the weakest layer to contain the same two elements always. The second condition is more tricky, but once again can be broken down into several sub-conditions in the layered case. We leave the formal work for the main paper. We conclude with mentioning the known fact that getting single-peaked preferences via deliberation would pave the way of using aggregation rules which will lead to collective decision making avoiding the impossibility results of Arrow and others.

References


Dynamic Term-Modal Logic Revisited

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Term-modal logic, is a first-order modal logic, where the modalities are indexed by terms of first-order language. This allows quantification over modalities and such. Dynamic term-modal logic further generalizes this by allowing PDL-like constructions over these term-modalities. In the talk we will examine to what extent this provides new insights to well-known philosophical problems having to do with first-order modal logic: existence, identity, quantification, de-re versus de dicto, etc.
Cut-free and Analytic Sequent Calculus of First-Order Intuitionistic Epistemic Logic

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[Artemov and Protopopescu, 2016] gave an intuitionistic epistemic logic based on a verification reading of the intuitionistic knowledge in terms of Brouwer-Heyting-Kolmogorov interpretation. According to this interpretation, a proof of $A \supset B$ is a construction such that when a proof of $A$ is given, a proof of $B$ can be constructed. [Artemov and Protopopescu, 2016] proposed that a proof of a formula $KA$ (read “it is known that $A$”), is the conclusive verification of the existence of a proof of $A$. Then $A \supset KA$ expresses that, when a proof of $A$ is given, the conclusive verification of the existence of the proof of $A$ can be constructed. Since a proof of $A$ itself is the conclusive verification of the existence of a proof of $A$, they claim that $A \supset KA$ is valid. But $KA \supset A$ (usually called factivity or reflection) is not valid, since the verification does not always give a proof. They provided a Hilbert system of intuitionistic epistemic logic $IEL$ as the intuitionistic propositional logic plus the axioms schemes $K(A \supset B) \supset KA \supset KB$, $A \supset KA$ and $\neg KA \perp$. Moreover they gave $IEL$ the following Kripke semantics. We say that $M = (W, \leq, R, V)$ is a Kripke model for $IEL$ if $(W, \leq, V)$ is a Kripke model for intuitionistic propositional logic and $R$ is a binary relation such that $R \subseteq \leq$, $\leq; R \subseteq R$ and $R$ satisfies the seriality. Then $KA$ is true on a state $w$ of $M$ if and only if for any $v$, $wRv$ implies $A$ is true in $v$ of $M$. [Artemov and Protopopescu, 2016] also proved that their Hilbert system is sound and complete.

The study of $IEL$ also casts light on the study of the knowability paradox. The knowability paradox, also known as the Fitch-Church paradox, states that, if we claim the knowability principle: every truth is knowable ($A \supset \Box KA$), then we are forced to accept the omniscience principle: every truth is known ($A \supset KA$) [Fitch, 1963]. This paradox is commonly recognized as a threat to Dummett’s semantic anti-realism. It is because the semantic anti-realists claim the knowability principle but they do not accept the omniscience principle. However, as Dummett admitted that he had taken some of intuitionistic basic features as a model for an anti-realist view [Dummett, 1978, p.164], it is reasonable to consider an intuitionistic logic as a basis. In this sense, if we employ BHK-interpretation of $KA$ as above to accept the $IEL$ in the study of the knowability paradox, $A \supset KA$ becomes valid and the knowability paradox is trivialized.

Proof-theoretical studies of $IEL$ have been investigated. In Krupski and Yatmanov [2016], the sequent calculus of $IEL$ has been given, though an inference rule corresponding to $KA \supset \neg
\neg A$ in their system for $IEL$ does not satisfy a desired syntactic property, i.e., the subformula property. In Protopopescu [2015], a Gödel-McKinsey-Tarski translation from the intuitionistic epistemic propositional logic to the bimodal expansion of the classical modal logic $S4$ has been studied.

In this paper, we study the first-order expansion $QIEL$ of intuitionistic epistemic logic of $IEL$. Artemov and Protopopescu mentioned that the notion of the intuitionistic knowledge
capture both mathematical knowledge and empirical knowledge. When we consider the mathematical knowledge, quantifiers become inevitable. Moreover when we are concerned with the empirical knowledge, we recall that Hintikka had given arguments for first-order epistemic logic in Hintikka [2005]. He mentioned that if we want to deal with the locutions like “knows who,” “knows when,” “knows where,” we can translate these expressions into a language with quantifiers. For example, about “who” we can have variables ranging over the human being, about “where” over the location in space. In this sense, our first-order expansion can provide a fundamental basis when we concern the intuitionistic mathematical and empirical knowledge.

We give the first-order expansion of IEL as QIEL. An expanded Kripke model $M = (W, \leq, R, D, I)$ is obtained by adding $D$ and $I$ into the Kripke model for IEL. Here $D$ is a function which assigns a nonempty domain $D(w)$ to $w \in W$ such that, for any $w, v \in W$, if $w \leq v$ then $D(w) \subseteq D(v)$. Moreover $I$ is an interpretation such that $I(c) \in D(w)$ for all $w \in W$ for any constant symbol $c$ and $I(P, w) \subseteq D(w)^n$ for every $w \in W$ and every $n$-arity predicate $P$ such that if $u \leq v$ then $I(P, u) \subseteq I(P, v)$ for all $u, v \in W$.

We also propose the sequent calculus for QIEL. The sequent calculus for IEL has been given by Krupski and Yatmanov [2016]. Their sequent calculus is obtained from the propositional part of Gentzen’s sequent calculus LJ (with structural rules of weakening and contraction) for the intuitionistic logic plus the following two inference rules on the knowledge operator:

$$
\frac{\Gamma_1, \Gamma_2 \Rightarrow A}{\Gamma_1, K\Gamma_2 \Rightarrow KA} \quad \frac{\Gamma \Rightarrow \bot}{\Gamma \Rightarrow F.} \quad (K\text{IEL}) \quad (U)
$$

where a sequent $\Gamma \Rightarrow A$ (where $\Gamma$ is a finite multiset of formulas) can be read as “if all of $\Gamma$ hold then $A$ holds.” They established the cut-elimination theorem of the calculus, i.e., a derivable sequent in their system is derivable without any application of the following cut rule:

$$
\frac{\Gamma \Rightarrow B \quad B, \Sigma \Rightarrow \Delta}{\Gamma, \Sigma \Rightarrow \Delta} \quad (\text{Cut})
$$

where $\Delta$ contains at most one formula. It is remarked, however, that this system does not enjoy the subformula property. That is, in the rule of $(U)$, we have a formula $K\bot$ which might not be a subformula of a formula in the lower sequent of the rule $(U)$.

This talk gives a new cut-free and analytic sequent calculus $\mathcal{G}(QIEL)$ of the first-order intuitionistic epistemic logic, which is obtained from adding the following rule $(K\text{IEL})$ into Gentzen’s LJ with quantifiers:

$$
\frac{\Gamma_1, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, K\Gamma_2 \Rightarrow KA} \quad (\text{KIEL})
$$

where $\Delta$ contains at most one formula. This rule is equivalent to the rules from Krupski and Yatmanov [2016] in a propositional setting. Moreover it is easy to see that $(K\text{IEL})$ satisfies the subformula property.

Let $\mathcal{G}^-(QIEL)$ be the system $\mathcal{G}(QIEL)$ without the cut rule. By the standard syntactic argument, we can establish the following fundamental proof-theoretic result.

**Theorem 1 (Cut-Elimination)** If $\mathcal{G}(QIEL) \vdash \Gamma \Rightarrow \Delta$ then $\mathcal{G}^-(QIEL) \vdash \Gamma \Rightarrow \Delta$.

**Corollary 1 (Disjunction Property, Existence Property, Craig Interpolation Theorem)** As a corollary of cut-elimination theorem we have:

1. If $\Rightarrow A \lor B$ is derivable in $\mathcal{G}(QIEL)$, then either $\Rightarrow A$ or $\Rightarrow B$ is derivable in $\mathcal{G}(QIEL)$. 


2. For any formula of the form $\exists x A$, if $\vdash \exists x A$ is derivable in $G(QIEL)$ then there exists a term $t$ such that $\vdash A(t/x)$ is derivable in $G(QIEL)$.

3. If $\vdash A \supset B$ is derivable in $G(QIEL)$, then there exists a formula $C$ such that $\vdash A \supset C$ and $\vdash C \supset B$ are also derivable in $G(QIEL)$, and all free variables, predicate symbols and constant symbols of $C$ are shared by both $A$ and $B$.

Given a sequent $\Gamma \Rightarrow \Delta$, $\Gamma^*$ denotes the conjunction of all formulas in $\Gamma$ ($\Gamma^* \equiv \top$ if $\Gamma$ is empty) and $\Delta^*$ denotes the unique formula in $\Delta$ if $\Delta$ is non-empty; it denotes $\bot$ otherwise. We say that a sequent $\Gamma \Rightarrow \Delta$ is valid in a class $M$ of models (denoted by $M \models \Gamma \Rightarrow \Delta$), if $\Gamma^* \supset \Delta^*$ is satisfied in any states of any Kripke models.

**Theorem 2 (Soundness of $G(QIEL)$)** Let $\Gamma \Rightarrow \Delta$ be any sequent. If $G(QIEL) \vdash \Gamma \Rightarrow \Delta$ then $M \models \Gamma \Rightarrow \Delta$.

With the method from Hermant [2005], we prove the following:

**Theorem 3 (Completeness of $G^{-}(QIEL)$)** Let $\Gamma \Rightarrow \Delta$ be a sequent. If $M \models \Gamma \Rightarrow \Delta$ then $G^{-}(QIEL) \vdash \Gamma \Rightarrow \Delta$.

**Corollary 2** The following are all equivalent.

1. $M \models A,$
2. $G(QIEL) \vdash \Rightarrow A,$
3. $G^{-}(QIEL) \vdash \Rightarrow A,$

In particular, we can also prove the cut elimination theorems semantically by Theorem 2 and Theorem 3.

**References**


1 Introduction

“Distributed knowledge” is a notion developed in the community of multi-agent epistemic logic [1, 8]. In [1, p. 3], the notion is explained informally as follows:

A group has distributed knowledge of a fact $\varphi$ if the knowledge of $\varphi$ is distributed among its members, so that by pooling their knowledge together the members of the group can deduce $\varphi$, even though it may be the case that no member of the group individually knows $\varphi$.

For example, a group consisting of $a$ and $b$ has distributed knowledge of a fact $q$, when $a$ knows $p \rightarrow q$ and $b$ knows $p$. Formally, “A group $G$ has distributed knowledge of a fact $\varphi$.” is written as “$D_G\varphi$”, whose meaning is usually defined in a Kripke model. Let $W$ be a possibly countable set of states, $\text{Agt}$ be a finite set of agents, $(R_a)_{a \in \text{Agt}}$ be a family of binary relations on $W$, indexed by agents, and $V$ be a valuation function $\text{Prop} \rightarrow \mathcal{P}(W)$, where $\text{Prop}$ is a countable set of propositional variables. We call a tuple $M = (W, (R_a)_{a \in \text{Agt}}, V)$ a (Kripke) model. For a group $G \subseteq \text{Agt}$, satisfaction of $D_G\varphi$ at a state $w$ in a model $M$ is defined as follows:

$$M, w \models D_G\varphi \iff \text{for all } v, \text{ if } (w, v) \in \bigcap_{a \in G} R_a \text{ then } M, v \models \varphi$$

It is clear from the definition that the operator $D_{\{a\}}$ behaves the same as $K_a$, a box-like operator for an agent $a$, usually defined in multi-agent epistemic logic. Therefore, we do not introduce $K_a$-like operator as a primitive one in this abstract.

The study of distributed knowledge so far is mainly model-theoretic [16, 13, 4, 15] and proof-theoretic study has been not so active. As far as we know, existing sequent calculi for logic with distributed knowledge are presented only in [6] and [5]. The former contains a natural G3-style formalization, in which each formula has a label and the latter contains Getzen-style and Kanger-style sequent calculus for logic with distributed knowledge operator which is simpler than the one we are interested in, in that the operator is not parameterized by group $G$.

We propose Gentzen-style sequent calculi (without label) for five kinds of multi-agent epistemic propositional logics with distributed knowledge operators, parameterized by
groups, which are reasonable generalization of sequent calculi for basic modal logic and prove the cut elimination theorem for four of them. Using a method described in [7], Craig’s interpolation theorem is also established for the four system, in which not only condition of propositional variables but also that of agents is taken into account. This is a new result for logic for distributed knowledge, as far as we know.

In the following, we briefly sketch our proof systems, and then state and comment on the theorems we have on the systems.

2 Proof Systems

We denote a finite set of agents by \( \text{Agt} \). We call a nonempty subset of \( \text{Agt} \) “group” and denote it by \( G, H \), etc. Let \( \text{Prop} \) be a countable set of propositional variables and \( \text{Form} \) be the set of formulas defined inductively by the following clauses (\( \lor \) and \( \neg \) are defined in the same way as the classical propositional logic):

\[
\text{Form} \ni \varphi ::= p \in \text{Prop} \mid \bot \mid \top \mid \varphi \land \varphi \mid \varphi \rightarrow \varphi \mid D_G \varphi
\]

First, we explain known Hilbert-style axiomatization for logics with \( D_G \) operator (for detail, refer to [1]). The following are axioms for the logics:

- (Taut) all instantiations of propositional tautology
- (Incl) \( D_G \varphi \rightarrow D_H \varphi \) \((G \subseteq H)\)
- (K) \( D_G (\varphi \rightarrow \psi) \rightarrow D_G \varphi \rightarrow D_G \psi \)
- (T) \( D_G \varphi \rightarrow \varphi \)
- (4) \( D_G \varphi \rightarrow D_G D_G \varphi \)
- (5) \( \neg D_G \varphi \rightarrow D_G \neg D_G \varphi \)

An axiom system \( \text{H(K)}_D \) (\( \text{H(KT)}_D, \text{H(K4)}_D \), \( \text{H(S4)}_D \), and \( \text{H(S5)}_D \)) is a collection of the inference rules of modus ponens (“from \( \varphi \rightarrow \psi \) and \( \varphi \) infer \( \psi \)” and necessitation (“from \( \varphi \) infer \( D_G \varphi \”)), axioms (Taut) and (Incl) (common to all the five systems), and (an) axiom(s) (K) ((K) and (T); (K) and (4); (K), (T), and (4); and (K), (T), and (5), respectively).

We now propose our sequent calculi for the logics for distributed knowledge. To the ordinary LK system [2, 3], we add some of the following rules for each logic:

\[
\frac{\varphi_1, \cdots, \varphi_n \Rightarrow \psi \quad \left( \bigcup_{i=1}^n G_i \subseteq G \right)}{D_{G_i} \varphi_1, \cdots, D_{G_n} \varphi_n \Rightarrow D_G \psi} \quad (D_G)
\]

\[
\frac{\varphi, \Gamma \Rightarrow \Delta \quad (D_G \Rightarrow)}{D_{G \varphi}, \Gamma \Rightarrow \Delta}
\]
Theorem 1 (Equivalence between Hilbert-style and Gentzen-style axiomatization)

Let $X$ be any of $K_D$, $KT_D$, $K4_D$, $S4_D$, and $S5_D$. Then, the following hold.

1. If $\vdash_{H(X)} \varphi$, then $\vdash_G(X) \Rightarrow \varphi$

2. If $\vdash_G(X) \Gamma \Rightarrow \Delta$, then $\vdash_{H(X)} \bigwedge \Gamma \Rightarrow \bigvee \Delta$

We have the cut elimination theorem for our sequent calculi, except for $G(S5_D)$.

Theorem 2 (Cut elimination) Let $X$ be any of $K_D$, $KT_D$, $K4_D$, and $S4_D$. Then, the following holds: If $\Gamma \vdash_{G(X)} \Gamma \Rightarrow \Delta$, then $\Gamma \vdash_{G^{-}(X)} \Gamma \Rightarrow \Delta$, where $G^{-}(X)$ denotes a system “$G(X)$ minus cut rule”.

Flexibility of choice of groups occurring in the left side of the lower sequent in the rule $(D_G)$ and the three $(\Rightarrow D_G)$-type rules is a key to the result. The reason why cut elimination theorem does not hold for $G(S5_D)$ is that the sequent calculus for basic $S5$, on which $G(S5_D)$ is based, is not cut-free [9].

As an application of the cut elimination theorem, Craig’s interpolation theorem can be derived, using a method described in [7]. (Application of the method to basic modal logic is also described in [10].)

Theorem 3 (Craig’s interpolation theorem) Let $X$ be any of $K_D$, $KT_D$, $K4_D$, and $S4_D$. Given that $\vdash_{G(X)} \varphi \Rightarrow \psi$, there exists a formula $\chi$ satisfying the following conditions:

1. $\vdash_{G(X)} \varphi \Rightarrow \chi$ and $\vdash_{G(X)} \chi \Rightarrow \psi$. 

The sequent calculus $G(K_D)$, $G(KT_D)$, $G(K4_D)$, $G(S4_D)$, and $G(S5_D)$ is LK with the rule(s) $(D_G)$, $(D_G \Rightarrow)$, $(\Rightarrow D_G)$, $(\Rightarrow D_G)$, and $(\Rightarrow D_G)$, respectively.

The idea underlying the rule $(D_G)$ is similar to that of an inference rule called “R12” described in [12, section 4]. Our calculi $G(KT_D)$, $G(K4_D)$, $G(S4_D)$, and $G(S5_D)$ are constructed based on the known sequent calculus for $KT$, $K4$, $S4$, and $S5$ (surveyed in [11, 14]).

We note that for any logic $X$ of the logics described above, $H(X)$ and $G(X)$ are equivalent in derivability of formulas, and hence that each system $G(X)$ deserves its own name.

Let $X$ be any of $K_D$, $KT_D$, $K4_D$, $S4_D$, and $S5_D$. Then, the following hold.

1. If $\vdash_{H(X)} \varphi$, then $\vdash_G(X) \Rightarrow \varphi$

2. If $\vdash_G(X) \Gamma \Rightarrow \Delta$, then $\vdash_{H(X)} \bigwedge \Gamma \Rightarrow \bigvee \Delta$

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1. If $\vdash_{H(X)} \varphi$, then $\vdash_G(X) \Rightarrow \varphi$

2. If $\vdash_G(X) \Gamma \Rightarrow \Delta$, then $\vdash_{H(X)} \bigwedge \Gamma \Rightarrow \bigvee \Delta$

We have the cut elimination theorem for our sequent calculi, except for $G(S5_D)$.
2. \( V(\chi) \subseteq V(\varphi) \cap V(\psi) \), where \( V(\rho) \) denotes the set of propositional variables occurring in formula \( \rho \).

3. \( A(\chi) \subseteq A(\varphi) \cap A(\psi) \), where \( A(\rho) \) denotes the set of agents occurring in formula \( \rho \).

We note that not only the condition for propositional variables but also the condition for agents can be satisfied.

References


