SOCREAL 2019

Abstracts

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About SOCREAL 2019

Since the last years of the 20th century, a number of attempts have been made in order to model various aspects of social interaction among agents including individual agents, organizations, and individuals representing organizations. The aim of SOCREAL Workshop is to bring together researchers working on diverse aspects of such interaction in logic, philosophy, ethics, computer science, cognitive science and related fields in order to share issues, ideas, techniques, and results.

The earlier editions of SOCREAL Workshop has been held in March 2007, March 2010, October 2013, and October 2016. Building upon the success of these editions, its fifth edition will be held from 15 November till 17 November 2019 under the auspices of Philosophy and Ethics Laboratory at Faculty of Humanities and Human Sciences, Hokkaido University, CAEP (Center for Applied Ethics and Philosophy) at Faculty of Humanities and Human Sciences, Hokkaido University, and LOG-UCI (An interdisciplinary study of the logical dynamics of the interaction between utterances and social contexts), a research project funded by JSPS (JSPS KAKENHI Grant Number JP 17H02258).

SOCREAL 2019 will consist of keynote lectures by invited speakers and presentations of submitted papers. Researchers from various fields, including logic, philosophy, ethics, computer science, cognitive science were invited to submit an extended abstract (up to two thousand words) by 30 August 2019. We received 16 abstracts and each of them was peer-reviewed by the program committee of the workshop. Here you will find abstracts of 8 accepted papers and 4 keynote lectures.

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The Program & The Table of Contents

13:05 – 13:10 [Closing] W409
Katsuhiko Sano, Hokkaido University, Japan

What is and what might have been

Jeremy Seligman
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A central theme of Arthur Prior's 1956 Locke Lectures on "Time and Modality" is the trouble raised for logic by the contingency of existence. His tentative solution was to abandon bivalence with the curious and poorly-understood System Q. I present a simple alternative that has been overlooked by the mainstream development of modal predicate logic, and which provides an easy way of combing actualism (the view that all that exists actually exists) with contingentism (the view that there might have been things other than there are). Timothy Williamson's argument for the necessity of existence is then re-examined. For those familiar with my work, this relates directly to "common sense" modal predicate logic, but focusses more on the philosophical context.

A Sequent Calculus for K-restricted Common Sense Modal Predicate Logic

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Abstract

In recent years, Common sense Modal Predicate Calculus (CMPC) has been proposed by J. van Benthem in [4, pp. 120–121] and further developed by J. Seligman in [1, 3, 2]. It allows us to 'take $\exists$ to mean just "exists" while denying the Constant Domain thesis' [1, p. 8].[1] This is done in terms of *talking about only things in each world in which they exist*. From a proof-theoretical view, the Hilbert-style system for CMPC given by Seligman is a system for modal predicate logic **S5** *which has the following axiom* K_{inv} *instead of axiom* K:

$$\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi) \quad \textit{provided that all free variables in } \varphi \textit{ are free variables in } \psi.$$

It is quite interesting because it might make a clean sweep of all philosophical discussions on possible world semantics between actualists and possibilists. However, neither van Benthem nor Seligman have developed K-restricted CMPC and expansions of the logic with some well known axioms. Moreover, proof-theoretic studies for such logics have not been done yet.

In this talk, I shall propose a sequent calculus for K-restricted CMPC. The main mathematical contributions of this talk are the completeness result (Theorem 1) and cut elimination theorem (Theorem 2) for the calculus. If time allows I shall also introduce sequent calculi for K-restricted CMPC with T axiom and D-like axioms. In what follows, I will outline the contents of this talk.

The language $\mathcal{L}$ of K-restricted Common sense Modal Predicate Calculus **CK** consists of a countably infinite set $\mathsf{Var} = \{x, y, \ldots\}$ of variables, a countably infinite set $\mathsf{Pred} = \{P, Q, \ldots\}$ of predicate symbols each of which has a fixed finite

[1] The Constant Domain thesis is a thesis that '[e]very possible world has exactly the same objects as every other possible world.' [1, p. 5]

arity, and logical symbols, $\bot, \supset, \Box, \forall$. The set Form of formulas of $\mathcal{L}$ is defined recursively as follows:

$$\mathsf{Form} \ni \varphi ::= Px_1 \dots x_n \mid \bot \mid (\varphi \supset \varphi) \mid \forall x\varphi \mid \Box\varphi$$

where P is a predicate symbol with arity n and $x, x_1, \dots, x_n$ are variables. The other connectives are defined as usual. We also define the sets $\mathsf{FV}(\varphi)$ and $\mathsf{FV}(\Gamma)$ of free variables in a formula φ and a set Γ of formulas, respectively, as usual.

Semantics for **CK** is given as follows. A frame is a tuple (W, R, D), where W is a nonempty set; R is a binary relation on W; D is a W-indexed family $\{D_w\}_{w\in W}$ of nonempty sets. Thus R does not need to satisfy the *inclusion requirement*: if wRv then $D_w \subseteq D_v$. A model is a tuple (F, V), where F is a frame and V is a valuation that maps each world w and each predicate P to a subset $V_w(P)$ of D_w. An *assignment* α is a partial function from variables to entities and $\alpha(x|d)$ stands for the same assignment as α except for assigning d to x. In addition to these notions, we follow [1, p. 15] and say that a formula φ is an α_w*-formula* if $\alpha(x) \in D_w$ for any variable $x \in \mathsf{FV}(\varphi)$. Then, similarly as in [1, pp. 15–16], the satisfaction relation and validity are defined as follows.

Definition 1 (Satisfaction relation)**.** Let M be a model, α be an assignment, and w be a world in W. The *satisfation relation* $M, \alpha, w \models \varphi$ between M, α, w and an α_w-formula φ is defined as follows:

$$\begin{array}{lll}
M, \alpha, w \models Px_1 \dots x_n & \text{iff} & (\alpha(x_1), \dots, \alpha(x_n)) \in V_w(P) \\
M, \alpha, w \not\models \bot & & \\
M, \alpha, w \models \psi \supset \gamma & \text{iff} & M, \alpha, w \models \psi \text{ implies } M, \alpha, w \models \gamma \\
M, \alpha, w \models \forall x\psi & \text{iff} & M, \alpha(x|d), w \models \psi \quad \text{for any } d \in D_w \\
M, \alpha, w \models \Box\psi & \text{iff} & M, \alpha, v \models \psi \\
 & & \text{for any } v \text{ such that } wRv \text{ and } \psi \text{ is an } \alpha_v\text{-formula}
\end{array}$$

Definition 2 (Validity)**.** Let $\Gamma \cup \{\varphi\}$ be a set of formulas. We say that φ *is valid in a frame* if for any model M based on the frame, assignment α and world w such that φ is an α_w-formula, $M, \alpha, w \models \varphi$. We also say that φ *is valid in a class of frames* if φ is valid in all frames in the class.

The following propositions that Seligman proves in [1, pp. 16–17] are noteworthy[2].

Proposition 3 (Converse Barcan formula)**.** A formula $\Box\forall x\varphi \supset \forall x\Box\varphi$ is valid in the class of all frames.

Proof. Fix any model M, assignment α, world w such that $\Box\forall x\varphi \supset \forall x\Box\varphi$ is an α_w-formula. Suppose $M, \alpha, w \models \Box\forall x\varphi$ and fix any element $d \in D_w$, any world v such that wRv and φ is an $\alpha(x|d)_v$-formula. We show $M, \alpha(x|d), v \models \varphi$. Since

[2] Strictly speaking, he considers the dual formulas of those in Proposition 3,4.

$\mathsf{FV}(\forall x\varphi) \subseteq \mathsf{FV}(\varphi)$ and φ is an $\alpha(x|d)_v$-formula, we have that $\forall x\varphi$ is an $\alpha(x|d)_v$-formula and thus that $\forall x\varphi$ is an α_v-formula. Hence we get $M, \alpha, v \models \forall x\varphi$ so $M, \alpha(x|d), v \models \varphi$. ■

Proposition 4. A formula $\forall x\Box\varphi \supset \Box\forall x\varphi$ is not valid in the class $\mathbb{F}$ of all frames $F = (W, R, D)$ such that R is an equivalence relation.

Proof. Consider a model $M = (W, R, D, V)$, where $W = \{0, 1\}$; $R = W \times W$; $D_0 = \{a\}$ and $D_1 = \{b\}$; $V_0(P) = \{a\}$ and $V_1(P) = \varnothing$ for some predicate symbol P with arity 1, and $V_i(Q) = \varnothing$ for the other predicate symbols Q with arity n. Then, we can establish $M, \alpha, 0 \models \forall x\Box Px$ but $M, \alpha, 0 \not\models \Box\forall x Px$. Therefore, $\forall x\Box\varphi \supset \Box\forall x\varphi$ is not valid in $\mathbb{F}$. ■

Given finite multisets Γ, Δ of formulas, we call an expression $\Gamma \Rightarrow \Delta$ a sequent. Then a sequent calculus G(**CK**) for **CK** is given in Table 1. The rule $\Box_{inv}$ in it plays roles of axiom K_{inv} and the necessitation rule in the Hilbert-style system for CMPC given by Seligman. The notion of a derivation in G(**CK**) is defined as usual.

Table 1: A Sequent Calculus G(**CK**) for **CK**

Initial Sequents	
$\varphi \Rightarrow \varphi$	$\bot \Rightarrow$
Structural Rules	
$\dfrac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \varphi} \ (\Rightarrow w)$	$\dfrac{\Gamma \Rightarrow \Delta}{\varphi, \Gamma \Rightarrow \Delta} \ (w \Rightarrow)$
$\dfrac{\Gamma \Rightarrow \Delta, \varphi, \varphi}{\Gamma \Rightarrow \Delta, \varphi} \ (\Rightarrow c)$	$\dfrac{\varphi, \varphi\Gamma \Rightarrow \Delta}{\varphi, \Gamma \Rightarrow \Delta} \ (c \Rightarrow)$
$\dfrac{\Gamma \Rightarrow \Delta, \varphi \quad \varphi, \Theta \Rightarrow \Sigma}{\Gamma, \Theta \Rightarrow \Delta, \Sigma} \ (Cut)$	
Logical Rules	
$\dfrac{\varphi, \Gamma \Rightarrow \Delta, \psi}{\Gamma \Rightarrow \Delta, \varphi \supset \psi} \ (\Rightarrow \supset)$	$\dfrac{\Gamma \Rightarrow \Delta, \varphi \quad \psi, \Theta \Rightarrow \Sigma}{\varphi \supset \psi, \Gamma, \Theta \Rightarrow \Delta, \Sigma} \ (\supset \Rightarrow)$
$\dfrac{\Gamma \Rightarrow \Delta, \varphi(y/x)}{\Gamma \Rightarrow \Delta, \forall x\varphi} \ (\Rightarrow \forall^{\dagger})$	$\dfrac{\varphi(t/x), \Gamma \Rightarrow \Delta}{\forall x\varphi, \Gamma \Rightarrow \Delta} \ (\forall \Rightarrow)$
$\dfrac{\Gamma \Rightarrow \varphi}{\Box\Gamma \Rightarrow \Box\varphi} \ (\Box^{\ddagger}_{inv})$	
†: y does not occur in $\Gamma, \Delta, \forall x\varphi$.	‡: $\mathsf{FV}(\Gamma) \subseteq \mathsf{FV}(\varphi)$.

We also say that *a sequent* $\Gamma \Rightarrow \Delta$ *is valid if* $(\gamma_1 \wedge \cdots \wedge \gamma_m) \supset (\delta_1 \vee \cdots \vee \delta_n)$ *is valid*, where $\Gamma = \{\gamma_1, \ldots, \gamma_m\}$ and $\Delta = \{\delta_1, \ldots, \delta_n\}$. Then, the following theorems hold under the settings above.

Theorem 1 (Completeness)**.** Let $\Gamma \cup \{ \varphi \}$ be a set of formulas. If $\Gamma \Rightarrow \varphi$ is valid in the class of all frames, then $\Gamma \Rightarrow \varphi$ is derivable in G(**CK**).

Theorem 2 (Cut elimination)**.** Let Γ, Δ be finite multisets of formulas. If $\Gamma \Rightarrow \Delta$ is derivable in G(**CK**), then it is also derivable in G(**CK**) without any application of *Cut*.

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[2] Jeremy Seligman. Common Sense Modal Predicate Logic. Presentation, 2017. Non-classical Modal and Predicate Logics: The 9th International Workshop on Logic and Cognition, Guangzhou, China (4th Dec. 2017).

[3] Jeremy Seligman. Common Sense Modal Predicate Logic. 2nd draft, November 2017.

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There Are no Empty Groups

Extended Abstract

David Strohmaier, University of Cambridge, davidstrohmaier92@gmail.com

Mereological group ontology analyses groups as wholes composed of physical parts. Such mereological accounts have enjoyed renewed popularity in recent years (Sheehy 2003, 2006a, 2006b; Ritchie 2013, 2015, 2018; Hawley 2017; Strohmaier 2018), but they have not yet addressed a crucial challenge. How can such accounts respond to proposed instances of groups without members or any other parts? The literature suggests that the Supreme Court can persist when all judges serving on it resign simultaneously (Epstein 2015) and that corporations do not require any physical parts (Smith 2003).

If these supposed examples of empty groups held up to scrutiny, it would undermine the mereological approach to group ontology. Groups cannot be wholes without having parts. In the present paper, I show how mereological accounts can face this challenge and dispel the force of these counterexamples.

To specify neo-Aristotelian mereology, I use Fine's hylomorphic approach as laid out in his 1999 paper *Things and their Parts* (see also Uzquiano 2018). In this text he introduces rigid and variable embodiments. A rigid embodiment has a constituent structure that can be represented as "<a, b, c…/R>", where a, b, c… are objects, R is a relation and the "/" denotes the primitive relation of rigid embodiment. The objects as well as the relations are timeless parts of the whole. According to Fine, a ham sandwich can be analysed as a rigid embodiment of three timeless parts, two pieces of bread and a ham piece. The relation R would be the ham being between the bread pieces.

Variable embodiments are represented as "f=/F/", where F is a principle and there are a series of manifestations f_t. F picks out the objects as manifestations of the embodiment and can be thought of as a function from world-times to things (Fine 1999: 69). The water in the Thames can be analysed as such a variable embodiment where a principle, presumably related to the riverbed, picks out various quantities of water as manifestations. These manifestations are a part of the variable embodiment at the time of manifestations and the embodiment exists whenever a manifestation of it exists.

Fine analyses a car using both resources. It is a variable embodiment picking out a rigid embodiment of a motor, a chassis and other parts at a world-time. The combination of variable and rigid embodiments allows the car to undergo changes in parts, while also capturing that there is a structuring relation to the whole. At each point in time during which the car exists it is manifested by a rigid embodiment of parts, but over time these can be different embodiments. The wheels might be replaced, which entails a change of rigid embodiments, while the car persists.

Following Fine, one can analyse groups the same way as cars (see also Uzquiano 2018). A reading group is a variable embodiment /G/ manifested by rigid embodiments, that is the manifestation of the group at a world-time is a whole with parts standing in a particular relation to each other. If Rory, Paris, and Doyle are the members of the reading group today, there is a rigid embodiment <Rory, Paris, Doyle/R>. Should at a later point Marty join the reading group, then another rigid embodiment <Rory, Paris, Doyle, Marty/R> would manifest the group at that later world-time. The relation could also change, for example, because Paris appoints herself successfully leader of the reading group. Again, that would lead to a different embodiment <Rory, Paris, Doyle, Marty/R'> manifesting the group at that point. In virtue of being a variable embodiment the group can persist through all these changes of rigid embodiments that manifest it. The theory, however, requires physical manifestations of groups at each point of their existence.

I focus in my discussion on the case of the Supreme Court. It has been a prime example in group ontology (see Uzquiano 2004) and Brian Epstein has suggested it as a candidate for a group which exists at some world-time without members. In his book the Ant Trap, he raises the question whether the Supreme Court existed, "with no members, when the Constitution was ratified, or when the Judiciary Act of 1789 was passed?" (Epstein 2015: 158) In an illustrative example, Epstein also assumes that "once [the Supreme Court] has come to exist, it continues to exist in perpetuity" (Epstein 2015: 159). Accordingly, the Supreme Court will also persist without any members.

This proposal of an empty group is motivated within Epstein's approach to social ontology. According to Epstein, facts about the Supreme Court are not exclusively grounded in facts about its members and some of them are not even partially grounded in facts about the members. The Supreme Court has certain powers, such as revoking the decisions of lower court, independently of its members and one might propose that the Supreme Court has these powers even when it does not have any members. But the Supreme Court can have such powers only while it exists.

As can be seen, Epstein raises multiple questions, which can receive different responses. For the present purposes, the decisive questions are (1) whether the Supreme Court can exist at any world-time without having members; (2) whether the Supreme Court can exist at any world-time without having parts; and (3), if the response to either of these questions is affirmative, whether this is compatible with a mereological account of groups.

One simple solution I will neglect here is to argue that the Supreme Court has other material parts than its members, e.g. the Supreme Court Building. While this would address the difficulty in the specific case of the Supreme Court, it would be easy to adapt the supposed counterexamples to evade this response. It would be too much to require that all candidate groups have non-member material parts.

Instead, I propose, the case of the Supreme Court is best dealt with by comparing it with the case of the US President. The US Presidency was also created prior to the first President being sworn in and the role of the Presidency would also continue if the President unexpectedly died and before anyone else replaced them. Nonetheless, in neither case would one say that there exists a US President although there is no person who is the President. Since the US presidency does not confer immortality, there is no President persisting throughout the history of the United States.

What would persist, I suggest, is a whole that allows for someone to be sworn in and become the US President under appropriate conditions. The federal government is a mereological composite that typically manifests as a rigid embodiment including a President. Likewise, prior to any Supreme Court justice being confirmed, what exists is a whole which allows for a group, the actual Supreme Court, to fill a specific role. The Judiciary Act of 1789 established the federal judiciary and thereby changed the federal government to enable groups to serve as the Supreme Court.

The proposal of groups without members rests on a confusion between type and token. The type Supreme Court can exist in the absence of any justices serving on the court, because it can exist in the absence of a token group. The type Supreme Court has certain powers lacking a material token, in the same way the US President has pardoning power even when there is no token President. It is a power conferred to tokens in virtue of being to a certain type. As soon as we remind ourselves of the type-token distinction, there are no special mysteries here.

What makes the confusion between types and tokens so tempting is that there can always only be one realising token of the type Supreme Court. But that is also the case for the Presidency, where only one person can be the US President at a world-time. Nonetheless, no one would suppose that the US Presidents can persist independently of a physical object and there exists a separate term for the type.

Having accounted for the intuitions in the case of the Supreme Court, I also address how this solution can be generalised to other groups, including limited liability corporations. As a consequence of my proposal, one needs to distinguish a specific type for each corporation from its tokens. There is a Microsoft type in addition to a Microsoft token. Such a multiplication of types might appear counter-intuitive, but I propose it is not unusual within the field of social ontology. We can set up kinds such as the US Presidency without much ado (cf. Epstein 2015).

I will also address the problem that some ordinary discourse seems to presuppose the existence of a group token despite the lack of physical parts. One might say of a corporation lacking any physical embodiment that it still owns abstract assets such as patents. My account is committed to not take these utterances at face value. I will discuss how such property ascriptions can be re-interpreted and propose that we understand them as falling under a subjunctive conditional; that is if the type were to be instantiated, the token would have these properties.

By dispelling examples of groups seemingly lacking members, my paper resolves one of the few remaining challenges to mereological accounts of groups. Consequently, group mereology exerts great appeal and can serve as the foundation of future research, empirical as well as philosophical.

Count-as Conditionals in Channel Theory

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Searle's theory of institutional facts seems to be of vital importance in understanding social reality. According to his summary, the following "rather simple set of equivalences and logical implications" hold (2010, p.23):

institutional facts = status function → deontic powers → desire-independent reason for action.

Status functions are functions people, things, processes, and so on have, not in virtue of their "sheer physical features" but in virtue of certain status they are recognized to have, and deontic powers are things like "rights, duties, obligations, requirements, permissions, authorizations, entitlements, and so on" (2010, p. 9). In simple cases, institutional facts are generated in accordance to the constitutive rules of the following form (Searle 1969, pp. 51-52, 1995, pp. 28, 41-51):

X counts as Y in context C,

where the term Y stands for the relevant status which the status function accompanies.

Recently several attempts to capture the logic of count-as conditionals have been made in the deontic logic literature. For example, Jones and Sergot (1996) includes the following principle as one of the logical principles for the logic of count-as (Jones and Sergot, 1996, p. 436).

$$(A \Rightarrow_s B) \rightarrow ((B \Rightarrow_s C) \rightarrow (A \Rightarrow_s C)),$$

where the expression ``$\Rightarrow_s$'' is used to represent the special kind of conditional such that the sentence ``$A \Rightarrow_s B$'' intuitively means that A counts as B in the institution s.

The importance of having such a logic is of course clear, but if we are to analyze the logic of social reality, it seems that we need to go further for at least two reasons. First, even if an entity of type X counts as Y in context C, another entity of type X may fail to do so in other contexts. Thus, we need to be able to talk about background conditions that characterize C.

Second, constitutive rules can be hierarchically structured, and so an entity e of type X which counts as Y in a context c of type C may further count as Z in that context if any entity of type Y counts as Z in context D and c is also of type D. The Purpose of this paper is to show how the logic of such phenomena can be captured in Channel Theory developed in Barwise and Seligman (1997).

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Constitutive Rules of Social Practices: Definitional or Essential?

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The aim of this paper is to distinguish between two types of constitutive rules that I will call "definitional" vs. "essential" constitutive rules, to make sense of the distinction, and further to try to show its implications for an account of social reality. Very roughly, constitutive rules are rules that constitute social reality and play a role in the determination of what social practices are. What I want to argue is that we have to distinguish two ways in which constitutive rules can make sense of social practices: some constitutive rules are there to give meaning to activities within those practices and to define those activities; others operate on a deeper level and underlie, in an essential way, those social practices themselves. I will call the first type of constitutive rules "definitional" and the second type "essential", and I try to give a possible explanation of this distinction.

How I will proceed is the following: I start by introducing the distinction between the two types of constitutive rules through speech act theories and games via which I came to this distinction, by reference to two conflated ways of characterizing constitutive rules in speech act theories: the Searlian characterization (Searle J., 1969) and the Williamsonian one (Williamson T., 2000). According to both authors, speech acts, as well as games, are governed by constitutive rules, but whereas a constitutive rule in the Williamsonian sense is a rule that is *essential* to an act, such that it necessarily governs every performance of the act (Williamson T., 2000: 239), a constitutive rule in the Searlian sense is a rule that is *tautological* in character, such that it can be seen, now as a rule, now as an analytic truth based on the meaning of the activity term in question. (Searle J., 1969: 34). Therefrom, I introduce the distinction between "definitional" vs. "essential" constitutive rules. Definitional rules correspond to the Searlian sense of 'constitutive', and essential rules correspond to the Williamsonian sense. The difference lies in the fact that if a constitutive rule is definitional, we do not engage in the act of which the rule is definitional if we do not act in accordance with the rule, but if a constitutive rule is essential, obeying it is not a necessary condition for performing the act which is constituted by that rule. I argue that whereas competitive games are governed by definitional rules, speech acts are governed by essential rules.

I then suggest a possible way to trace this distinction in an institutional framework, by introducing a parallel distinction between intra-institutional concepts and trans-institutional concepts (Miller D., 1981). The former are concepts that are entirely defined or that exist only in virtue of a rule within a certain institution, and the latter are somewhat-floating concepts used in different institutions. (Miller D., 1981) I then suggest that there is a parallel between intra- vs. trans-institutional concepts and the definitional vs. essential constitutive rules, which can help us find an explanation, in the institutional framework, of the distinction between the two types of constitutive rules: essential rules are those in the formulation of which a trans-institutional concept is used and which give the point and significance of the practice of which they are constitutive; definitional rules are those constitutive rules which do not involve any trans-institutional concept, and involve at least one intra-institutional concept.

In a second part I try to situate these distinctions in two different frameworks of accounting for social practices: First, an essentialist framework though the work of A. Reinach, and then a conventionalist framework through the work of A. Marmor. According to Reinach (Reinach A. 1983), social and legal entities form a specific ontological category of temporal objects which have their own independent being and are governed by what he calls “essential laws”. I aim to situate the distinction between the two types of constitutive rules by reference to two characteristics of Reinachien essential laws: their immediate intelligibility and their non-forgettability. I will then compare this “essentialist” account with Marmor’s account of social conventions (Marmor A., 2009), according to which social practices are results of [constitutive] conventions, and he distinguishes between two types of conventions in these domains: surface conventions and deep conventions. I again aim to situate the distinction between the two types of constitutive rules with respect to Marmorian surface and deep conventions. I conclude that in whichever way we want to defend the emergence of social and legal institutions, we had better be disposed with the distinction between definitional and essential rules.

Keywords: Constitutive Rules, Speech Acts, Social Practices and Legal Institutions.

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Extended abstract of:
A classification of discursive references to settle what is modified by talking and why it is so.
Dra. Maribel Narváez Mora (Universitat de Girona)

I have claimed (Narvaéz, 2018) that in order to build a dynamic model of sense/knowledge transformation, we need to reshape our notion of discursive reference. In this presentation, I will introduce a classification of discursive references or modes of aboutness. My aim is to offer a scalable tool to settle what can be modify by talking in communicative interactions, and why it is so.

Over forty years ago, the Canadian philosopher Ian Hacking explained how language came to matter to philosophy while the notion of knowledge was being transformed (Hacking, 1975, p. 186). Platonic ideas as perfect objects, or thoughts as a type of Cartesian substance, left room for statements and propositions. When statements were considered a suitable way of representing and expressing knowledge, their meanings –propositions- became the contents of justified true beliefs. In the analytical tradition, to speak about this transformation, the phrase linguistic turn, first popularised by Richard Rorty, is of common use.[1]

However, the pioneering discussions that strengthened the relationship between language and knowledge took place in Vienna during the late nineteenth and early twentieth centuries, and they had to do directly with meaning (*Bedeutung)* (Mulligan, 2012, p. 109 et seq.). The debate between those who, like Brentano, maintained the meaning of a word was the object named and those like Husserl, who rejected such a position, allowed Mauthner to state, 'Philosophy is theory of knowledge and the theory of knowledge is critic of language' (Mauthner, 1901). To be sure, this was the thematization that involved Frege, Wittgenstein[2] and Russell in their respective works, establishing the relationships between word-meaning-object.[3]

As intuitive as these relationships may seem,[4] the truth is that the issue remains problematic despite the long and sophisticated discussion about them. The readjustments

[1] What Richard Rorty called linguistic philosophy was 'the view that philosophical problems are problems that may be solved (or dissolved) either by reforming language or by understanding more about the language we presently use' (Rorty, 1967, p. 3). Although it was Rorty who made popular the *linguistic turn* label, nowadays, the discussion about its history and scope is well alive. See Koopman (2007), Wagner (2010) and Hacker (2013). The latter paper includes an illustrative graphic about the linguistic turn history located as a gif image at the following:
http://www.oxfordhandbooks.com/view/10.1093/oxfordhb/9780199238842.001.0001/oxfordhb-9780199238842-oxfordhb_9780199238842_graphic_018-full.gif.

[2] When Wittgenstein moves away from the criticism of language from Mauthner's view, saying it explicitly in the Tractatus (4.0031), and accepts the criticism of language in the sense of Russell, he takes sides with a type of purification to distinguish the linguistic appearances of the logical form of the proposition. A presentation of the similarities and differences between Mauthner and Wittgenstein can be found in Santibáñez (2007).

[3] The most common way of introducing the relationships (of correction, adequacy or truth) between symbols (words), thoughts and references (meanings) and referents (objects) is the semiotic triangle of Ogden and Richards (1923).

[4] Probably, the relationship between word-meaning-object is familiar to us from the work of Ferdinand de Saussure, *Cours de Linguistique Générale* (1916). However, the famous significant-signifier pair is not at all equivalent to the pair word-meaning. In Saussure's linguistic treatment, words are an inseparable union of significant-signifier. The significant was conceived as an acoustic image of the word and the signifier as the concept that the sign expresses. This is important because the semantic relations that are contemplated are made arbitrary when associated with an acoustic image or signify a certain concept or meaning but not when it comes to the association of a meaning to a word. Put in a different way, to be a word is not to be an acoustic or graphic image.

between conceptions of language and epistemological positions seem to have no end. Under certain conditions, we can say that the meaning of words determines what we are talking about, but what we talk about seems to determine the meaning of the words.

In philosophical treatments of language to refer to something is to relate linguistic representations with what they represent.[5] This relationship is influenced by the model of the proper name[6] in which a word refers to an individual entity, and under this influence, the idea that words are names that represent and bring into the discourse[7] more or less complex entities (objects, properties, situations, states of affairs, processes, events, classes and so forth) keeps exercising a strong influence. The sentences in which (and by which) something about those entities is predicated are then considered their descriptions. Of course, many other functions of language are admitted in addition to the descriptive one, but independently of the speech act enacted, the truth is that the very same way of referring is presupposed. Regardless of the purposes for and the ways of using language, the semantic value (*Bedeutung*) of the terms and phrases that refer[8] is the entity brought into the discourse, that is, its referent. In this model, naming, referring and representing are closely related semantic relationships. The (logical) name is used to represent a referent, so a part of the problem in the philosophy of language has to do with how to disentangle this relationship.[9]

In this presentation borrowing to some degree the structure of Horwick's argumentation (Horwick, 1990, 1998) about a minimalist conception of truth – as within the realm of deflationists theories – "*to refer to*" becomes a transparent element but not a redundant one[10]. The role of the binary predicate '… refers to…' is expressive and inferential. Asserting that a statement – sentence, or speech – refers to something is to take it as a statement – sentence or speech. To carry out this project, as I said, a classification for discursive references will be advanced. This classification covers discursive references or modes of aboutness, not types of referents seen as a function of whatsoever modes of existence. As such, it has to be a useful and scalable *tool to talk about what we talk about* in any communicative interaction.

The main criterion used here to discriminate discursive references is given by the predicate '…is expressible' and its negation '…is not expressible'.[11] Being expressible is a constitutive feature of some discursive references. The relationship stated in an assertion between a discursive reference and the predicate '…is expressible' is definitional and not attributive, and here, internal is preferred. By opposition, the same happens with non-

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[5] A full development of those debates can be seen in Reimer and Michaeldenson (2016).

[6] (Donnellan, 1966, 1970)

[7]From a lexical etiology perspective, to have a name is not a condition that can be referred to. The detection of patterns, their recurrence and the interactions with them can give a name in an efficient manner to manage information.

[8] The identity of the semantic content requires referents being the semantic value of referring expressions. However, note that the phrase 'semantic value' coined by Miller (1998, 2007, pp. 7, 9, 340) to clarify the Fregean notion *Bedeutung* is used to justify that the same semantic content can be asserted, ordered and asked. So far in the text, we have seen that *Bedeutung* is understood in some cases as meaning, in others as reference and eventually as semantic value hesitating from being a concept to being an object.

[9] A paradigmatic case is the treatment of fictions in the philosophy of language; see García-Carpintero and Martí (2014). The title of this collection of works is empty representations trying to highlight a problem. Because proper names in the logical sense serve '[…] to pick out an object, to bring it into our talk or thought, to call our attention to it for further representational purposes such as saying something about it, asking about it or giving directions concerning it' how can it be explained as representational and referential functions in front of non-existent objects?

[10] See also McDonald 2011 for a minimalist approach of properties and facts.

[11] There is no impediment assuming that our criterion is the presence or absence of a property: the property of being expressible. In any case, the relationship between predicates and properties functions in parallel to the relationship between references and referent.

expressible discursive references. In order to keep this explication in the domain of discourse and contrary to what happened within the framework of the linguistic turn, we will not move from language to epistemological and ontological spheres. In any case, the main implications will turn to be epistemological and ontological.

When referring is already considered a (semantic) relation establishing a connection between language and reality (Robertson, 2012, p. 189; Martí, 2012, p. 106), it becomes impossible to avoid the usual recalcitrant dualities classifying references – referents actually – using some fundamental ontological or epistemological criteria. As soon as referents – extra-linguistic realities to which we refer to – come into play, we stop classifying discursive references, and start to classify reality according to several modes of existence, hence, we take an ontological perspective.[12] Then, as far as those referents cannot ontologically be material, physical, empirical, factual or whatever the sustained ontological naturalized commitment advocates for, they are postulated, by exclusion, to be immaterial, spiritual, abstract or ideal provided that those types of reality are accepted; otherwise, they are eliminated as inexistent or reduced to existent ones. I submit that the other way around is possible: if what we refer to can be said (narrated, explained, etc.), then it is expressible;[13] otherwise, we are talking about language independent realities – of a variety of types and scales to be discussed. Because the predicate used here to classify discursive references points to the verbal ability of expressing by language, the assumptions made about expressibility are shared among the speakers of the linguistic community. Consequently, our semantic interpretation of *reference* – namely our accepted semantic substitute[14] for it – will be 'what we talk (it talks) about' and what we talk about is expressible or non-expressible.

When what we talk (it talks) about is fully expressible: a norm, an idea, a concept, a story, our way of representing it is linguistic[15] and our way of experiencing it is cognitive and emotional, which is where its motivational power emerges. This power can be understood as exhibiting top-down causation (Ellis, 2016). When what we talk (it talks) about is non-expressible: a cell, a hurricane, a bike, our way of representing it can be linguistic, using criteria which encapsulates some natural or conventional description of that extension (entity, pattern, process). Our way of experiencing it, besides being, to some extent, cognitive and emotional, is physical too when actualised at certain scale. These references have an impact, affecting our bodies physically in a biological manner or aided by technology.[16]

Only to the extent to which a discursive reference is non-expressible does it make sense to treat it as extra-linguistic and to assume ontological and epistemic naturalised commitments.[17] However, to the extent to which a discursive reference is expressible, its

[12] What an ontological perspective is before the linguistic turn has nothing to do with the Quinean demand of ontological commitment.
[13] Following Terrence W. Deacon's (2013) proposal in *Incomplete Nature*, expressible references are ententional, and the central feature of ententional phenomena is to produce a limitation or constraint, being absences 'are intrinsically incomplete in the sense of being in relationship to, constituted by, or organized to achieve something non-intrinsic' (p. 549).
[14] The concept of semantic interpretation here follows the Wittgensteinian one (1953 PI §201; 1958: 63: 1967: PG §229, 41e; 1974 Z:§9) and is something that is given in signs, substitutes one expression for another or adds a new symbol to an old one.
[15] When Wittgenstein points out the problem of ostensive definitions is recognizing that *having a name* is already a move into a language game.
[16] To deal with problems of existence and actualisation, the concept of scale, introduced later on in the presentation, will play a key role.
[17] According to Price's terminology, it can be said that objective naturalism has a place in a subjective naturalistic project.

naturalisation will have a more limited scope.[18] There is an internal relationship between sharing a language and converging on whether what is talked about is or is not expressible. Any question as to why a reference is expressible points to a constitutive feature of it: norms, principles, ideas and concepts are expressible; otherwise, they would not be norms, principles, ideas or concepts. Although we represent and name norms, principles, ideas and concepts, only non-expressible references can be described in spite of the traditional considerations of philosophy of language.

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[18] It is worth mentioning Terrence W. Deacon's work again because the naturalised understanding of ententional phenomena is something difficult to achieve because his research shows us an emergentist explanation of mind and meaning. Here, we do not deal with the nature of a concept, a norm or a story precisely because the only feature of expressible references we are interested in is their expressible character.

The Dynamics of Group Knowledge and Belief

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Principles of reasoning about group knowledge and belief have received attention over the past decade, in particular in the context of reasoning about the dynamics of interaction. In the talk I will review some of this work, hopefully provide some new insights, and pose some open problems. I will focus on formalisations in modal logic.

What we mean when we say that a group knows something can be radically different depending on context. Well-known notions of group knowledge that have been proposed in the literature include general knowledge (everybody-knows), distributed knowledge, common knowledge, relativised common knowledge. What group belief is, however, is murkier. Applying the same definitions to belief, group belief is not actually always belief. The existence of group belief depends on the particular properties one assumes of belief, and I will map out different possible notions of group belief under different notions of belief. I will also discuss intermediate notions of group belief between distributed and common belief.

Moving to dynamics, we first look at the consequences of adding new group knowledge operators to dynamic epistemic logics such as public announcement logic and action model logic. The relationship between distributed and common knowledge har been of special interest in the dynamic setting, an intuitive idea being that distributed knowledge is potential common knowledge. However this idea is clearly false: it is possible to have distributed knowledge of a Moore-like sentence, which can never even become individual knowledge. I will discuss a dynamic operator that exactly captures what is true after the group have shared all their information with each other; this is what we call resolving distributed knowledge. Intuitions about group knowledge, such as the one just mentioned, are often related to group ability; which states of knowledge a group can make come about. I will thus discuss group knowledge first in the context of general group ability operators such as those found in Alternating-time Temporal Logic and Coalition Logic, and then circle back to dynamic epistemic logics again and discuss cases where ability means ability to achieve some state of knowledge by using public announcements. I will have something to say about how all these different static and dynamic takes on group knowledge and belief are tied together.

A Measurement-Theoretic Modification of Harvey's Aggregation Theorem (Extended Abstract)

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1 Measurement-Theoretic Considerations of Harsanyi's Aggregation Theorem

Harsanyi [5] attempts to develop expected utility theory of von Neumann and Morgenstern [10] to provide a formalization of *(weighted) utilitarianism*. Weymark [15] refers to this result as Harsanyi's Aggregation Theorem. Here we would like to define such measurement-theoretic concepts as

1. scale types,
2. representation and uniqueness theorems, and
3. measurement types

on which the argument of this paper is based: *First*, we classify *scale types* in terms of the class of *admissible transformations* φ. A scale is a triple $\langle \mathfrak{U}, \mathfrak{B}, f \rangle$ where $\mathfrak{U}$ is an observed relational structure that is qualitative, $\mathfrak{B}$ is a numerical relational structure that is quantitative, and f is a homomorphism from $\mathfrak{U}$ into $\mathfrak{B}$.

- A is the domain of $\mathfrak{U}$ and B is the domain of $\mathfrak{B}$. When the admissible transformations are all the functions $\varphi : f(A) \to B$, where $f(A)$ is the range of f, of the form $\varphi(x) := \alpha x; \alpha > 0$. φ is called a *similarity transformation*, and a scale with the similarity transformations as its class of admissible transformations is called a *ratio scale*. Length is an example of a ratio scale.
- When the admissible transformations are all the functions $\varphi : f(A) \to B$ of the form $\varphi(x) := \alpha x + \beta; \alpha > 0$, φ is called a *positive affine transformation*, and a corresponding scale is called an *interval scale*. Temperature on the Fahrenheit scale and temperature on the Celsius scale are examples of interval scales.
- When a scale is unique up to order, the admissible transformations are *monotone increasing functions* φ satisfying the condition that $x \geq y$ iff $\varphi(x) \geq \varphi(y)$. Such scales are called *ordinal scales*. The Mohs scale is an example of a ordinal scale.
- A scale is called a *log-interval scale* if the admissible transformations are functions φ of the form $\varphi(x) := \alpha x^{\beta}; \alpha, \beta > 0$. Psychophysical functions are examples of log-interval scales.

Second, we state about *representation* and *uniqueness theorems*. There are two main problems in measurement theory:

1. the *representation problem*: Given a numerical relational structure $\mathfrak{B}$, find conditions on an observed relational structure $\mathfrak{A}$ (necessary and) sufficient for the *existence* of a homomorphism f from $\mathfrak{A}$ to $\mathfrak{B}$ that preserves all the relations and operations in $\mathfrak{A}$.
2. the *uniqueness problem*: Find the transformation of the homomorphism f under which all the relations and operations in $\mathfrak{A}$ are preserved.

A solution to the former can be furnished by a *representation theorem* that specifies conditions on $\mathfrak{A}$ are (necessary and) sufficient for the existence of f. A solution to the latter can be furnished by a *uniqueness theorem* that specifies the transformation up to which f is unique. *Third*, we classify *measurement types*.

1. *ordinal measurement*
2. *cardinal measurement*
   (a) extensive measurement
   (b) difference measurement
      i. *algebraic-difference measurement*
      ii. positive-difference measurement
      iii. absolute-difference measurement

Suppose A is a set, $\succ$ is a binary relation on A, $\bigcirc$ is a binary operation on A, $\succ'$ is a quaternary relation on A, and f is a real-valued function. Then we call

the representation $a \succ b$ iff $f(a) > f(b)$

ordinal measurement. When such f exists, then $\langle \mathfrak{A}, \mathfrak{B}, f \rangle$ is an *ordinal scale*. We call

the representation $a \succ b$ iff $f(a) > f(b)$ and $f(a \bigcirc b) = f(a) + f(b)$

extensive measurement. When such f exists, then $\langle \mathfrak{A}, \mathfrak{B}, f \rangle$ is a *ratio scale*. We call

the representation $(a, b) \succ' (c, d)$ iff $f(a) - f(b) > f(c) - f(d)$

, when the direction of differences is taken into consideration, *positive-difference measurement*, when the direction of differences is not taken into consideration, *algebraic-difference measurement*. In the latter case, when such f exists, then $\langle \mathfrak{A}, \mathfrak{B}, f \rangle$ is a *interval scale*. We call

the representation $(a, b) \succ' (c, d)$ iff $|f(a) - f(b)| > |f(c) - f(d)|$

absolute-difference measurement. In terms of these measurement-theoretic concepts, Harsanyi's Aggregation Theorem can be stated in the following way:

Theorem 1 (Harsanyi's Aggregation Theorem). *Suppose that individual and social binary preference relations $\succsim_i$ $(i = 1, \ldots, n)$ and $\succsim$ on the set of lotteries satisfy von Neumann-Morgenstern axioms, and also suppose that $\succsim_i$ and $\succsim$ satisfy the Strong Pareto condition. Furthermore, suppose that $\succsim_i$ and $\succsim$ are represented by individual and* *social expected utility functions* $U_i (i = 1, \ldots, n)$ *and* U *respectively. Then, there are real numbers* $\alpha_i (> 0)(i = 1, \ldots, n)$ *and* β *such that*

$$U(p) = \sum_{i=1}^{n} \alpha_i U_i(p) + \beta,$$

for any lottery p.

The next corollary directly follows from this theorem:

Corollary 1 (Weighted Utilitarianism on Set of Lotteries). *Lotteries are socially ranked according to a weighted utilitarian rule:*

$$U(p) \geq U(q) \text{ iff } \sum_{i=1}^{n} \alpha_i U_i(p) \geq \sum_{i=1}^{n} \alpha_i U_i(q),$$

for any lotteries $p.q$.

Harsanyi's Aggregation Theorem follows from the next lemmas:

Lemma 1 (Representation). *Suppose that* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *satisfy Weak Order, Continuity, and Independence. Then, there exist individual and social expected utility functions* $U_i (i = 1, \ldots, n)$ *and* U *such that*

$$\begin{cases} p \succsim_i q \text{ iff } U_i(p) \geq U_i(q), \\ p \succsim q \text{ iff } U(p) \geq U(q), \end{cases}$$

for any lotteries p, q.

Lemma 2 (Uniqueness). *Suppose that* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of lotteries satisfy not only the conditions for the representation above but also Nondegeneracy. Then, the individual and social expected utility functions* U_i *and* U *are unique up to a positive affine transformation.*

There are *at least two* well-known criticisms on Harsanyi's Aggregation Theorem. The *first* criticism is by Sen [14]: Von Neumann-Morgenstern axioms on individual binary preference relations in Lemma 1 are for *ordinal measurement* and, therefore, any *monotone increasing (even non-affine) transform* of an expected utility function is a satisfactory representation of individual binary preference relations. However, (weighted) utilitarianism requires a theory of *cardinal utility*, and so Harsanyi is not justified in giving his theorems utilitarian interpretations. The *second* criticism is based on the following probability agreement theorem that is provided by Broome [2]:

Theorem 2 (Probability Agreement Theorem). *Suppose that individual and social binary preference relations* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of lotteries satisfy von Neumann-Morgenstern axioms. Then,* $\succsim_i$ *and* $\succsim$ *cannot satisfy the strong Pareto condition unless every individual agrees about the probability of every elementary event.*

In fact, under many circumstances, the members of a society have different beliefs (probabilities).

2 Harvey's Aggregation Theorem and Cardinal Utility

In order to escape these two criticisms, we *might* resort to Harvey's Aggregation Theorem ([6]) that has *quaternary preference relations* as primitive that can be represented by *utility differences*, and is concerned only with quaternary preference relations on the set of *outcomes* but is not concerned with binary preference relations on the set of lotteries in Harsanyi's Aggregation Theorem. Lange [8] is the first to connect formally the ranking of *utility differences* with *positive affine transformations* of utility functions. However, he does not use the expression "cardinal utility".[1] Alt [1] is considered to be the first to prove the representation theorem for quaternary preference relations that can be represented by utility differences, and the uniqueness theorem on the uniqueness of the utility functions up to positive affine transformations. However, he also dose not connect utility differences with the expression "cardinal utility". Samuelson [12] is the first to connect utility differences in which utility functions are unique up to positive affine transformations "cardinal utility", though he takes a negative toward cardinal utility. Harvey [6, p.69] defines *difference-worth conditions* as follows: We will use conditions on a quaternary preference relation $\succsim$ as any set of conditions that are satisfied iff there exists a *worth function* w such that

$$(a,b) \succsim (c,d) \text{ iff } w(a) - w(b) \geq w(c) - w(d)$$

for any outcome a, b, c, d, and we will refer to any such conditions as a set of difference-worth conditions. Then Harvey's Aggregation Theorem can be stated in the following way:

Theorem 3 (Harvey's Aggregation Theorem). *Suppose that individual and social quaternary preference relations* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of outcomes satisfy a certain set of difference-worth conditions. Then,* $\succsim_i$ *and* $\succsim$ *satisfy the strong Pareto condition iff there are real numbers* $\alpha_i (> 0)(i = 1, \ldots, n)$ *and* β *such that*

$$w(a) = \sum_{i=1}^{n} \alpha_i w_i(a) + \beta,$$

for any outcome a.

The next corollary directly follows from this theorem:

Corollary 2 (Weighted Utilitarianism on Set of Outcomes). *Outcomes are socially ranked according to a weighted utilitarian rule.*

Harvey's Aggregation theorem follows directly from the next lemmas:

Lemma 3 (Representation). *Suppose that* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of outcomes satisfy a certain set of difference-worth conditions. Then, there exist individual and social worth functions* $w_i (i = 1, \ldots, n)$ *and* w *such that*

$$(1) \begin{cases} (a,b) \succsim_i (c,d) \textit{ iff } w_i(a) - w_i(b) \geq w_i(c) - w_i(d), \\ (a,b) \succsim (c,d) \textit{ iff } w(a) - w(b) \geq w(c) - w(d), \end{cases}$$

[1] About the history of cardinal utility, refer to [9, pp.95–116].

for any outcome a, b, c, d.

Lemma 4 (Uniqueness). *Suppose that $\succsim_i$ $(i = 1, \ldots, n)$ and $\succsim$ on the set of outcomes satisfy the conditions for the representation above. Then, $w_i(i = 1, \ldots, n)$ and w are unique up to a positive affine transformation.*

Because any set of difference-worth conditions is for *algebraic-difference measurement* that is a kind of *cardinal measurement*, this theorem *might* escape the first criticism. When Hammond [4] attempts to salvage utilitarianism in the way that the (strong) Pareto condition can apply only to *outcomes*. Harvey takes the same position as Hammond that *might* enable this theorem to escape the second criticism.

3 Our Two Criticisms on Harvey's Aggregation Theorem from Measurement-Theoretic Point of View

Now we inspect Harvey's Aggregation Theorem from a measurement-theoretic point of view. We offer *two* criticisms on Harvey's Aggregation Theorem: The *first* criticism is as follows: As Roberts [11, p.139] says, the only set of necessary and sufficient difference-worth conditions is due to Scott [13], and requires the assumption that the set of outcomes is *finite*. So when there is no domain-size limitation, the set of necessary and sufficient difference-worth conditions is still unknown. The *second* criticism is as follows: The most essential task of aggregation theorem from a measurement-theoretic point of view is to prove the existence of individual and social worth functions that represent individual and social quaternary preference relations which satisfy *not only* difference-worth conditions *but also* the strong Pareto condition. However, Harvey's Aggregation Theorem is not of such a form. For, in Lemma 3, individual and social quaternary preference relations satisfy *only* difference-worth conditions. So the existence of individual and social worth functions that represent individual and social quaternary preference relations which satisfy both difference-worth conditions and the strong Pareto condition is not guaranteed. Harvey [6, p.72] comments on the feature of his own theorem:

> I view the result in Harsanyi [5] and the result presented here as uniqueness results rather than as existence results. ... an expected-utility function or a worth function is unique up to a positive affine function.

Then, does what Harvey says keep to the point? What should be proved is the uniqueness of individual and social worth functions that represent individual and social quaternary preference relations which satisfy *not only* difference-worth conditions *but also* the strong Pareto condition. However, in Lemma 4, individual and social quaternary preference relations satisfy also *only* difference-worth conditions. So the uniqueness of individual and social worth functions that represent individual and social quaternary preference relations which satisfy both difference-worth conditions and the strong Pareto condition is not guaranteed either. After all, Harvey's Aggregation Theorem can give any answer neither to the *representation problem* nor to the *uniqueness problem*.

4 Our Aggregation Theorems

The *aim* of this paper is that we prove new aggregation theorems, which escape these two criticisms, inspired by Harvey's Aggregation Theorem. Our aggregation representation and uniqueness theorems (main results) can be stated in the following way:

Theorem 4 (Aggregation Representation Theorem (Main Result 1)). *Suppose that individual and social quaternary preference relations $\succsim_i$ $(i = 1, \ldots, n)$ and $\succsim$ on the set of outcomes satisfy Weak Order, Order Reversal, Weak Monotonicity, Soluvability and Archimedean condition in Krantz et al. [7], and also suppose that $\succsim_i$ and $\succsim$ satisfy the strong Pareto condition. Then, there exist individual and social utility functions $u_i (i = 1, \ldots, n)$ and u such that*

$$(1) \quad \begin{cases} (a, b) \succsim_i (c, d) \text{ iff } u_i(a) - u_i(b) \geq u_i(c) - u_i(d), \\ (a, b) \succsim (c, d) \text{ iff } u(a) - u(b) \geq u(c) - u(d), \end{cases}$$

for any outcome a, b, c, d and there are real numbers $\alpha_i (> 0)(i = 1, \ldots, n)$ and β such that

$$u(a) = \sum_{i=1}^{n} \alpha_i u_i(a) + \beta,$$

for any outcome a.

One of key techniques for proving this theorem is a version of *Moment Theorem* in abstract linear spaces in Domotor [3]. The next corollary directly follows from this theorem.

Corollary 3 (Weighted Utilitarianism on Set of Outcomes). *Outcomes are socially ranked according to a weighted utilitarian rule.*

Theorem 5 (Aggregation Uniqueness Theorem (Main Result 2)). *Suppose that $\succsim_i$ $(i = 1, \ldots, n)$ and $\succsim$ on the set of outcomes satisfy the conditions for the representation above. Then, $u_i (i = 1, \ldots, n)$ and u are unique up to a positive affine transformation.*

Because our aggregation theorems do not include any set of necessary and sufficient difference-worth (algebraic difference) conditions but include only some sufficient conditions, it escapes the first criticism. Because our aggregation representation theorem guarantees the existence of individual and social utility functions that represent individual and social quaternary preference relations which satisfy *not only* difference-worth (algebraic difference) conditions *but also* the strong Pareto condition, and our aggregation uniqueness theorem guarantees the uniqueness of such functions, they escape the second criticism. Finally, we would like to discuss the following possible criticism, which is similar to the first criticism on Harsanyi's Aggregation Theorem, from a measurement-theoretic point of view to our aggregation representation and uniqueness theorems. We can prove the following propositions similar to Lemma 1 and Lemma 2 of Harsanyi's Aggregation Theorem:

Proposition 1 (Representation). *Suppose that individual and social quaternary preference relations* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of outcomes satisfy Weak Order, Order Reversal, Weak Monotonicity, Soluvability and Archimedean condition in Krantz et al. [7]. Then, there exist individual social utility functions* $u_i(i = 1, \ldots, n)$ *and* u *such that*

$$(2) \quad \begin{cases} (a,b) \succsim_i (c,d) \textit{ iff } \dfrac{u_i(a)}{u_i(b)} \geq \dfrac{u_i(c)}{u_i(d)}, \\ (a,b) \succsim (c,d) \textit{ iff } \dfrac{u(a)}{u(b)} \geq \dfrac{u(c)}{u(d)}, \end{cases}$$

for any outcome a, b, c, d.

Proposition 2 (Uniqueness). *Suppose that* $\succsim_i$ $(i = 1, \ldots, n)$ *and* $\succsim$ *on the set of outcomes satisfy the conditions for the representation above. Then,* $u_i(i = 1, \ldots, n)$ *and* u *are unique up to a transformation of functions of the form* $\alpha x^{\beta}; \alpha, \beta > 0$.

These propositions imply that Weak Order, Order Reversal, Weak Monotonicity, Soluvability and Archimedean condition in Krantz et al. [7] can satisfy not only (1) but also (2). So our aggregation theorems cannot justify weighted utilitarianism. How can we escape this criticism? Von Neumann-Morgenstern axioms on individual binary preference relations in Lemma 1 are considered, as we have argued earlier, to be for ordinal measurement according to the first criticism by Sen. In this criticism, the fact that any monotone increasing (even non-affine) transform of an expected utility function is a satisfactory representation of individual binary preference relations is used to prove that von Neumann-Morgenstern axioms on individual binary preference relations in Lemma 1 are not for cardinal measurement but for *ordinal measurement*. In Lemma 2, von Neumann-Morgenstern axioms together with the Us being expected utility functions imply the cardinality of Us. So von Neumann-Morgenstern axioms only does not justify the cardinality of Us. On the other hand, because our axioms on individual quaternary preference relations are in nature for *utility-difference measurement (algebraic-difference measurement)* that is a kind of *cardinal measurement*, only our axioms justify the cardinality of ws. Propositions 1 and 2 are not about utility-difference measurement. So we do not have to take these propositions into consideration.

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Single-peakedness of preferences via deliberation: A formal study
(An extended abstract)

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1 Introduction

There are two important aspects of any democratic decision: aggregation of preferences and deliberation about preferences. They are essential and complementary components of any decision making process. While the well-studied process of aggregation focuses on accumulating individual preferences without discussing their origin [4], deliberation can be seen as a conversation through which individuals justify their preferences, a process that might lead to changes in their opinions as they get influenced by one another. Till now, there has been a lot of work on the 'aggregation' aspect (e.g., [12, 14, 6]). However, some recent work has focussed on the deliberation aspect as well [8, 9, 10, 15].

Sometimes, deliberation does not lead to unanimity in preferences, but the discussion can lead to some 'preference uniformity' (see how deliberation can help in bypassing social choice theory's impossibility results in [5]), which might facilitate their eventual aggregation. In addition, the combination of both processes provides a more realistic model for decision making scenarios. In light of this status quo, our focus is on the formal study of achieving such preference uniformities, e.g., single-peaked, single-caved, single-crossing, value-restricted, best-restricted, worst-restricted, medium-restricted, or group-separable profiles. In this short abstract we provide our preliminary ideas towards achieving single-peakedness of preference profiles via deliberation.

In what follows, we define two preference upgrade operators based on [8, 9] and provide a preliminary discussion on how single-peaked preference profiles can be achieved through such operations. We will delve into the details of the logical language in the main paper.

2 Basic concepts

The focus of this work is public deliberation, so let Ag be a *finite non-empty* set of agents with $|Ag| = n \geq 2$ (if $n = 1$, there is no scope for joint discussion). Below we present the most important definitions of this framework.

Definition 1 (PR frame)**.** *A* preference and reliability (PR) frame F *is a tuple* $\langle W, \{\leq_i, \preccurlyeq_i\}_{i \in Ag} \rangle$ *where*

- W *is a finite non-empty set of worlds;*
- $\leq_i \subseteq (W \times W)$ *is a total preorder (a total, reflexive and transitive relation), agent i's* preference relation *over worlds in W ($u \leq_i v$ is read as "world v is at least as preferable as world u for agent i");*
- $\preccurlyeq_i \subseteq (Ag \times Ag)$ *is a total order (a total, reflexive, transitive and antisymmetric relation), agent i's* reliability relation *over agents in Ag ($j \preccurlyeq_i j'$ is read as "agent j' is at least as reliable as agent j for agent i").*

Some further useful definitions are given below.

Definition 2. *Let $F = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in Ag} \rangle$ be a PR frame.*

- $u <_i v$ *("u is less preferred than v for agent i") iff$_{def}$ $u \leq_i v$ and $v \not\leq_i u$.*
- $u \simeq_i v$ *("u and v are equally preferred for agent i") iff$_{def}$ $u \leq_i v$ and $v \leq_i u$.*
- $j \prec_i j'$ *("j is less reliable than j' for agent i") iff$_{def}$ $j \preccurlyeq_i j'$ and $j' \not\preccurlyeq_i j$.*
- $\mathrm{mr}(i) = j$ *(j is agent i's most reliable agent) iff$_{def}$ $j' \preccurlyeq_i j$ for every $j' \in Ag$.*
- $\mathrm{Max}_{\leq_i}(U)$*, the set containing agent i's most preferred worlds among those in $U \subseteq W$, is formally defined as $\{v \in U \mid u \leq_i v \text{ for every } u \in U\}$.*

3 Preference dynamics: lexicographic upgrade

Intuitively, a public announcement of the agents' individual preferences might induce an agent i to adjust her own preferences according to what has been announced and the reliability she assigns to the set of agents.[1] Thus, agent i's preference ordering *after* such announcement, $\leq'_i$, can be defined in terms of the just announced preferences (the agents' preferences *before* the announcement, $\leq_1, \ldots, \leq_n$) and how much i relied on each agent (i's reliability *before* the announcement, $\preccurlyeq_i$): $\leq'_i := f(\leq_1, \ldots, \leq_n, \preccurlyeq_i)$ for some function f. Below, we define a general upgrade operation based on agent reliabilities from [8].

[1] Note that we do not study the formal representation of such announcement, but rather the representation of its effects.

Definition 3 (General lexicographic upgrade)**.** *A* lexicographic list $\mathcal{R}$ *over* W *is a finite non-empty list whose elements are indices of preference orderings over* W*, with* $|\mathcal{R}|$ *the list's length and* $\mathcal{R}[k]$ *its* k*th element* ($1 \leq k \leq |\mathcal{R}|$)*. Intuitively,* $\mathcal{R}$ *is a priority list of preference orderings, with* $\leq_{\mathcal{R}[1]}$ *the one with the highest priority. Given* $\mathcal{R}$*, the preference ordering* $\leq_{\mathcal{R}} \subseteq (W \times W)$ *is defined as*

$$u \leq_{\mathcal{R}} v \quad \textit{iff}_{def} \quad \underbrace{\Big(u \leq_{\mathcal{R}[\,|\mathcal{R}|\,]} v \ \wedge \ \bigwedge_{k=1}^{|\mathcal{R}|-1} u \simeq_{\mathcal{R}[k]} v\Big)}_{1} \ \vee \ \underbrace{\bigvee_{k=1}^{|\mathcal{R}|-1} \Big(u <_{\mathcal{R}[k]} v \ \wedge \ \bigwedge_{l=1}^{k-1} u \simeq_{\mathcal{R}[l]} v\Big)}_{2}$$

Thus, $u \leq_{\mathcal{R}} v$ *holds if this agrees with the least prioritised ordering* ($\leq_{\mathcal{R}[|\mathcal{R}|]}$) *and for the rest of them* u *and* v *are equally preferred (part 1), or if there is an ordering* $\leq_{\mathcal{R}[k]}$ *with a strict preference for* v *over* u *and all orderings with higher priority see* u *and* v *as equally preferred (part 2).*

Proposition 1. *Let* $\mathcal{R}$ *be a lexicographic list over* W*. If* every *ordering* $\mathcal{R}[k]$ ($1 \leq k \leq |\mathcal{R}|$) *is reflexive (transitive, total, respectively), then so is* $\leq_{\mathcal{R}}$*.*

As a consequence of this proposition, the general lexicographic upgrade preserves total preorders (and thus our class of semantic models) *when every preference ordering in* $\mathcal{R}$ *satisfies the requirements.*

Even though the general lexicographic upgrade covers many natural upgrades [8], there are also 'reasonable' policies that fall outside its scope. Sometimes we are not interested in considering the complete order among the choices of the most reliable agent, but only her most preferred choices. To model such upgrades, as mentioned in [9] we provide the following preference upgrade definition.

Definition 4 (General layered upgrade)**.** *A* layered list $\mathcal{S}$ *over* W *is a finite (possibly empty) list of pairwise disjoint subsets of* W *together with a default preference ordering over* W*. The list's length is denoted by* $|\mathcal{S}|$*, its* k*th element is denoted by* $\mathcal{S}[k]$ *(with* $1 \leq k \leq |\mathcal{S}|$*), and* $\leq^{\mathcal{S}}_{\text{def}}$ *is its default preference ordering. Intuitively,* $\mathcal{S}$ *defines layers of elements of* W *in the new preference ordering* $\leq_{\mathcal{S}}$*, with* $\mathcal{S}[1]$ *the set of worlds that will be in the topmost layer and* $\leq^{\mathcal{S}}_{\text{def}}$ *the preference ordering that will be applied to each individual set and to those worlds not in* $\bigcup_{k=1}^{|\mathcal{S}|} \mathcal{S}[k]$*. Formally, given* $\mathcal{S}$*, the ordering* $\leq_{\mathcal{S}} \subseteq (W \times W)$ *is defined as*

$$u \leq_{\mathcal{S}} v \quad \textit{iff}_{def} \quad \underbrace{\Big(u \leq^{\mathcal{S}}_{\text{def}} v \ \wedge \ \big(\{u,v\} \cap \bigcup_{k=1}^{|\mathcal{S}|} \mathcal{S}[k] = \emptyset \vee \bigvee_{k=1}^{|\mathcal{S}|} \{u,v\} \subseteq \mathcal{S}[k]\big)\Big)}_{1} \ \vee \ \underbrace{\bigvee_{k=1}^{|\mathcal{S}|} \Big(v \in \mathcal{S}[k] \ \wedge \ u \notin \bigcup_{l=1}^{k} \mathcal{S}[l]\Big)}_{2}$$

Thus, $u \leq_{\mathcal{S}} v$ holds if this agrees with the default ordering $\leq^{\mathcal{S}}_{\mathrm{def}}$ and either neither u nor v are in any of the specified sets in $\mathcal{S}$ or else both are in the same set (part 1), or if there is a set $\mathcal{S}[k]$ in which v appears and u appears neither in the same set (a case already covered in part 1) nor in one with higher priority (part 2).

Proposition 2. *Let $\mathcal{S}$ be a layered list over W. If $\leq^{\mathcal{S}}_{\mathrm{def}}$ is reflexive (transitive, total, respectively), then so is $\leq_{\mathcal{S}}$.*

Definition 5. *Let $M = \langle W, \{\leq_i, \preccurlyeq_i\}_{i \in Ag}, V\rangle$ be a PR model.*

- *Let $\mathcal{S}$ be a layered list whose default ordering is reflexive, transitive and total; let $j \in Ag$ be an agent. The PR model $\mathrm{gy}^j_{\mathcal{S}}(M) = \langle W, \{\leq'_i, \preccurlyeq_i\}_{i \in Ag}, V\rangle$ is such that, for every agent $i \in Ag$, $\leq'_i := \leq_{\mathcal{S}}$ if $i = j$, and $\leq'_i := \leq_i$ otherwise.*
- *Let $\boldsymbol{\mathcal{S}}$ be a list of $|Ag|$ layered lists whose default ordering are reflexive, transitive and total, with $\boldsymbol{\mathcal{S}}_i$ its ith element. The PR model $\mathrm{gy}_{\boldsymbol{\mathcal{S}}}(M) = \langle W, \{\leq'_i, \preccurlyeq_i\}_{i \in Ag}, V\rangle$ is such that, for every agent $i \in Ag$, $\leq'_i := \leq_{\boldsymbol{\mathcal{S}}_i}$*

We have proposed different preference upgrade operators based on agent reliabilities. Now, the question is under what conditions these upgrade operators may lead to single-peakedness of agent preferences.

4 Deliberating towards single-peakedness

On the one hand we have the general lexicographic upgrade operation which considers a particular list to define the upgraded preferences. On the other hand we have this layered upgrade operation which is based on arbitrary subsets of choices and providing an order between them. There is a whole territory of possible upgrade operators in between these possibilities that is uncharted as of now. We would like to focus on charting the territory with a special emphasis on single-peakedness. We now assume the preference orderings to be asymmetric in addition to being total and transitive. Each agent is endowed with such a preference relation over the worlds.

Definition 6. *A preference profile is single-peaked if there exists a world w_i for each agent i and a linear order L such that $w_i L w' L w''$ or $w'' L w' L w_i$ imply $w' <_i w''$.*

Ballester and Haeringer [2] showed that the following two conditions characterize single-peakedness.

- For any subset of worlds the set of worlds considered as the worst by all agents cannot contains more than 2 elements (known as the worst-restricted condition in the literature).

- There cannot be four worlds w_1, w_2, w_3, w_4 and two agents i, j such that $w_1 <_i w_2 <_i w_3$, $w_3 <_j w_2 <_j w_1$, and $w_4 <_i w_2$, $w_4 <_j w_2$. In other words, two agents cannot disagree on the relative ranking of two alternatives with respect to a third alternative but agree on the (relative) ranking of a fourth one.

Our task is to investigate that under what conditions the given deliberation processes can achieve these properties. The first one should be easy to get: Since the orderings are asymmetric, the lexicographic upgrade policy will be identified with the drastic upgrade policy [8] which would lead to unanimity or oscillation. If unanimity is reached, we have single-peakedness trivially. In case of oscillation, we need to make sure that whichever be the agents included in oscillation for each agent, the least preferred world can only vary between (at most) two of the given worlds. For the layered upgrade ordering we will have a more interesting property of ensuring the weakest layer to contain the same two elements always. The second condition is more tricky, but once again can be broken down into several sub-conditions in the layered case. We leave the formal work for the main paper. We conclude with mentioning the known fact that getting single-peaked preferences via deliberation would pave the way of using aggregation rules which will lead to collective decision making avoiding the impossibility results of Arrow and others.

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Dynamic Term-Modal Logic Revisited

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Term-modal logic, is a first-order modal logic, where the modalities are indexed by terms of first-order language. This allows quantification over modalities and such. Dynamic term-modal logic further generalizes this by allowing PDL-like constructions over these term-modalities. In the talk we will examine to what extent this provides new insights to well-known philosophical problems having to do with first-order modal logic: existence, identity, quantification, de-re versus de dicto, etc.

Cut-free and Analytic Sequent Calculus of First-Order Intuitionistic Epistemic Logic

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[Artemov and Protopopescu, 2016] gave an intuitionistic epistemic logic based on a verification reading of the intuitionistic knowledge in terms of Brouwer-Heyting-Kolmogorov interpretation. According to this interpretation, a proof of $A \supset B$ is a construction such that when a proof of A is given, a proof of B can be constructed. [Artemov and Protopopescu, 2016] proposed that a proof of a formula KA (read "it is known that A"), is the conclusive verification of the existence of a proof of A. Then $A \supset KA$ expresses that, when a proof of A is given, the conclusive verification of the existence of the proof of A can be constructed. Since a proof of A itself is the conclusive verification of the existence of a proof of A, they claim that $A \supset KA$ is valid. But $KA \supset A$ (usually called *factivity* or *reflection*) is not valid, since the verification does not always give a proof. They provided a Hilbert system of intuitionistic epistemic logic **IEL** as the intuitionistic propositional logic plus the axioms schemes $K(A \supset B) \supset KA \supset KB$, $A \supset KA$ and $\neg K\bot$. Moreover they gave **IEL** the following Kripke semantics. We say that $M = (W, \leq, R, V)$ is a *Kripke model* for **IEL** if $(W, \leq, V)$ is a Kripke model for intuitionistic propositional logic and R is a binary relation such that $R \subseteq \leq$, $\leq ; R \subseteq R$ and R satisfies the seriality. Then KA is true on a state w of M if and only if for any v, wRv implies A is true in v of M. [Artemov and Protopopescu, 2016] also proved that their Hilbert system is sound and complete.

The study of **IEL** also casts light on the study of the knowability paradox. The knowability paradox, also known as the Fitch-Church paradox, states that, if we claim the knowability principle: every truth is knowable ($A \supset \Diamond KA$), then we are forced to accept the omniscience principle: every truth is known ($A \supset KA$) [Fitch, 1963]. This paradox is commonly recognized as a threat to Dummett's semantic anti-realism. It is because the semantic anti-realists claim the knowability principle but they do not accept the omniscience principle. However, as Dummett admitted that he had taken some of intuitionistic basic features as a model for an anti-realist view [Dummett, 1978, p.164], it is reasonable to consider an intuitionistic logic as a basis. In this sense, if we employ BHK-interpretation of KA as above to accept the **IEL** in the study of the knowability paradox, $A \supset KA$ becomes valid and the knowability paradox is trivialized.

Proof-theoretical studies of **IEL** have been investigated. In Krupski and Yatmanov [2016], the sequent calculus of **IEL** has been given, though an inference rule corresponding to $KA \supset \neg\neg A$ in their system for **IEL** does not satisfy a desired syntactic property, i.e., the subformula property. In Protopopescu [2015], a Gödel-McKinsey-Tarski translation from the intuitionistic epistemic propositional logic to the bimodal expansion of the classical modal logic **S4** has been studied.

In this paper, we study the first-order expansion **QIEL** of intuitionistic epistemic logic of **IEL**. Artemov and Protopopescu mentioned that the notion of the intuitionistic knowledge capture both mathematical knowledge and empirical knowledge. When we consider the mathematical knowledge, quantifiers become inevitable. Moreover when we are concerned with the empirical knowledge, we recall that Hintikka had given arguments for first-order epistemic logic in Hintikka [2005]. He mentioned that if we want to deal with the locutions like "knows who," "knows when," "knows where," we can translate these expressions into a language with quantifiers. For example, about "who" we can have variables ranging over the human being, about "where" over the location in space. In this sense, our first-order expansion can provide a fundamental basis when we concern the intuitionistic mathematical and empirical knowledge.

We give the first-order expansion of **IEL** as **QIEL**. An expanded Kripke model $M = (W, \leq, R, D, I)$ is obtained by adding D and I into the Kripke model for **IEL**. Here D is a function which assigns a nonempty domain $D(w)$ to $w \in W$ such that, for any $w, v \in W$, if $w \leq v$ then $D(w) \subseteq D(v)$. Moreover I is an interpretation such that $I(c) \in D(w)$ for all $w \in W$ for any constant symbol c and $I(P, w) \subseteq D(w)^n$ for every $w \in W$ and every n-arity predicate P such that if $u \leq v$ then $I(P, u) \subseteq I(P, v)$ for all $u, v \in W$.

We also propose the sequent calculus for **QIEL**. The sequent calculus for **IEL** has been given by Krupski and Yatmanov [2016]. Their sequent calculus is obtained from the propositional part of Gentzen's sequent calculus **LJ** (with structural rules of weakening and contraction) for the intuitionistic logic plus the following two inference rules on the knowledge operator:

$$\frac{\Gamma_1, \Gamma_2 \Rightarrow A}{\Gamma_1, K\Gamma_2 \Rightarrow KA}\ (KI) \qquad \frac{\Gamma \Rightarrow K\bot}{\Gamma \Rightarrow F.}\ (U)$$

where a sequent $\Gamma \Rightarrow A$ (where Γ is a finite multiset of formulas) can be read as "if all of Γ hold then A holds." They established the cut-elimination theorem of the calculus, i.e., a derivable sequent in their system is derivable without any application of the following cut rule:

$$\frac{\Gamma \Rightarrow B \quad B, \Sigma \Rightarrow \Delta}{\Gamma, \Sigma \Rightarrow \Delta}\ (Cut),$$

where Δ contains at most one formula. It is remarked, however, that this system does not enjoy the subformula property. That is, in the rule of (U), we have a formula $K\bot$ which might not be a subformula of a formula in the lower sequent of the rule (U).

This talk gives a new cut-free and analytic sequent calculus $\mathscr{G}(\mathbf{QIEL})$ of the first-order intuitionistic epistemic logic, which is obtained from adding the following rule (K_{IEL}) into Gentzen's **LJ** with quantifiers:

$$\frac{\Gamma_1, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, K\Gamma_2 \Rightarrow K\Delta}\ (K_{IEL})$$

where Δ contains at most one formula. This rule is equivalent to the rules from Krupski and Yatmanov [2016] in a propositional setting. Moreover it is easy to see that (K_{IEL}) satisfies the subformula property.

Let $\mathscr{G}^-(\mathbf{QIEL})$ be the system $\mathscr{G}(\mathbf{QIEL})$ without the cut rule. By the standard syntactic argument, we can establish the following fundamental proof-theoretic result.

Theorem 1 (Cut-Elimination) *If* $\mathscr{G}(\mathbf{QIEL}) \vdash \Gamma \Rightarrow \Delta$ *then* $\mathscr{G}^-(\mathbf{QIEL}) \vdash \Gamma \Rightarrow \Delta$.

Corollary 1 (Disjunction Property, Existence Property, Craig Interpolation Theorem) *As a corollary of cut-elimination theorem we have:*

1. *If* $\Rightarrow A \vee B$ *is derivable in* $\mathscr{G}(\mathbf{QIEL})$*, then either* $\Rightarrow A$ *or* $\Rightarrow B$ *is derivable in* $\mathscr{G}(\mathbf{QIEL})$.

2. *For any formula of the form $\exists x A$, if $\Rightarrow \exists x A$ is derivable in $\mathscr{G}(\mathbf{QIEL})$ then there exists a term t such that $\Rightarrow A(t/x)$ is derivable in $\mathscr{G}(\mathbf{QIEL})$.*

3. *If $\Rightarrow A \supset B$ is derivable in $\mathscr{G}(\mathbf{QIEL})$, then there exists a formula C such that $\Rightarrow A \supset C$ and $\Rightarrow C \supset B$ are also derivable in $\mathscr{G}(\mathbf{QIEL})$, and all free variables, predicate symbols and constant symbols of C are shared by both A and B.*

Given a sequent $\Gamma \Rightarrow \Delta$, Γ_* denotes the conjunction of all formulas in Γ ($\Gamma_* \equiv \top$ if Γ is empty) and Δ^* denotes the unique formula in Δ if Δ is non-empty; it denotes $\perp$ otherwise. We say that a sequent $\Gamma \Rightarrow \Delta$ is valid in a class $\mathbb{M}$ of models (denoted by $\mathbb{M} \models \Gamma \Rightarrow \Delta$), if $\Gamma_* \supset \Delta^*$ is satisfied in any states of any Kripke models.

Theorem 2 (Soundness of $\mathscr{G}(\mathbf{QIEL})$) *Let $\Gamma \Rightarrow \Delta$ be any sequent. If $\mathscr{G}(\mathbf{QIEL}) \vdash \Gamma \Rightarrow \Delta$ then $\mathbb{M} \models \Gamma \Rightarrow \Delta$.*

With the method from Hermant [2005], we prove the following:

Theorem 3 (Completeness of $\mathscr{G}^-(\mathbf{QIEL})$) *Let $\Gamma \Rightarrow \Delta$ be a sequent. If $\mathbb{M} \models \Gamma \Rightarrow \Delta$ then $\mathscr{G}^-(\mathbf{QIEL}) \vdash \Gamma \Rightarrow \Delta$.*

Corollary 2 *The following are all equivalent.*

1. $\mathbb{M} \models A$,
2. $\mathscr{G}(\mathbf{QIEL}) \vdash \Rightarrow A$,
3. $\mathscr{G}^-(\mathbf{QIEL}) \vdash \Rightarrow A$,

In particular, we can also prove the cut elimination theorems semantically by Theorem 2 and Theorem 3.

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Sequent Calculi for Multi-Agent Epistemic Logics for Distributed Knowledge

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1 Introduction

"Distributed knowledge" is a notion developed in the community of multi-agent epistemic logic [1, 8]. In [1, p. 3], the notion is explained informally as follows:

> A group has distributed knowledge of a fact φ if the knowledge of φ is distributed among its members, so that by pooling their knowledge together the members of the group can deduce φ, even though it may be the case that no member of the group individually knows φ.

For example, a group consisting of a and b has distributed knowledge of a fact q, when a knows $p \to q$ and b knows p. Formally, "A group G has distributed knowledge of a fact φ." is written as "$D_G\varphi$", whose meaning is usually defined in a Kripke model. Let W be a possibly countable set of states, Agt be a finite set of agents, $(R_a)_{a\in\mathsf{Agt}}$ be a family of binary relations on W, indexed by agents, and V be a valuation function $\mathsf{Prop} \to \mathcal{P}(W)$, where Prop is a countable set of propositional variables. We call a tuple $M = (W, (R_a)_{a\in\mathsf{Agt}}, V)$ a (Kripke) model. For a group $G \subseteq \mathsf{Agt}$, satisfaction of $D_G\varphi$ at a state w in a model M is defined as follows:

$$M, w \models D_G\varphi \Longleftrightarrow \text{for all } v, \text{ if } (w, v) \in \bigcap_{a\in G} R_a \text{ then } M, v \models \varphi$$

It is clear from the definition that the operator $D_{\{a\}}$ behaves the same as K_a, a box-like operator for an agent a, usually defined in multi-agent epistemic logic. Therefore, we do not introduce K_a-like operator as a primitive one in this abstract.

The study of distributed knowledge so far is mainly model-theoretic [16, 13, 4, 15] and proof-theoretic study has been not so active. As far as we know, existing sequent calculi for logic with distributed knowledge are presented only in [6] and [5]. The former contains a natural G3-style formalization, in which each formula has a label and the latter contains Getzen-style and Kanger-style sequent calculus for logic with distributed knowledge operator which is simpler than the one we are interested in, in that the operator is not parameterized by group G.

We propose Gentzen-style sequent calculi (without label) for five kinds of multi-agent epistemic propositional logics with distributed knowledge operators, parameterized by groups, which are reasonable generalization of sequent calculi for basic modal logic and prove the cut elimination theorem for four of them. Using a method described in [7], Craig's interpolation theorem is also established for the four system, in which not only condition of propositional variables but also that of agents is taken into account. This is a new result for logic for distributed knowledge, as far as we know.

In the following, we briefly sketch our proof systems, and then state and comment on the theorems we have on the systems.

2 Proof Systems

We denote a finte set of agents by Agt. We call a nonempty subset of Agt "group" and denote it by G, H, etc. Let Prop be a countable set of propositional variables and Form be the set of formulas defined inductively by the following clauses ($\vee$ and $\neg$ are defined in the same way as the classical propositional logic):

$$\mathsf{Form} \ni \varphi ::= p \in \mathsf{Prop} \mid \bot \mid \top \mid \varphi \wedge \varphi \mid \varphi \to \varphi \mid D_G\varphi$$

First, we explain known Hilbert-style axiomatization for logics with D_G operator (for detail, refer to [1]). The following are axioms for the logics:

- (Taut) all instantiations of propositional tautology
- (Incl) $D_G\varphi \to D_H\varphi$ $(G \subseteq H)$
- (K) $D_G(\varphi \to \psi) \to D_G\varphi \to D_G\psi$
- (T) $D_G\varphi \to \varphi$
- (4) $D_G\varphi \to D_G D_G\varphi$
- (5) $\neg D_G\varphi \to D_G\neg D_G\varphi$

An axiom system $\mathsf{H}(\mathbf{K}_D)$ ($\mathsf{H}(\mathbf{KT}_D)$,$\mathsf{H}(\mathbf{K4}_D)$, $\mathsf{H}(\mathbf{S4}_D)$, and $\mathsf{H}(\mathbf{S5}_D)$) is a collection of the inference rules of modus ponens ("from $\varphi \to \psi$ and φ infer ψ") and necessitation ("from φ infer $D_G\varphi$"), axioms (Taut) and (Incl) (common to all the five systems), and (an) axiom(s) (K) ((K) and (T); (K) and (4); (K), (T), and (4); and (K), (T), and (5), respectively).

We now propose our sequent calculi for the logics for distributed knowledge. To the ordinary LK system [2, 3], we add some of the following rules for each logic:

$$\frac{\varphi_1, \cdots, \varphi_n \Rightarrow \psi \quad (\bigcup_{i=1}^n G_i \subseteq G)}{D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow D_G\psi} \ (D_G)$$

$$\frac{\varphi, \Gamma \Rightarrow \Delta}{D_G\varphi, \Gamma \Rightarrow \Delta} \ (D_G \Rightarrow)$$

$$\frac{\varphi_1, \cdots, \varphi_n, D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow \psi \quad (\bigcup_{i=1}^n G_i \subseteq G)}{D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow D_G\psi} \ (\Rightarrow {D_G}^{\mathbf{K4}_D})$$

$$\frac{D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow \psi \quad (\bigcup_{i=1}^n G_i \subseteq G)}{D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow D_G\psi} \ (\Rightarrow {D_G}^{\mathbf{S4}_D})$$

$$\frac{\varphi_1, \cdots, \varphi_n \Rightarrow \psi_1, \cdots, \psi_m, \chi \quad (\bigcup_{i=1}^n G_i \cup \bigcup_{j=1}^m H_j \subseteq G)}{D_{G_1}\varphi_1, \cdots, D_{G_n}\varphi_n \Rightarrow D_{H_1}\psi_1, \cdots, D_{H_m}\psi_m, D_G\chi} \ (\Rightarrow {D_G}^{\mathbf{S5}_D})$$

A sequent calculus $\mathsf{G}(\mathbf{K}_D)$ ($\mathsf{G}(\mathbf{KT}_D)$,$\mathsf{G}(\mathbf{K4}_D)$, $\mathsf{G}(\mathbf{S4}_D)$, and $\mathsf{G}(\mathbf{S5}_D)$) is LK with the rule(s) (D_G) ((D_G) and $(D_G \Rightarrow)$; $(\Rightarrow {D_G}^{\mathbf{K4}_D})$; $(D_G \Rightarrow)$ and $(\Rightarrow {D_G}^{\mathbf{S4}_D})$; and $(D_G \Rightarrow)$ and $(\Rightarrow {D_G}^{\mathbf{S5}_D})$, respectively).

The idea underlying the rule (D_G) is similar to that of an inference rule called "$R12$" described in [12, section 4]. Our calculi $\mathsf{G}(\mathbf{KT}_D)$,$\mathsf{G}(\mathbf{K4}_D)$, $\mathsf{G}(\mathbf{S4}_D)$, and $\mathsf{G}(\mathbf{S5}_D)$ are constructed based on the known sequent calculus for $\mathbf{KT}, \mathbf{K4}, \mathbf{S4}$, and $\mathbf{S5}$ (surveyed in [11, 14]).

We note that for any logic $\mathbf{X}$ of the logics described above, $\mathsf{H}(\mathbf{X})$ and $\mathsf{G}(\mathbf{X})$ are equivalent in derivability of formulas, and hence that each system $\mathsf{G}(\mathbf{X})$ deserves its own name.

Theorem 1 (Equivalence between Hilbert-style and Gentzen-style axiomatization) *Let $\mathbf{X}$ be any of $\mathbf{K}_D$, $\mathbf{KT}_D$, $\mathbf{K4}_D$, $\mathbf{S4}_D$, and $\mathbf{S5}_D$. Then, the following hold.*

1. *If $\vdash_{\mathsf{H}(\mathbf{X})} \varphi$, then $\vdash_{\mathsf{G}(\mathbf{X})} \Rightarrow \varphi$*
2. *If $\vdash_{\mathsf{G}(\mathbf{X})} \Gamma \Rightarrow \Delta$, then $\vdash_{\mathsf{H}(\mathbf{X})} \bigwedge \Gamma \to \bigvee \Delta$*

We have the cut elimination theorem for our sequent calculi, except for $\mathsf{G}(\mathbf{S5}_D)$.

Theorem 2 (Cut elimination) *Let $\mathbf{X}$ be any of $\mathbf{K}_D$, $\mathbf{KT}_D$, $\mathbf{K4}_D$, and $\mathbf{S4}_D$. Then, the following holds: If $\vdash_{\mathsf{G}(\mathbf{X})} \Gamma \Rightarrow \Delta$, then $\vdash_{\mathsf{G}^-(\mathbf{X})} \Gamma \Rightarrow \Delta$, where $\mathsf{G}^-(\mathbf{X})$ denotes a system "$\mathsf{G}(\mathbf{X})$ minus cut rule".*

Flexibility of choice of groups occurring in the left side of the lower sequent in the rule (D_G) and the three $(\Rightarrow D_G)$-type rules is a key to the result. The reason why cut elimination theorem does not hold for $\mathsf{G}(\mathbf{S5}_D)$ is that the sequent calculus for basic $\mathbf{S5}$, on which $\mathsf{G}(\mathbf{S5}_D)$ is based, is not cut-free [9].

As an application of the cut elimination theorem, Craig's interpolation theorem can be derived, using a method described in [7]. (Application of the method to basic modal logic is also described in [10].)

Theorem 3 (Craig's interpolation theorem) *Let $\mathbf{X}$ be any of $\mathbf{K}_D$, $\mathbf{KT}_D$, $\mathbf{K4}_D$, and $\mathbf{S4}_D$. Given that $\vdash_{\mathsf{G}(\mathbf{X})} \varphi \Rightarrow \psi$, there exists a formula χ satisfying the following conditions:*

1. *$\vdash_{\mathsf{G}(\mathbf{X})} \varphi \Rightarrow \chi$ and $\vdash_{\mathsf{G}(\mathbf{X})} \chi \Rightarrow \psi$.*

2. *$V(\chi) \subseteq V(\varphi) \cap V(\psi)$, where $V(\rho)$ denotes the set of propositional variables occuring in formula ρ.*

3. *$A(\chi) \subseteq A(\varphi) \cap A(\psi)$, where $A(\rho)$ denotes the set of agents occuring in formula ρ.*

We note that not only the condition for propositional variables but also the condition for agents can be satisfied.

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