

# Acts of requesting in a dynamic logic of knowledge and obligation

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# Outline

- 1 Introduction
- 2 DEL and A dynamic logic of acts of commanding
- 3 Refinements and Variations
  - Conflicting commands
  - Acts of commanding and promising
  - Obligations and preferences
  - Assertions, concessions and their withdrawals
- 4 Acts of requesting
  - Selecting base logic (Steps 1 and 2 of the recipe)
  - Dynamifying MEDL (Step 3)
  - Dynamic logic DMEDL (Steps 4 & 5)

# The gap

## Van Benthem & Liu (2007) on commanding

For instance, intuitively, a command

“See to it that  $\varphi$ !”

makes worlds where  $\varphi$  holds preferred over those where it does not - **at least, if we accept the preference induced by the issuer of the command.**

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# Austin's Distinction (1955, pp.101-3.)

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He said to me "Shoot her!" meaning by 'shoot' shoot and referring by 'her' to her.

## Illocutionary Act

He urged (advised, ordered, etc.) me to shoot her.

## Perlocutionary Act

- (a) He persuaded me to shoot her.
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- Strawson (1964) observed that the kind of conventional effects involved in the examples used by Austin are dependent on special extralinguistic conventions.
- He then argued that there are many other illocutionary acts that do not seem to be dependent on any such special extralinguistic conventions.
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## Conventional effects vs. utterers' intentions

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- Searle criticized Grice (and Strawson) for treating meaning as “a matter of intending to perform a perlocutionary acts”,
- but agreed with Strawson in seeing Austin’s notion of conventional effect as an overgeneralization (1971 → 1979, p.7).
- Searle sees conventionality of illocutionary acts as a matter of meaning, and denied the distinction between locutionary acts and illocutionary acts.
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- Indeed, even typical illocutionary acts such as acts of promising, which both Strawson and Searle see not conventional in what they take to be Austin's sense, seem to involve more than the mere securing of uptake.
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# What Austin's Earlier Answer Enables us to See

## Perlocutionary acts

Since perlocutionary acts are acts that really produce real effects, they cannot be completed without really producing them.

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# The problem

- Is it possible to develop this conception of illocutionary acts into a general theory of illocutionary acts?
- In order to do so, we have to
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# The plan

- The recent development of Dynamic Epistemic Logics suggests a recipe for developing logics that can capture effects of various speech acts.
- We have developed dynamic logics that can deal with acts of commanding, promising, asserting, conceding, and withdrawing according to this recipe (Yamada 07a, 07b, 08a, 08b, To appear).
- We will briefly review these developments.
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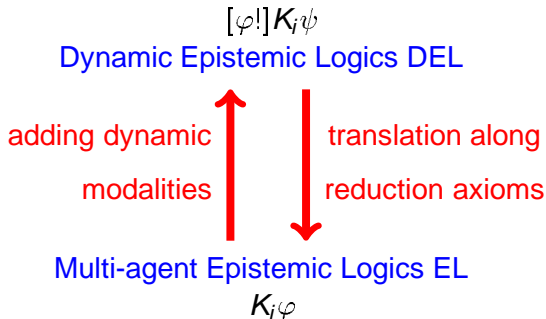
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# The developments of dynamic epistemic logics



Cf. Plaza (1989), Gerbrandy & Groeneveld (1997), Gerbrandy (1999), Baltag, Moss, & Solecki (1999), Kooi & van Benthem (2004), van Ditmarsch, Kooi, and van der Hoek (2007)



## Two points to be noted

The formulas of the form  $\varphi \rightarrow [\varphi!]K_i\varphi$  are shown to be valid for any  $i \in I$  if no operators of the form  $K_i$  occur in  $\varphi$ .

- This is too strong for interpreting natural language public announcements.
- A gap similar to the one we have seen is also present here.

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# The recipe (Yamada, to appear)

- 1 Carefully identify the aspects affected by the speech acts you want to study
- 2 find the modal logic that characterizes these aspects
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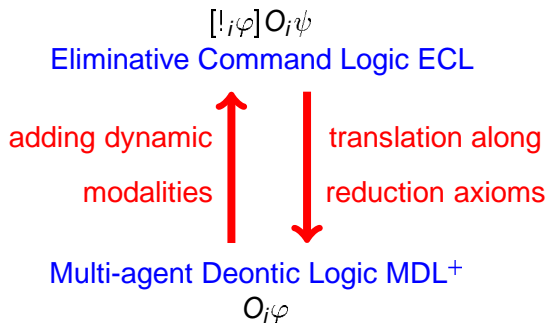
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# This recipe works for acts of commanding

(Yamada, 2007a)



# The language of multi-agent deontic logic

## Definition

Take a countably infinite set  $A_{\text{prop}}$  of proposition letters and a finite set  $I$  of agents, with  $p$  ranging over  $A_{\text{prop}}$  and  $i$  over  $I$ . The multi-agent monadic deontic language  $\mathcal{L}_{\text{MDL}^+}$  is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi$$

$O_a\varphi$  It is obligatory upon an agent  $a$  to see to it that  $\varphi$ .

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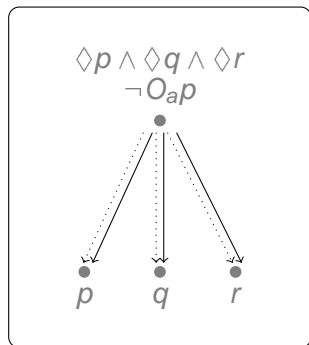
# $\mathcal{L}_{\text{MDL}^+}$ -models

## Definition

By an  $\mathcal{L}_{\text{MDL}^+}$ -model, we mean a tuple  
 $M = \langle W^M, \Rightarrow^M, \{\cup_i^M \mid i \in I\}, V^M \rangle$  where:

- (i)  $W^M$  is a non-empty set (heuristically, of ‘possible worlds’),
- (ii)  $\Rightarrow^M \subseteq W^M \times W^M$ ,
- (iii)  $\cup_i^M \subseteq \Rightarrow^M$  for each  $i \in I$ ,
- (iv)  $V^M$  is a function that assigns a subset  $V^M(p)$  of  $W^M$  to each proposition letter  $p \in \text{Aprop}$ .

# Example 1: on a hot day in a shared office

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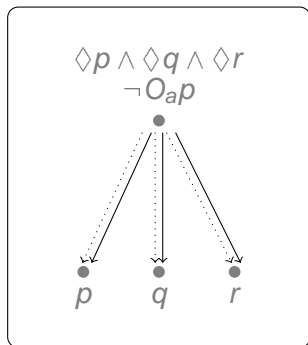
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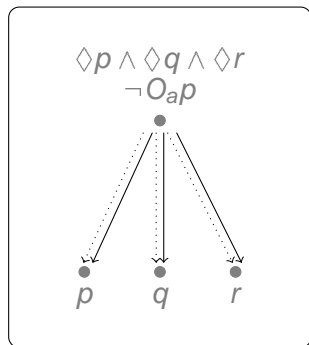
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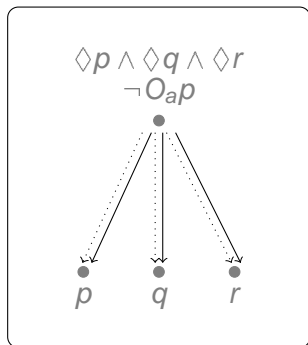
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# The language of command logic

## Definition

Take the same countably infinite set  $Aprop$  of proposition letters and the same finite set  $I$  of agents as before, with  $p$  ranging over  $Aprop$ , and  $i$  over  $I$ . The language  $\mathcal{L}_{ECL}$  of eliminative command logic ECL is given by:

$$\begin{aligned} \varphi &::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi \mid [\pi]\varphi \\ \pi &::= !_i\varphi \end{aligned}$$

$[\!_a\psi]O_a\varphi$  After every effective act of commanding an agent  $a$  to see to it that  $\psi$ , it is obligatory upon  $a$  to see to it that  $\varphi$ .

# The truth definition for $\mathcal{L}_{\text{ECL}}$

## Definition

Let  $M$  be an  $\mathcal{L}_{\text{MDL}^+}$ -model and  $w$  a point in  $M$ . If  $p \in \text{Aprop}$ , and  $i \in I$ , then the truth definition for  $\mathcal{L}_{\text{ECL}}$  is given by expanding that of  $\mathcal{L}_{\text{MDL}^+}$  mutatis mutandis with the following new clause:

$$(g) \quad M, w \models_{\text{ECL}} [!i\chi]\varphi \text{ iff } M_{!i\chi}, w \models_{\text{ECL}} \varphi,$$

where  $M_{!i\chi}$  is the  $\mathcal{L}_{\text{MDL}^+}$ -model obtained from  $M$  by replacing  $\cup_i^M$  with  $\{\langle x, y \rangle \in \cup_i^M \mid M, y \models_{\text{ECL}} \chi\}$ .

# The truth definition for $\mathcal{L}_{\text{ECL}}$

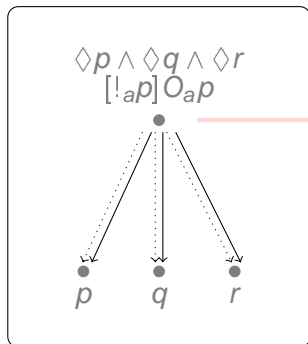
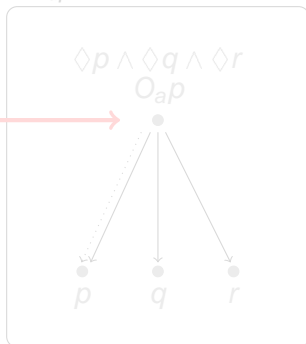
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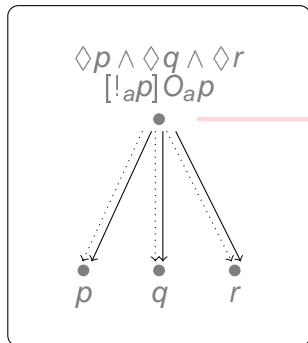
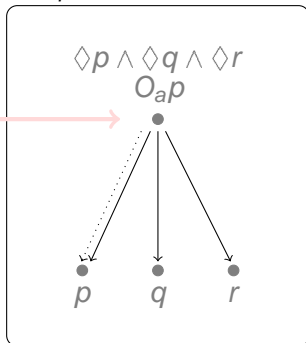
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# Your boss's act of commanding in ECL

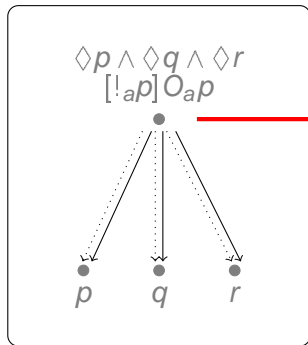
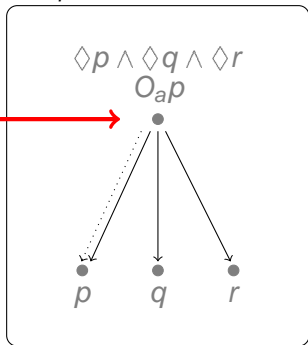
 $\mathcal{M}$ 

 $!_{ap}$ 
 $\mathcal{M}_{!_{ap}}$ 


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# Some interesting principles

## CUGO Principle

If  $\varphi$  is a formula of  $\mathcal{L}_{\text{MDL}^+}$  and is free of occurrences of modal formulas of the form  $O_i$ , then  $[!_i\varphi]O_i\varphi$  is valid.

## Dead End Principles

$[!_i(\varphi \wedge \neg\varphi)]O_i\psi$  is valid.

## Restricted Sequential Conjunction

If  $\varphi$  and  $\psi$  are formulas of  $\mathcal{L}_{\text{MDL}^+}$  and are free of occurrences of modal formulas of the form  $O_i$ , then  $[!_i\varphi][!_i\psi]\chi \leftrightarrow [!_i(\varphi \wedge \psi)]\chi$  is valid.

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# The proof system for ECL

## Definition

The proof system for ECL includes all the axioms and all the rules of the proof system for MDL<sup>+</sup>, and in addition, the following rule and axioms:

$$(!\text{-nec}) \quad \frac{\psi}{[!i\varphi]\psi} \quad (\text{for each } i \in I)$$

(To be continued)

# The proof system for ECL (continued)

## Continued

- (!1)  $[!i\varphi]p \leftrightarrow p$
- (!2)  $[!i\varphi]\top \leftrightarrow \top$
- (!3)  $[!i\varphi]\neg\psi \leftrightarrow \neg[!i\varphi]\psi$
- (!4)  $[!i\varphi](\psi \wedge \chi) \leftrightarrow [!i\varphi]\psi \wedge [!i\varphi]\chi$
- (!5)  $[!i\varphi]\Box\psi \leftrightarrow \Box[!i\varphi]\psi$
- (!6)  $[!i\varphi]O_j\psi \leftrightarrow O_j[!i\varphi]\psi \quad (i \neq j)$
- (!7)  $[!i\varphi]O_i\psi \leftrightarrow O_i(\varphi \rightarrow [!i\varphi]\psi)$

# Translation from $\mathcal{L}_{ECL}$ to $\mathcal{L}_{MDL+}$

## Definition

$$t(p) = p$$

$$t(\top) = \top$$

$$t(\neg\varphi) = \neg t(\varphi)$$

$$t(\varphi \wedge \psi) = t(\varphi) \wedge t(\psi)$$

$$t(\Box\varphi) = \Box t(\varphi)$$

$$t(O_i\varphi) = O_i t(\varphi)$$

$$t([!_i\varphi]p) = p$$

$$t([!_i\varphi]\top) = \top$$

$$t([!_i\varphi]\neg\psi) = \neg t([!_i\varphi]\psi)$$

$$t([!_i\varphi](\psi \wedge \chi)) = t([!_i\varphi]\psi) \wedge t([!_i\varphi]\chi)$$

$$t([!_i\varphi]\Box\psi) = \Box t([!_i\varphi]\psi)$$

$$t([!_i\varphi]O_j\psi) = O_j t([!_i\varphi]\psi) \quad (i \neq j)$$

$$t([!_i\varphi]O_i\psi) = O_i(t(\varphi) \rightarrow t([!_i\varphi]\psi))$$

$$t([!_i\varphi][!_j\psi]\chi) = t([!_i\varphi]t([!_j\psi]\chi))$$

(for any  $j \in I$ )

# Some results (Yamada, 2007a)

## Theorem

*There is a complete axiomatization of ECL.*



- 1 Introduction
- 2 DEL and A dynamic logic of acts of commanding
- 3 **Refinements and Variations**
  - Conflicting commands
  - Acts of commanding and promising
  - Obligations and preferences
  - Assertions, concessions and their withdrawals
- 4 Acts of requesting
  - Selecting base logic (Steps 1 and 2 of the recipe)
  - Dynamifying MEDL (Step 3)
  - Dynamic logic DMEDL (Steps 4 & 5)

# Contradictory commands from two distinct authorities

## A dilemma

$$[!(a,b)p][!(a,c)\neg p](O_{(a,b)}p \wedge O_{(a,c)}\neg p) \ .$$

Note that this does not lead to deontic explosion.

## Example 2: Conflicting commands from your boss and your guru

### A contingent dilemma

$$[!_{(a,b)}p][!_{(a,c)}q](O_{(a,b)}p \wedge O_{(a,c)}q) \wedge \neg(p \wedge q) .$$

$p$  You will attend the conference in São Paulo on 11 June 2012.

$q$  You will join the demonstration in Sapporo on 11 June 2012.

## Some results (Yamada, 2007b)

### CUGO Principle

If  $\varphi$  is a formula of MDL<sup>+</sup>II and is free of modal operators of the form  $O_{(i,j)}$ ,  $[!(i,j)\varphi]O_{(i,j)}\varphi$  is valid.

### Theorem

*There is a complete axiomatization of ECLII.*

## Some results (Yamada, 2007b)

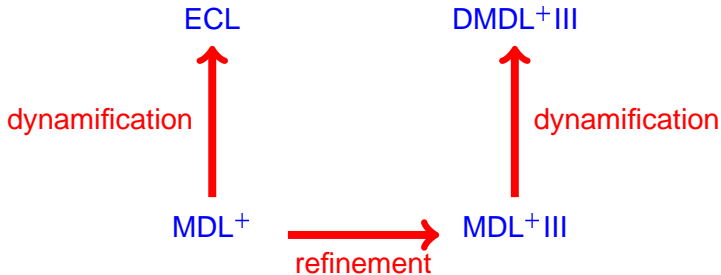
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# A further refinement and extension (Yamada 2008a)

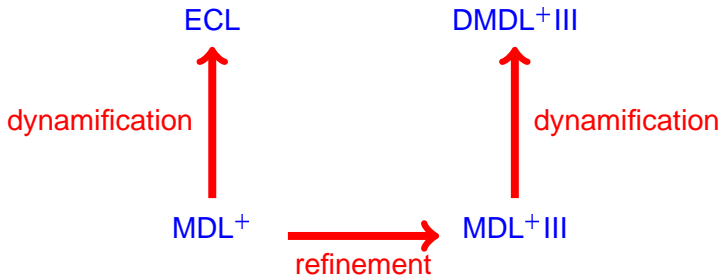


$O_{(i,j,k)}\varphi$  It is obligatory upon an agent  $i$  with respect to an obligee  $j$  in the name of  $k$  to see to it that  $\varphi$ .

$Com_{(i,j)}\varphi$  Act of commanding.

$Prom_{(i,j)}\varphi$  Act of promising.

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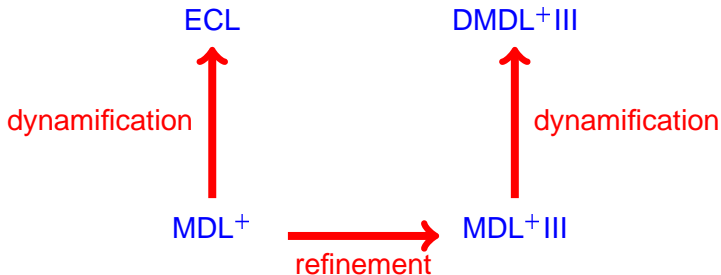


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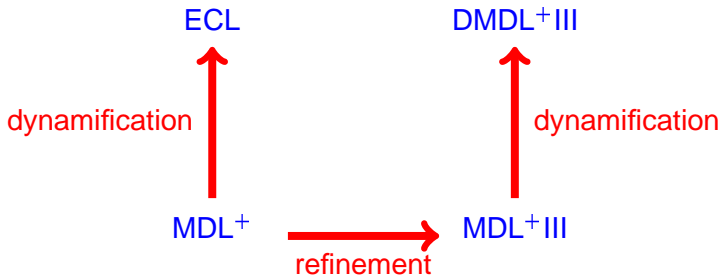
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## Example 3: a command and a promise can lead to a dilemma

### A contingent dilemma

$$[Prom_{(a,b)} p][Com_{(c,a)} q](O_{(a,b,a)} p \wedge O_{(a,c,c)} q) \wedge \neg(p \wedge q) .$$

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## Some results (Yamada, 2008a)

### CUGO Principle

If  $\varphi$  is a formula of MDL<sup>+</sup>III and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Com_{(i,j)}\varphi]O_{(j,i,i)}\varphi$  is valid.

### PUGO Principle

If  $\varphi$  is a formula of MDL<sup>+</sup>III and is free of modal operators of the form  $O_{(i,j,i)}$ ,  $[Prom_{(i,j)}\varphi]O_{(i,j,i)}\varphi$  is valid.

### Theorem

*There is a complete axiomatization of DMDL<sup>+</sup>III.*

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# The same strategy works for changing preferences (van Benthem and Liu, 2007) (Liu, 2008)

Dynamic Epistemic Upgrade Logic DEUL

adding dynamic  
modalities



translation along  
reduction axioms



Epistemic Preference Logic EPL

# Combining preference upgrades and deontic updates (Yamada 2008b)



# The language of DPL

## Definition

Take a set  $Aprop$  of proposition letters, and a set  $I$  of agents, with  $p$  ranging over  $Aprop$  and  $i, j$  over  $I$ . The deontic preference language is given by:

$$\varphi ::= \perp \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid U\varphi \mid [pref]_i\varphi \mid O_{(i,j)}\varphi$$



# The language of DDPL

## Definition

Take a set  $Aprop$  of proposition letters, and a set  $I$  of agents, with  $p$  ranging over  $Aprop$  and  $i, j$  over  $I$ . The dynamic deontic preference language is given by:

$$\begin{aligned} \varphi & ::= \perp \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid U\varphi \mid [pref]_i\varphi \mid O_{(i,j)}\varphi \mid [\pi]\varphi \\ \pi & ::= \#_i\varphi \mid !(i,j)\varphi \end{aligned}$$

## Some results (Yamada, 2008b)

### Theorem

*There is a complete axiomatization of DDPL.*

The following formulas are satisfiable.

$$O_{(i,j)}p \wedge U(p \rightarrow \langle \text{pref} \rangle_i \neg p) .$$

$$[!_{(i,j)}p]U(p \rightarrow \langle \text{pref} \rangle_i \neg p) .$$

$\langle \text{pref} \rangle_i \varphi$  is an abbreviation of  $\neg[\text{pref}]_i \neg \varphi$ .

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# The same recipe works for acts of asserting and conceding (Yamada, to appear)

## Dynamified Multiagent Propositional Commitment Logic

DMPCL

adding dynamic  
modalities



translation along  
reduction axioms



MPCL

Multi-agent Propositional Commitment Logic

# Walton & Krabbe (1995)

## Three Kinds of propositional commitments

- commitments incurred by making concessions
- commitments called assertions
- participant's dark-side commitments

Since dark-side commitments are hidden commitments and supposed to be fixed, we will ignore them.

We call the remaining two kinds of commitments c-commitments and a-commitments respectively.

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# A-commitments and c-commitments

According to Walton and Krabbe (1995, p.186)

Propositional commitments constitute a special case of commitments to a course of action.

- an agent who has an a-commitment to the proposition  $p$  is obliged to defend it if the other party in the dialogue require her to justify it
- an agent who has a c-commitments to  $p$  is only obliged to allow the other party to use it in the arguments.

As anyone who asserts that  $p$  will be obliged to allow the other party to use it in the arguments, a-commitments imply c-commitments.



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# The language of MPCL

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Take a countably infinite set  $Aprop$  of proposition letters, and a finite set  $I$  of agents, with  $p$  ranging over  $Aprop$ , and  $i$  over  $I$ . The language  $\mathcal{L}_{MPCL}$  of the multi-agent propositional commitment logic MPCL is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid [a-cmt]_i\varphi \mid [c-cmt]_i\varphi$$

$[a-cmt]_i\varphi$ : an agent  $i$  has an a-commitment to the proposition  $\varphi$ ,

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# P-commitments are different from knowledge

The following formulas are not valid.

$$[a-cmt]_i \varphi \rightarrow \varphi$$

$$[c-cmt]_i \varphi \rightarrow \varphi$$

Cf.  $K_i \varphi \rightarrow \varphi$

# P-commitments are different from belief

The following formulas are not valid.

$$\neg[a\text{-cmt}]_i \perp$$

$$\neg[c\text{-cmt}]_i \perp$$

Cf.  $\neg B_i \perp$



# $\mathcal{L}_{\text{MPCL}}$ -models

## Definition

By an  $\mathcal{L}_{\text{MPCL}}$ -model, we mean a tuple

$M = \langle W^M, \{\triangleright_i^M \mid i \in I\}, \{\blacktriangleright_i^M \mid i \in I\}, V^M \rangle$  where:

- (i)  $W^M$  is a non-empty set (heuristically, of ‘possible worlds’),
- (ii)  $\triangleright_i^M \subseteq W^M \times W^M$  for each  $i \in I$ ,
- (iii)  $\blacktriangleright_i^M \subseteq \triangleright_i^M$  for each  $i \in I$ ,
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- (iii)  $\blacktriangleright_i^M \subseteq \triangleright_i^M$  for each  $i \in I$ ,
- (iv)  $V^M$  is a function that assigns a subset  $V^M(p)$  of  $W^M$  to each proposition letter  $p \in \text{Aprop}$ .

# Truth definition for $\mathcal{L}_{MPCL}$ (crucial part)

In addition to the standard clauses for proposition letters and Boolean operations,

- (e)  $M, w \models_{MPCL} [a\text{-cmt}]_i \varphi$  iff for every  $v$  such that  
 $\langle w, v \rangle \in \triangleright_i^M, M, v \models_{MPCL} \varphi$
- (f)  $M, w \models_{MPCL} [c\text{-cmt}]_i \varphi$  iff for every  $v$  such that  
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# The Proof system for MPCL

## Definition

The proof system for MPCL includes (i) all instantiations of propositional tautologies over the present language, (ii) K-axioms for  $[a\text{-cmt}]_i$ -modality and  $[c\text{-cmt}]_i$ -modality for each  $i \in I$ , (iii) modus ponens, and (iv) necessitation rules for  $[a\text{-cmt}]_i$ -modality and  $[c\text{-cmt}]_i$ -modality for each  $i \in I$ , in addition to the axiom of the following form for each  $i \in I$ :

$$\text{(Mix)} \quad [a\text{-cmt}]_i \varphi \rightarrow [c\text{-cmt}]_i \varphi$$

## Theorem (Completeness of MPCL)

MPCL is strongly complete with respect to  $\mathcal{L}_{\text{MPCL}}$ -models.

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# Closure

Propositional commitments are closed with respect to the logical consequence.

$$([\mathbf{a-cmt}]_i \varphi \wedge [\mathbf{a-cmt}]_i (\varphi \rightarrow \psi)) \rightarrow [\mathbf{a-cmt}]_i \psi$$

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Rational agents should withdraw at least one of their assertions or concessions if some unwanted consequences are derived from what they have explicitly asserted or conceded.

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# The language of DMPCL

## Definition

Take the same countably infinite set  $Aprop$  of proposition letters and the same finite set  $I$  of agents as before, with  $p$  ranging over  $Aprop$ , and  $i$  over  $I$ . The language  $\mathcal{L}_{\text{DMPCL}}$  of dynamified multi-agent propositional commitment logic DMPCL is given by:

$$\begin{aligned} \varphi & ::= \top \mid \mathbf{p} \mid \neg\varphi \mid \varphi \wedge \psi \mid [\mathbf{a-cmt}]_i\varphi \mid [\mathbf{c-cmt}]_i\varphi \mid [\pi]\varphi \\ \pi & ::= \mathbf{assert}_i\varphi \mid \mathbf{concede}_i\varphi \end{aligned}$$

# The truth definition for $\mathcal{L}_{\text{DMPCL}}$

## Definition

Let  $M$  be an  $\mathcal{L}_{\text{MPCL}}$ -model and  $w$  a point in  $M$ . If  $p \in \text{Aprop}$ , and  $i \in I$ , then the truth definition for  $\mathcal{L}_{\text{DMPCL}}$  is given by expanding that of  $\mathcal{L}_{\text{MPCL}}$  mutatis mutandis with the following new clause:

(g)  $M, w \models_{\text{DMPCL}} [\text{assert}_i \chi] \varphi$  iff  $M_{\text{assert}_i \chi}, w \models_{\text{DMPCL}} \varphi$

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# The proof system for $\mathcal{L}_{\text{DMPCL}}$

## Definition

The proof system for DMPCL includes all the axioms and all the rules of the proof system for MPCL, and in addition, necessitation rules for assertion modality and concession modality for each  $i \in I$ , and the following axioms:

- |      |  |                   |  |                |
|------|--|-------------------|--|----------------|
| (A1) | $[\text{assert}_i \varphi] p$                        | $\leftrightarrow$ | $p$  |                |
| (A2) | $[\text{assert}_i \varphi] \top$                     | $\leftrightarrow$ | $\top$   |                |
| (A3) | $[\text{assert}_i \varphi] \neg \psi$                | $\leftrightarrow$ | $\neg [\text{assert}_i \varphi] \psi$                                      |                |
| (A4) | $[\text{assert}_i \varphi] (\psi \wedge \chi)$       | $\leftrightarrow$ | $[\text{assert}_i \varphi] \psi \wedge [\text{assert}_i \varphi] \chi$     |                |
| (A5) | $[\text{assert}_i \varphi] [\mathbf{a-cmf}]_j \psi$  | $\leftrightarrow$ | $[\mathbf{a-cmf}]_j [\text{assert}_i \varphi] \psi$                        | $(i \neq j)$   |
| (A6) | $[\text{assert}_i \varphi] [\mathbf{a-cmf}]_i \psi$  | $\leftrightarrow$ | $[\mathbf{a-cmf}]_i (\varphi \rightarrow [\text{assert}_i \varphi] \psi)$  |                |
| (A7) | $[\text{assert}_i \varphi] [\mathbf{c-cmf}]_j \psi$  | $\leftrightarrow$ | $[\mathbf{c-cmf}]_j [\text{assert}_i \varphi] \psi$                        | $(i \neq j)$   |
| (A8) | $[\text{assert}_i \varphi] [\mathbf{c-cmf}]_i \psi$  | $\leftrightarrow$ | $[\mathbf{c-cmf}]_i (\varphi \rightarrow [\text{assert}_i \varphi] \psi)$  |                |
| (C1) | $[\text{concede}_i \varphi] p$                       | $\leftrightarrow$ | $p$  |                |
| (C2) | $[\text{concede}_i \varphi] \top$                    | $\leftrightarrow$ | $\top$   |                |
| (C3) | $[\text{concede}_i \varphi] \neg \psi$               | $\leftrightarrow$ | $\neg [\text{concede}_i \varphi] \psi$                                     |                |
| (C4) | $[\text{concede}_i \varphi] (\psi \wedge \chi)$      | $\leftrightarrow$ | $[\text{concede}_i \varphi] \psi \wedge [\text{concede}_i \varphi] \chi$   |                |
| (C5) | $[\text{concede}_i \varphi] [\mathbf{a-cmf}]_j \psi$ | $\leftrightarrow$ | $[\mathbf{a-cmf}]_j [\text{concede}_i \varphi] \psi$                       | (for any $j$ ) |
| (C6) | $[\text{concede}_i \varphi] [\mathbf{c-cmf}]_j \psi$ | $\leftrightarrow$ | $[\mathbf{c-cmf}]_j [\text{concede}_i \varphi] \psi$                       | $(i \neq j)$   |
| (C7) | $[\text{concede}_i \varphi] [\mathbf{c-cmf}]_i \psi$ | $\leftrightarrow$ | $[\mathbf{c-cmf}]_i (\varphi \rightarrow [\text{concede}_i \varphi] \psi)$ |                |

# Translation from $\mathcal{L}_{\text{DMPCL}}$ to $\mathcal{L}_{\text{MPCL}}$

## Definition

The translation function that takes a formula from  $\mathcal{L}_{\text{DMPCL}}$  and yields a formula in  $\mathcal{L}_{\text{MPCL}}$  is defined as follows:

$t(p)$	$=p$	$t([\text{assert}_i\varphi]p)$	$=p$
		$t([\text{concede}_i\varphi]p)$	$=p$
$t(\top)$	$=\top$	$t([\text{assert}_i\varphi]\top)$	$=\top$
		$t([\text{concede}_i\varphi]\top)$	$=\top$
$t(\neg\varphi)$	$=\neg t(\varphi)$	$t([\text{assert}_i\varphi]\neg\psi)$	$=\neg t([\text{assert}_i\varphi]\psi)$
		$t([\text{concede}_i\varphi]\neg\psi)$	$=\neg t([\text{concede}_i\varphi]\psi)$
$t(\varphi \wedge \psi)$	$=t(\varphi) \wedge t(\psi)$	$t([\text{assert}_i\varphi](\psi \wedge \chi))$	$=t([\text{assert}_i\varphi]\psi) \wedge t([\text{assert}_i\varphi]\chi)$
		$t([\text{concede}_i\varphi](\psi \wedge \chi))$	$=t([\text{concede}_i\varphi]\psi) \wedge t([\text{concede}_i\varphi]\chi)$
$t([\mathbf{a-cm\#}_i\varphi])$	$=[\mathbf{a-cm\#}_i]t(\varphi)$	$t([\text{assert}_i\varphi][\mathbf{a-cm\#}_j\psi])$	$=[\mathbf{a-cm\#}_j]t([\text{assert}_i\varphi]\psi) \quad (i \neq j)$
		$t([\text{assert}_i\varphi][\mathbf{a-cm\#}_j\psi])$	$=[\mathbf{a-cm\#}_j]t(\varphi \rightarrow [\text{assert}_i\varphi]\psi)$
		$t([\text{concede}_i\varphi][\mathbf{a-cm\#}_j\psi])$	$=[\mathbf{a-cm\#}_j]t([\text{concede}_i\varphi]\psi)$
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## Some results

### Proposition

If  $\varphi \in \mathcal{L}_{\text{MPCL}}$  is free of modalities indexed by  $i$ , the following formulas are valid:

$$\begin{aligned} & [\text{assert}_i\varphi][\mathbf{a-cmt}]_i\varphi \\ & [\text{assert}_i\varphi][\mathbf{c-cmt}]_i\varphi \\ & [\text{concede}_i\varphi][\mathbf{c-cmt}]_i\varphi . \end{aligned}$$

### Theorem

*There is a complete axiomatization of DMPCL.*

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### Theorem

*There is a complete axiomatization of DMPCL.*

# Does the same strategy work for acts of asserting and conceding combined with acts of withdrawing ?

## Dynamified Multiagent Propositional Commitment Logic with withdrawals DMPCL<sup>+</sup>

adding dynamic  
modalities



translation  
available ?



## Multi-agent Propositional Commitment Logic MPCL



# The language of DMPCL<sup>+</sup>

## Definition

Take the same countably infinite set  $Aprop$  of proposition letters and the same finite set  $I$  of agents as before, with  $p$  ranging over  $Aprop$ , and  $i$  over  $I$ . The language  $\mathcal{L}_{DPCMT^+}$  of dynamified multi-agent propositional commitment logic with withdrawals DMPCL<sup>+</sup> is given by:

$$\begin{aligned} \varphi &::= \top \mid \mathbf{p} \mid \neg\varphi \mid \varphi \wedge \psi \mid [\mathbf{a-cmf}]_i\varphi \mid [\mathbf{c-cmf}]_i\varphi \mid [\pi]\varphi \\ \pi &::= \text{assert}_i\varphi \mid \text{concede}_i\varphi \mid \bigcirc\text{assert}_i\varphi \mid \bigcirc\text{concede}_i\varphi \end{aligned}$$

# An update by withdrawing?

A sequence of acts:  $\dots, \text{assert}_i\chi, \text{assert}_j\xi, \text{assert}_i\eta, \dots$

$\Downarrow \text{Oassert}_j\xi$

A reduced sequence:  $\dots, \text{assert}_i\chi, \text{assert}_i\eta, \dots$

The set of propositional commitments agents bear after  $j$ 's act of withdrawing of the form  $\text{Oassert}_j\xi$  will be, other things being equal, the same as the set of propositional commitments they would bear if  $j$  had not asserted that  $\xi$ .

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## A positive commitment act sequence

If  $\sigma$  is a sequence of moves in an argumentation, it may involve not only acts of asserting and conceding but also acts of withdrawing. We call it a commitment affecting act sequence, or *caa*-sequence for short.

We will first consider a special kind of sequences, namely, a sequence  $\sigma = \langle \pi_1, \pi_2, \dots, \pi_n \rangle$  of speech acts  $\pi_j$  ( $1 \leq j \leq n$ ) such that each  $\pi_j$  is either of the form  $\text{assert}_i \varphi$  for some  $i \in I$  or of the form  $\text{concede}_i \varphi$  for some  $i \in I$ . We call such a sequence a positive commitment act sequence, or a *pca*-sequence for short.

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# Reduced positive commitment act sequence

## Definition

Let  $\sigma$  be a (possibly empty) positive commitment act sequence  $\langle \pi_1, \dots, \pi_n \rangle$  such that each  $\pi_j$  ( $1 \leq j \leq n$ ) is of the form  $\text{assert}_i \varphi$  for some  $i \in I$  or of the form  $\text{concede}_i \varphi$  for some  $i \in I$ . We define the reduced sequence  $\sigma \upharpoonright \text{Oassert}_i \varphi$  ( $\sigma \upharpoonright \text{Oconcede}_i \varphi$ ) obtained by withdrawing every occurrence of an act of type  $\text{assert}_i \varphi$  ( $\text{concede}_i \varphi$ ) from  $\sigma$  as follows:

(To be continued)



## Reduced pca-sequence (continued)

$$\sigma \mid \mathcal{O} \text{assert}_i \varphi$$

$$= \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \pi_1, \dots, \pi_{n-1} \rangle \mid \mathcal{O} \text{assert}_i \varphi & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \text{assert}_i \varphi \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle \mid \mathcal{O} \text{assert}_i \varphi, \pi_n \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n \neq \text{assert}_i \varphi \end{cases}$$

and

$$\sigma \mid \mathcal{O} \text{concede}_i \varphi$$

$$= \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \pi_1, \dots, \pi_{n-1} \rangle \mid \mathcal{O} \text{concede}_i \varphi & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \text{concede}_i \varphi \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle \mid \mathcal{O} \text{concede}_i \varphi, \pi_n \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n \neq \text{concede}_i \varphi . \end{cases}$$

# How to work with arbitrary sequence

## definition

Given an arbitrary caa-sequence  $\sigma$  possibly involving acts of withdrawing as well as acts of asserting and acts of conceding, we define its corresponding pca-sequence  $\sigma^*$  as follows:

$$\sigma^* = \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle^*, \text{assert}_i \varphi \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \text{assert}_i \varphi \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle^*, \text{concede}_i \varphi \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \text{concede}_i \varphi \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle^* | \odot \text{assert}_i \varphi \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \odot \text{assert}_i \varphi \\ \langle \langle \pi_1, \dots, \pi_{n-1} \rangle^* | \odot \text{concede}_i \varphi \rangle & \text{if } \sigma = \langle \pi_1, \dots, \pi_n \rangle, \text{ and } \pi_n = \odot \text{concede}_i \varphi \end{cases}$$

# The Problem of Notation

Given a pca-sequence  $\sigma = \langle \pi_1, \dots, \pi_n \rangle$ , the model obtained by updating  $M$  with  $\sigma$  is denoted by  $(\dots (M_{\pi_1}) \dots)_{\pi_n}$  in the notation of the truth definition for  $\mathcal{L}_{\text{DMPCL}}$ .

This notation leads to a paradox when we deal with withdrawals. Let abbreviate  $(\dots (M_{\pi_1}) \dots)_{\pi_n}$  as  $M_\sigma$ . Now there may be another model  $N$  and a pcs-sequence  $\tau$  such that  $N_\tau = M$ . Then we might have

$$(N_\tau)_\sigma = M_\sigma \text{ but } ((N_\tau)_\sigma)_{\circlearrowleft \text{concede}_i \varphi} \neq (M_\sigma)_{\circlearrowleft \text{concede}_i \varphi}.$$

# Truth Definition 1/4

## Definition

Let  $M$  be an  $\mathcal{L}_{\text{MPCL}}$ -model,  $\sigma$  an arbitrary caa-sequence,  $\sigma^*$  the corresponding pca-sequence of  $\sigma$ , and  $w$  a point in  $M$ . If  $p \in \text{Aprop}$ , and  $i \in I$ , then:

- (a)  $M, \sigma, w \models_{\text{DMPCL}+} p$  iff  $w \in V^M(p)$
- (b)  $M, \sigma, w \models_{\text{DMPCL}+} \top$
- (c)  $M, \sigma, w \models_{\text{DMPCL}+} \neg \varphi$  iff it is not the case that  
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- (d)  $M, \sigma, w \models_{\text{DMPCL}+} (\varphi \wedge \psi)$  iff  $M, \sigma, w \models_{\text{DMPCL}+} \varphi$  and  
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# Truth Definition 2/4

- (e)  $M, \sigma, w \models_{\text{DMPCL}^+} [\mathbf{a-cmt}]_i \varphi$  iff for all  $v$  s. t.  $\langle w, v \rangle \in \triangleright_i^M \upharpoonright \sigma^*$ ,  
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- (f)  $M, \sigma, w \models_{\text{DMPCL}^+} [\mathbf{c-cmt}]_i \varphi$  iff for all  $v$  s. t.  $\langle w, v \rangle \in \blacktriangleright_i^M \upharpoonright \sigma^*$ ,  
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- (g)  $M, \sigma, w \models_{\text{DMPCL}^+} [\mathbf{assert}]_i \chi$  iff  $M, \langle \sigma, \mathbf{assert}_i \chi \rangle$ ,  
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- (h)  $M, \sigma, w \models_{\text{DMPCL}^+} [\mathbf{concede}]_i \chi$  iff  $M, \langle \sigma, \mathbf{concede}_i \chi \rangle$ ,  
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# Truth Definition 3/4

- (i)  $M, \sigma, w \models_{\text{DMPCL}^+} [\odot \text{assert}_i \chi] \varphi$  iff  $M, \sigma^* \uparrow \odot \text{assert}_i \chi,$   
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# Truth Definition 4/4

$$\triangleright_i^M \upharpoonright \sigma^* = \begin{cases} \triangleright_i^M & \text{if } \sigma^* \text{ is empty,} \\ \{ \langle \mathbf{x}, \mathbf{y} \rangle \in \triangleright_i^M \upharpoonright \langle \pi_1, \dots, \pi_{n-1} \rangle \mid M, \langle \pi_1, \dots, \pi_{n-1} \rangle, \mathbf{y} \models_{\text{DMPCL}^+} \psi \} & \text{if } \sigma^* = \langle \pi_1, \dots, \pi_n \rangle \text{ and } \pi_n = \text{assert}_i \psi, \\ \triangleright_i^M \upharpoonright \langle \pi_1, \dots, \pi_{n-1} \rangle & \text{if } \sigma^* = \langle \pi_1, \dots, \pi_n \rangle \text{ and } \pi_n \neq \text{assert}_i \psi, \end{cases}$$

and

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# A result and an open problem

## A result

Acts of withdrawing behave slightly differently from contraction studied in belief revision. Let  $\mathcal{B}$  be a set of beliefs of an agent, say  $a$ . Then in the AGM approach, contraction  $\ominus$  is supposed to satisfy the postulate that  $\varphi \notin \mathcal{B} \ominus \varphi$  if  $\not\vdash \varphi$ , but we have, for example,  $M, \sigma \models \text{assert}_a p, w \models_{\text{DMPCL}^+} [\text{a-cmt}]_a p$  if  $\sigma$  include  $\text{assert}_a q$  and  $\text{assert}_a(q \rightarrow p)$ .

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- 1 Introduction
- 2 DEL and A dynamic logic of acts of commanding
- 3 Refinements and Variations
  - Conflicting commands
  - Acts of commanding and promising
  - Obligations and preferences
  - Assertions, concessions and their withdrawals
- 4 Acts of requesting
  - Selecting base logic (Steps 1 and 2 of the recipe)
  - Dynamifying MEDL (Step 3)
  - Dynamic logic DMEDL (Steps 4 & 5)

# Motivations

In order to develop Austinian conception of illocutionary acts into a general theory, we have to

- specify conventional effects of a sufficiently rich variety of illocutionary acts, and
- develop a theory in which these illocutionary acts are shown to be fully characterized in terms of those conventional effects.

Although it seems intuitively clear that acts of requesting are different from acts of commanding (or ordering), it is not very easy to state their differences exactly.

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## Motivations (2)

If we have a good analysis of acts of requesting, it will yield a straightforward formulation of acts of asking questions as requests for information (or knowledge).

### Asking yes-no questions

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# Preliminary analysis of the effects of acts of requesting

In the case of acts of requesting, but not in the case of acts of commanding, refusals are among legitimate responses. In this sense, an act of requesting does not generate an obligation to do what is requested.

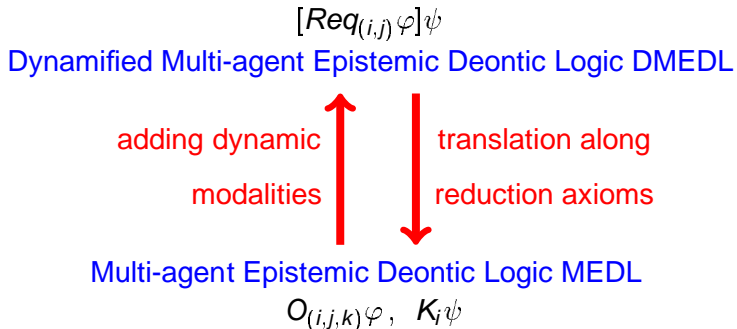
But when you are requested to do something, it would not be fully unproblematic for you to ignore the request without giving any response. At least you have to decide whether you should accept the request or not, and let the requester know your decision.

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# A plan of DMEDL





# The language of MEDL

We extend the language of MDL<sup>+</sup> III by adding an epistemic operator  $K_i$  for each agent  $i \in I$ .

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid O_{(i,j,k)}\varphi \mid K_i\varphi$$

For simplicity, we ignore alethic modality here.

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$$\begin{aligned} \varphi &::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid O_{(i,j,k)}\varphi \mid K_i\varphi \mid [\pi]\varphi \\ \pi &::= Com_{(i,j)}\varphi \mid Prom_{(i,j)}\varphi \mid Req_{(i,j)}\varphi \end{aligned}$$

The formula of the form  $[Req_{(i,j)}\varphi]\psi$  means that after an agent  $i$ 's act of requesting  $j$  to see to it that  $\varphi$ ,  $\psi$  holds.

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# The language of DMEDL

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$$\begin{aligned} \varphi & ::= \top \mid \mathbf{p} \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{O}_{(i,j,k)}\varphi \mid \mathbf{K}_i\varphi \mid [\pi]\varphi \\ \pi & ::= \mathbf{Com}_{(i,j)}\varphi \mid \mathbf{Prom}_{(i,j)}\varphi \mid \mathbf{Req}_{(i,j)}\varphi \end{aligned}$$

The formula of the form  $[\mathbf{Req}_{(i,j)}\varphi]\psi$  means that after an agent  $i$ 's act of requesting  $j$  to see to it that  $\varphi$ ,  $\psi$  holds.

# CUGO principle and PUGO principle

## CUGO Principle

If  $\varphi$  is a formula of MEDL and is free of modal operators of the form  $O_{(j,i,i)}$ ,  $[Com_{(i,j)}\varphi]O_{(j,i,i)}\varphi$  is valid.

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# The effects of acts of requesting

The foregoing discussions suggest the following principle.

## RUGO Principle

If  $\varphi$  is a formula of  $MEDL^+ III$  and is free of modal operators of the form  $O_{(j,i,i)}$ , formulas of the following form are valid:

$$[Req_{(i,j)}\varphi]O_{(j,i,i)}(K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi) .$$

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# Acts of requesting in DMEDL

## Truth definition

$M, w \models [Req_{(i,j)}\varphi]\psi$  iff  $M_{Req_{(i,j)}\varphi}, w \models \psi$ ,

where  $M_{Req_{(i,j)}\varphi}$  is a model of DMEDL obtained from  $M$  by replacing deontic accessibility relation  $\sim_{(j,i,i)}^M$  with its subset  $\{\langle x, y \rangle \in \sim_{(j,i,i)}^M \mid M, y \models K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi\}$ .

# Requesting and commanding

## RUGO Principle and CUGO Principle

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By CUGO principle, we also have:

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Thus we have:

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# Equivalence?

$$M_{Req(i,j)}\varphi = M_{Com(i,j)}(K_i O_{(j,i,j)}\varphi \vee K_i \neg O_{(j,i,j)}\varphi)$$

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## Further extension

We have ignored the difference in the modes of achievement.  
Cf. Searle(1985), Geis(1995).

A further extension is necessary in order to differentiate acts of requesting from acts of commanding.

In fact, this extension is anyway necessary in order to differentiate acts of commanding from acts of ordering and other similar directive illocutionary acts.



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