Dynamic Logic of Propositional Commitments*

Tomoyuki Yamada

1 Introduction

A number of systems of dynamic epistemic logic have been developed recently as extensions of static epistemic logic by Plaza [11], Gerbrandy & Groeneveld [5], Gerbrandy [4], Baltag, Moss & Solecki [2], and Kooi & van Benthem [7] among others.[2] In these systems, dynamic changes brought about by various kinds of information transmission including public announcements as well as private communications are studied, and these communicative acts are interpreted as events that update epistemic states of some or all of the agents involved. More recently, inspired by these developments, acts of commanding and acts of promising have been modeled as updators of deontic statuses of various alternative courses of actions available to agents involved in social interactions, and "dynamified" deontic logics have been developed as extensions of multi-agent variants of static deontic logic by Yamada in [16], [17], [18] and [19]. The same strategy has also been applied in developing dynamic logics of preference change by van Benthem & Liu [13] and Liu [9].

These developments suggest the following general recipe for developing various logics that deal with particular kinds of speech acts:

- first, carefully identify the aspects of the situations affected by the speech acts that you want to study, and find or develop a static modal logic that characterizes the aspects identified,

Tomoyuki Yamada
Hokkaido University, Sapporo, Japan, e-mail: yamada@LET.hokudai.ac.jp

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[2] A detailed, state-of-the-art textbook exposition of dynamic epistemic logic can be found in van Ditmarsch, van der Hoek & Kooi [14].

- next, add dynamic modal operators that stand for the types of the speech acts being studied, and define model updating operations that interpret these speech acts as what update the very aspects,
- and then finally, if possible, find a complete set of reduction axioms which enables you to derive the completeness of the dynamified logic from the completeness of the static logic.

The purpose of this paper is to describe how the effects of acts of making assertions, acts of making concessions, and acts of withdrawing assertions and concessions can be captured by developing dynamic logics according to this recipe.[3]

As the recipe dictates, our first task is to identify the aspects affected by these speech acts, and find a modal logic that characterizes these aspects. We do this in Sect. 2. Our working hypothesis is that these acts update the sets of so called "propositional commitments" the agents bear, and we develop a static logic, MPCL, which deals with propositional commitments in a multi-agent environment.

According to the above recipe, our next task is to "dynamify" MPCL in order to characterize the logical dynamics of changing propositional commitments. For technical reasons, we do this in two steps. First, in Sect. 3 we extend MPCL into DMPCL, dynamified MPCL, by adding two kinds of modal operators standing for acts of asserting and acts of conceding respectively, and present a complete set of reduction axioms for it. Then, in Sect. 4, we further extend it into DMPCL^+, by adding another two kinds of modal operators standing for acts of withdrawing assertions and acts of withdrawing concessions respectively. As may be expected, the effects of acts of withdrawing turn out to be very difficult to capture, and the completeness problem for DMPCL^+ is still open. Yet the possibility of withdrawal seems to be a distinguishing characteristic common to a wide range of acts whose effects are conventional or institutional, and so the logical dynamics of withdrawal seem to be of considerable significance to the study of social interactions among rational agents. We make a brief comparison with the AGM approach to belief revision in the same section. And finally, in Sect. 5, we briefly consider an application of DMPCL^+ to scorekeeping for argumentation games.

2 The Static Base Logic MPCL

In the literature on speech act theory, agents who make assertions are usually said to be committed to the truth of their assertions (for example, see [12]). The kind of commitments incurred are sometimes called "propositional commitments" in argumentation theory. The notion of propositional commitment is introduced into the study of dialogue by Hamblin in [6], and is further studied by Walton and Krabbe in [15] with reference to a particular kind of dialogue called "persuasion dialogue". Walton and Krabbe recognize three types of commitment, namely, (1) commitments

[3] This recipe was presented at the XXII World Congress of Philosophy, 30 July - 5 August, 2008, Seoul, Korea.

incurred by making concessions, (2) commitments called assertions, and (3) participants' dark-side commitments ([15], pp.186-7). In this paper, we treat acts of asserting and conceding as updators of the sets of propositional commitments agents bear following Walton and Krabbe, but only consider the first two types of "commitments" as commitments. Since "dark-side commitments" are "hidden or veiled commitments" that are supposed to be fixed, they will not be affected by the kinds of speech acts to be studied; they can be modeled as hidden beliefs. We refer to the first type of commitments as "c-commitments" and the second type of commitments as "a-commitments", reserving the term "assertion" for acts of asserting.

According to Walton and Krabbe ([15], p.8), propositional commitments constitute a special case of commitment to a course of action. The main difference between c-commitments and a-commitments lies in the fact that an agent who has an a-commitment to the proposition p is obliged to defend it if the other party in the dialogue requires her to justify it, while an agent who has a c-commitment to the proposition p is only obliged to allow the other party to use it in the arguments ([15], p.186). As anyone who asserts that p will be obliged to allow the other party to use it in the arguments, a-commitments imply c-commitments.

Our next task is to develop a static modal logic that deals with a-commitments and c-commitments. First, we define the language.

Definition 1. Take a countably infinite set *Aprop* of proposition letters, and a finite set I of agents, with p ranging over *Aprop*, and i over I. The language $\mathscr{L}_{\mathsf{MPCL}}$ of the multi-agent propositional commitment logic MPCL is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid [\text{a-cmt}]_i\varphi \mid [\text{c-cmt}]_i\varphi\ .$$

Intuitively, a formula of the form $[\text{a-cmt}]_i\varphi$ means that the agent i has an a-commitment to the proposition φ, and a formula of the form $[\text{c-cmt}]_i\varphi$ means that i has a c-commitment to φ. We will also say that the agent i is a-committed and c-committed to φ when we have $[\text{a-cmt}]_i\varphi$ and $[\text{c-cmt}]_i\varphi$ respectively. We use $\langle\text{a-cmt}\rangle_i\varphi$ and $\langle\text{c-cmt}\rangle_i\varphi$ as the abbreviations of $\neg[\text{a-cmt}]_i\neg\varphi$ and $\neg[\text{c-cmt}]_i\neg\varphi$ respectively, in addition to the standard abbreviations such as $\vee$, $\rightarrow$, etc.

Next we examine some general principles captured in the language just defined. As anyone who asserts that p will be expected to believe or know that p, and anyone who concedes that p will be expected not to know or believe that $\neg p$, the logic of propositional commitments may be expected to be similar to epistemic logic and doxastic logic. But there are some differences. First, unlike the knowledge that p (but similar to the belief that p), one's propositional commitment to p does not entail p. Propositional commitments are not veridical. Thus the following formulas are not valid:

$$[\text{a-cmt}]_i\varphi \rightarrow \varphi$$
$$[\text{c-cmt}]_i\varphi \rightarrow \varphi\ .$$

This means that when we build possible worlds models for interpreting sentences of this language, we should not assume that the accessibility relations for propositional commitments are reflexive. Second, although one's beliefs are often supposed to be consistent in doxastic logic, one's set of propositional commitments can be inconsistent. Thus the following formulas are not valid:

$$\neg[\text{a-cmt}]_i\bot$$
$$\neg[\text{c-cmt}]_i\bot \ .$$

This means that we should not assume that the accessibility relations for propositional commitments are serial. Moreover, it is not clear whether the following analogues of the so-called positive and negative introspection axioms of epistemic and doxastic logics are valid or not:

$$[\text{a-cmt}]_i\varphi \to [\text{a-cmt}]_i[\text{a-cmt}]_i\varphi$$
$$\neg[\text{a-cmt}]_i\varphi \to [\text{a-cmt}]_i\neg[\text{a-cmt}]_i\varphi$$
$$[\text{c-cmt}]_i\varphi \to [\text{c-cmt}]_i[\text{c-cmt}]_i\varphi$$
$$\neg[\text{c-cmt}]_i\varphi \to [\text{c-cmt}]_i\neg[\text{c-cmt}]_i\varphi \ .$$

Leaving the discussion of these introspection principles for further research, we will only assume K-axioms and necessitation rules for a-commitments and c-commitments in addition to the assumption that each a-commitment implies its corresponding c-commitment.

Thus, we define:

Definition 2. An $\mathscr{L}_{\mathsf{MPCL}}$-model is a tuple $M = \langle W^M, \{\rhd_i^M \mid i \in I\}, \{\blacktriangleright_i^M \mid i \in I\}, V^M\rangle$ satisfying the following conditions:

(i) W^M is a non-empty set (heuristically, of 'possible worlds'),

(ii) $\rhd_i^M \subseteq W^M \times W^M$ for each $i \in I$,

(iii) $\blacktriangleright_i^M \subseteq \rhd_i^M$ for each $i \in I$,

(iv) V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in Aprop$.

We sometimes refer to a possible world $w \in W^M$ as a point in W^M or in M as well.

A truth definition for $\mathscr{L}_{\mathsf{MPCL}}$ can be given in a standard way by associating the modal operators $[\text{a-cmt}]_i$ and $[\text{c-cmt}]_i$ with $\rhd_i^M$ and $\blacktriangleright_i^M$ respectively. Thus:

Definition 3. Let M be an $\mathscr{L}_{\mathsf{MPCL}}$-model and w a point in W^M. If $p \in Aprop$, and $i \in I$, then:

(a)	$M,w \models_{\mathsf{MPCL}} p$	iff	$w \in V^M(p)$
(b)	$M,w \models_{\mathsf{MPCL}} \top$		
(c)	$M,w \models_{\mathsf{MPCL}} \neg\varphi$	iff	it is not the case that $M,w \models_{\mathsf{MPCL}} \varphi$
(d)	$M,w \models_{\mathsf{MPCL}} (\varphi \wedge \psi)$	iff	$M,w \models_{\mathsf{MPCL}} \varphi$ and $M,w \models_{\mathsf{MPCL}} \psi$
(e)	$M,w \models_{\mathsf{MPCL}} [\text{a-cmt}]_i\varphi$	iff	for every v such that $\langle w,v\rangle \in \rhd_i^M$, $M,v \models_{\mathsf{MPCL}} \varphi$
(f)	$M,w \models_{\mathsf{MPCL}} [\text{c-cmt}]_i\varphi$	iff	for every v such that $\langle w,v\rangle \in \blacktriangleright_i^M$, $M,v \models_{\mathsf{MPCL}} \varphi$.

A formula φ is true in an $\mathscr{L}_{\mathsf{MPCL}}$-model M at a point w of W^M if $M,w \models_{\mathsf{MPCL}} \varphi$. The semantic consequence relation and the notion of validity can also be defined in the standard way.

Now we define the proof system for MPCL.

Definition 4. The proof system for MPCL consists of the following axioms and rules:

(i) all instantiations of propositional tautologies over the present language,
(ii) K-axioms for the commitment modalities $[\text{a-cmt}]_i$ and $[\text{c-cmt}]_i$ for each $i \in I$,
(iii) modus ponens,
(iv) necessitation rules for the commitment modalities $[\text{a-cmt}]_i$ and $[\text{c-cmt}]_i$ for each $i \in I$,
(v) the axiom of the following form for each $i \in I$:

$$\text{(Mix)} \quad [\text{a-cmt}]_i\varphi \to [\text{c-cmt}]_i\varphi \ .$$

This proof system is easily seen to be sound, and its completeness can be proved in an entirely standard way.[4]

Theorem 1 (Completeness of MPCL). *The above proof system completely axiomatizes* MPCL.

Note that our minimal set of assumptions involves the assumption that the following proposition is true:

Proposition 1. *The set of a-commitments and the set of c-commitments of an agent are both closed under logical consequences.*

[4] Strictly speaking, the necessitation rule for c-commitment is redundant since Mix Axiom enables us to derive it from the necessitation rule for a-commitment. We list it here in order to record the fact that MPCL is normal.

The epistemic analogue of this feature is usually called "logical omniscience", and is sometimes criticized as unrealistic. In the case of propositional commitments, however, we find the closure under logical consequences non-problematic. Rational agents should withdraw at least one of their assertions or concessions if some unwanted consequences are derived from what they have explicitly asserted or conceded. They are taken to be responsible for the logical consequences of what they have said at least to this extent.

3 The Logic of Acts of Asserting and Conceding DMPCL

The formulas of MPCL can be used to talk about the situations before and after the performance of an act of asserting, an act of conceding, or an act of withdrawing by modeling relevant situations using $\mathscr{L}_{\mathsf{MPCL}}$-models. For example, let $\langle M, w\rangle$ and $\langle N, w\rangle$ be the situations before and after an agent i's act of asserting that p respectively. If i has never committed to the truth of p before, then we have:

$$M, w \not\models_{\mathsf{MPCL}} [\text{a-cmt}]_i p$$
$$N, w \models_{\mathsf{MPCL}} [\text{a-cmt}]_i p \ .$$

Thus MPCL can be used to characterize the sets of propositions to which agents are committed with respect to each stage of their interactions. But in the above example, neither the difference between the stages before and after i's act of asserting nor i's act of asserting that links them can be talked about in MPCL; they are talked about in the metalanguage. Thus our next task is to dynamify MPCL in order to have an object language that can be used to characterize the logical dynamics of changing propositional commitments. As was said before, we do this in two steps for technical reasons. First, we extend MPCL into DMPCL, dynamified MPCL, by adding dynamic modal operators standing for acts of asserting and acts of conceding in this section. Then, in the next section, we further extend it into DMPCL$^+$, by adding another set of dynamic modal operators standing for acts of withdrawing.

Now we extend the language:

Definition 5. Take the same countably infinite set *Aprop* of proposition letters and the same finite set I of agents as before, with p ranging over *Aprop*, and i over I. The language $\mathscr{L}_{\mathsf{DMPCL}}$ of dynamified multi-agent propositional commitment logic DMPCL is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid [\text{a-cmt}]_i\varphi \mid [\text{c-cmt}]_i\varphi \mid [\pi]\varphi$$
$$\pi ::= \text{assert}_i\varphi \mid \text{concede}_i\varphi \ .$$

Note that all the formulas of $\mathscr{L}_{\mathsf{MPCL}}$ are also formulas of $\mathscr{L}_{\mathsf{DMPCL}}$.

A truth definition for this language can be given with reference to $\mathscr{L}_{\mathsf{MPCL}}$-models by expanding the truth definition for $\mathscr{L}_{\mathsf{MPCL}}$ with two additional clauses for the new kinds of formulas as follows:

Definition 6. Let M be an $\mathscr{L}_{\mathsf{MPCL}}$-model and w a point in W^M. If $p \in Aprop$, and $i \in I$, then:

(a) $M,w \models_{\mathsf{DMPCL}} p$ iff $w \in V^M(p)$
(b) $M,w \models_{\mathsf{DMPCL}} \top$
(c) $M,w \models_{\mathsf{DMPCL}} \neg\varphi$ iff it is not the case that $M,w \models_{\mathsf{DMPCL}} \varphi$
(d) $M,w \models_{\mathsf{DMPCL}} (\varphi \wedge \psi)$ iff $M,w \models_{\mathsf{DMPCL}} \varphi$ and $M,w \models_{\mathsf{DMPCL}} \psi$
(e) $M,w \models_{\mathsf{DMPCL}} [\text{a-cmt}]_i\varphi$ iff for every v such that $\langle w,v\rangle \in \rhd_i^M$, $M,v \models_{\mathsf{DMPCL}} \varphi$
(f) $M,w \models_{\mathsf{DMPCL}} [\text{c-cmt}]_i\varphi$ iff for every v such that $\langle w,v\rangle \in \blacktriangleright_i^M$, $M,v \models_{\mathsf{DMPCL}} \varphi$
(g) $M,w \models_{\mathsf{DMPCL}} [\text{assert}_i\chi]\varphi$ iff $M_{\text{assert}_i\chi}, w \models_{\mathsf{DMPCL}} \varphi$
(h) $M,w \models_{\mathsf{DMPCL}} [\text{concede}_i\chi]\varphi$ iff $M_{\text{concede}_i\chi}, w \models_{\mathsf{DMPCL}} \varphi$,

where $M_{\text{assert}_i\chi}$ is the $\mathscr{L}_{\mathsf{MPCL}}$-model obtained from M by replacing $\rhd_i^M$ and $\blacktriangleright_i^M$ with their subsets $\{\langle x,y\rangle \in \rhd_i^M \mid M,y \models_{\mathsf{DMPCL}} \chi\}$ and $\{\langle x,y\rangle \in \blacktriangleright_i^M \mid M,y \models_{\mathsf{DMPCL}} \chi\}$ respectively, and $M_{\text{concede}_i\chi}$ is the $\mathscr{L}_{\mathsf{MPCL}}$-model obtained from M by replacing $\blacktriangleright_i^M$ with its subset $\{\langle x,y\rangle \in \blacktriangleright_i^M \mid M,y \models_{\mathsf{DMPCL}} \chi\}$. A formula φ is true in an $\mathscr{L}_{\mathsf{MPCL}}$-model M at a point w of W^M if $M,w \models_{\mathsf{DMPCL}} \varphi$. The semantic consequence relation and the notion of validity can also be defined in the standard way.

Note that the truth of the formula of the form $[\text{assert}_i\chi]\varphi$ and that of the formula of the form $[\text{concede}_i\chi]\varphi$ at w in M are defined in terms of the truth of the formula of the form φ at w in the updated models $M_{\text{assert}_i\chi}$ and $M_{\text{concede}_i\chi}$ respectively. Intuitively, the update by assert$_i\chi$ cuts every accessibility link $\langle x,y\rangle$ of $\rhd_i^M$ and $\blacktriangleright_i^M$ if χ doesn't hold at y in M, and the update by concede$_i\chi$ cuts every accessibility link $\langle x,y\rangle$ of $\blacktriangleright_i^M$ if χ doesn't hold at y in M. This guarantees that the accessibility relation associated with c-commitments of any agent $i \in I$ will always be a subset of the accessibility relation associated with i's a-commitments, and so the updated models $M_{\text{assert}_i\chi}$ and $M_{\text{concede}_i\chi}$ will also be $\mathscr{L}_{\mathsf{MPCL}}$-models.

Note also that the first six clauses reproduce the corresponding clauses in the truth definition for $\mathscr{L}_{\mathsf{MPCL}}$ faithfully. Thus we have:

Corollary 1. *Let M be an $\mathscr{L}_{\mathsf{MPCL}}$-model and w a point in M. Then for any $\varphi \in \mathscr{L}_{\mathsf{MPCL}}$, we have:*

$$M,w \models_{\mathsf{DMPCL}} \varphi \textit{ iff } M,w \models_{\mathsf{MPCL}} \varphi .$$

For each $i \in I$, a formula $\varphi \in \mathscr{L}_{\mathsf{MPCL}}$ is said to be i-free if neither the operator $[\text{a-cmt}]_i$ nor the operator $[\text{c-cmt}]_i$ occurs in it. The following corollary can be proved by induction on the length of ψ:

Corollary 2. *If $\psi \in \mathscr{L}_{\mathsf{MPCL}}$ is i-free , then for any $\varphi \in \mathscr{L}_{\mathsf{MPCL}}$, we have:*

$$M, w \models_{\mathsf{DMPCL}} \psi \;\; \textit{iff} \;\; M_{\text{assert}_i\varphi}, w \models_{\mathsf{DMPCL}} \psi$$
$$M, w \models_{\mathsf{DMPCL}} \psi \;\; \textit{iff} \;\; M_{\text{concede}_i\varphi}, w \models_{\mathsf{DMPCL}} \psi \, .$$

We also have:

Proposition 2. *If $\varphi \in \mathscr{L}_{\mathsf{MPCL}}$ is i-free, the following three principles are valid:*

$$[\text{assert}_i\varphi][\text{a-cmt}]_i\varphi$$
$$[\text{assert}_i\varphi][\text{c-cmt}]_i\varphi$$
$$[\text{concede}_i\varphi][\text{c-cmt}]_i\varphi \, .$$

These restricted principles partially characterize the workings of acts of asserting and acts of conceding: though not without exceptions, they usually generate corresponding propositional commitments.[5]

If $\varphi \in \mathscr{L}_{\mathsf{MPCL}}$ is both i-free and j-free, we have

$$[\text{assert}_i\varphi][\text{assert}_j\neg\varphi]([\text{a-cmt}]_i\varphi \wedge [\text{a-cmt}]_j\neg\varphi) \, .$$

This means that if an agent i asserts φ in $\langle M, w\rangle$, and another agent j asserts $\neg\varphi$ after that, we have

$$(M_{\text{assert}_i\varphi})_{\text{assert}_j\neg\varphi}, w \models_{\mathsf{DMPCL}} ([\text{a-cmt}]_i\varphi \wedge [\text{a-cmt}]_j\neg\varphi) \, .$$

Thus, even if two agents jointly make mutually incompatible assertions, we can use DMPCL to represent the resulting situation without falling into a contradiction. While $(\varphi \wedge \neg\varphi)$ is a contradiction, $([\text{a-cmt}]_i\varphi \wedge [\text{a-cmt}]_j\neg\varphi)$ is not. This feature can be important when we design information systems which have to deal with possibly conflicting inputs from multiple agents. We have to be able to accommodate differences of opinions among agents without making the whole system inconsistent.[6]

Note also that even $[\text{a-cmt}]_i(\varphi \wedge \neg\varphi)$ is not by itself a contradiction, although it ascribes a contradictory a-commitment to the agent i. Such commitment will be generated if i asserts both φ and $\neg\varphi$, for example. In such a situation, accessibility relations associated with i's a-commitments and c-commitments will become empty,

[5] The restriction on φ is motivated by the fact that the truth of φ at w in M does not guarantee the truth of φ at w in $M_{\text{assert}_i\varphi}$ or the truth of φ at w in $M_{\text{concede}_i\varphi}$ if φ is not i-free. For more on this point, see [16] (p. 9).

[6] An interesting discussion of the usefulness of explicit treatment of speech acts in such a system can be found in [10].

and i will be both a-committed and c-committed to every proposition. Since a set of propositional commitments an agent has can be inconsistent, we find it important for us to be able to talk about speech acts that lead to such inconsistencies. As the so-called D Axiom would preclude the very possibility of such situations, we have avoided including it in our proof system for MPCL.

The proof system for DMPCL is given by expanding that for MPCL.

Definition 7. The proof system for DMPCL comprises all the axioms and rules of the proof system for MPCL, the necessitation rules for assertion modality $[\text{assert}_i\varphi]$ and concession modality $[\text{concede}_i\varphi]$, and the following axioms:

(A1)	$[\text{assert}_i\varphi]p$	$\leftrightarrow$	p	
(A2)	$[\text{assert}_i\varphi]\top$	$\leftrightarrow$	$\top$	
(A3)	$[\text{assert}_i\varphi]\neg\psi$	$\leftrightarrow$	$\neg[\text{assert}_i\varphi]\psi$	
(A4)	$[\text{assert}_i\varphi](\psi\wedge\chi)$	$\leftrightarrow$	$[\text{assert}_i\varphi]\psi\wedge[\text{assert}_i\varphi]\chi$	
(A5)	$[\text{assert}_i\varphi][\text{a-cmt}]_j\psi$	$\leftrightarrow$	$[\text{a-cmt}]_j[\text{assert}_i\varphi]\psi$	$(i\neq j)$
(A6)	$[\text{assert}_i\varphi][\text{a-cmt}]_i\psi$	$\leftrightarrow$	$[\text{a-cmt}]_i(\varphi\to[\text{assert}_i\varphi]\psi)$	
(A7)	$[\text{assert}_i\varphi][\text{c-cmt}]_j\psi$	$\leftrightarrow$	$[\text{c-cmt}]_j[\text{assert}_i\varphi]\psi$	$(i\neq j)$
(A8)	$[\text{assert}_i\varphi][\text{c-cmt}]_i\psi$	$\leftrightarrow$	$[\text{c-cmt}]_i(\varphi\to[\text{assert}_i\varphi]\psi)$	
(C1)	$[\text{concede}_i\varphi]p$	$\leftrightarrow$	p	
(C2)	$[\text{concede}_i\varphi]\top$	$\leftrightarrow$	$\top$	
(C3)	$[\text{concede}_i\varphi]\neg\psi$	$\leftrightarrow$	$\neg[\text{concede}_i\varphi]\psi$	
(C4)	$[\text{concede}_i\varphi](\psi\wedge\chi)$	$\leftrightarrow$	$[\text{concede}_i\varphi]\psi\wedge[\text{concede}_i\varphi]\chi$	
(C5)	$[\text{concede}_i\varphi][\text{a-cmt}]_j\psi$	$\leftrightarrow$	$[\text{a-cmt}]_j[\text{concede}_i\varphi]\psi$	(for any j)
(C6)	$[\text{concede}_i\varphi][\text{c-cmt}]_j\psi$	$\leftrightarrow$	$[\text{c-cmt}]_j[\text{concede}_i\varphi]\psi$	$(i\neq j)$
(C7)	$[\text{concede}_i\varphi][\text{c-cmt}]_i\psi$	$\leftrightarrow$	$[\text{c-cmt}]_i(\varphi\to[\text{concede}_i\varphi]\psi)$.	

Axioms (A6), (A8) and (C7) are crucial here. They capture how the acts of asserting and conceding update the model. Consider (A6). The left-hand side of it says that $[\text{a-cmt}]_i\psi$ holds after the update by $\text{assert}_i\varphi$. The right-hand side of it specifies the necessary and sufficient conditions for this in terms of the conditions that hold before the update. Take an arbitrary $\mathscr{L}_{\mathsf{MPCL}}$-model M and an arbitrary world w of M. $[\text{a-cmt}]_i\psi$ holds at w in the updated model $M_{\text{assert}_i\varphi}$ iff ψ holds at every world v that is accessible with respect to i's a-commitment ($[\text{a-cmt}]_i$-accessible, hereafter) from w in $M_{\text{assert}_i\varphi}$. Since the update by $\text{assert}_i\varphi$ cuts every $[\text{a-cmt}]_i$-arrow arriving in non-φ-worlds in M, only the φ-worlds $[\text{a-cmt}]_i$-accessible from w in M remain $[\text{a-cmt}]_i$-accessible from w in $M_{\text{assert}_i\varphi}$. But ψ holds at such world v in the updated model $M_{\text{assert}_i\varphi}$ iff $[\text{assert}_i\varphi]\psi$ holds at v in M. Hence $[\text{a-cmt}]_i\psi$ holds at w in the updated model $M_{\text{assert}_i\varphi}$ iff $[\text{a-cmt}]_i(\varphi\to[\text{assert}_i\varphi]\psi)$ holds at w in M before the update. Thus (A6) says that the necessary and sufficient condition for $[\text{a-cmt}]_i\psi$ to hold at w in $M_{\text{assert}_i\varphi}$ is that every φ world $[\text{a-cmt}]_i$-accessible from w in M is a world where $[\text{assert}_i\varphi]\psi$ holds in M. Axioms (A8) and (C7) can be understood similarly.

Note that Axioms (A1), (A2), (C1), and (C2) enable us to eliminate each occurrence of assertion modalities and concession modalities prefixed to a proposition letter or the constant $\top$. The other axioms enable us to reduce the length of the subformula to which an assertion modality or a concession modality is prefixed. Thus, these axioms, sometimes called "reduction axioms", enable us to define a translation function that takes a formula of $\mathscr{L}_{\mathsf{DMPCL}}$ and yields a formula of $\mathscr{L}_{\mathsf{MPCL}}$ that is provably equivalent to the original formula.

Definition 8. The translation function that takes a formula of $\mathscr{L}_{\mathsf{DMPCL}}$ and yields a formula of $\mathscr{L}_{\mathsf{MPCL}}$ is defined as follows:

$$
\begin{aligned}
&t(p) &&= p\\
&t(\top) &&= \top\\
&t(\neg\varphi) &&= \neg t(\varphi)\\
&t(\varphi\wedge\psi) &&= t(\varphi)\wedge t(\psi)\\
&t([\text{a-cmt}]_i\varphi) &&= [\text{a-cmt}]_i t(\varphi)\\
&t([\text{c-cmt}]_i\varphi) &&= [\text{c-cmt}]_i t(\varphi)\\
&t([\text{assert}_i\varphi]p) &&= p\\
&t([\text{assert}_i\varphi]\top) &&= \top\\
&t([\text{assert}_i\varphi]\neg\psi) &&= \neg t([\text{assert}_i\varphi]\psi)\\
&t([\text{assert}_i\varphi](\psi\wedge\chi)) &&= t([\text{assert}_i\varphi]\psi)\wedge t([\text{assert}_i\varphi]\chi)\\
&t([\text{assert}_i\varphi][\text{a-cmt}]_j\psi) &&= [\text{a-cmt}]_j t([\text{assert}_i\varphi]\psi) \quad (i\neq j)\\
&t([\text{assert}_i\varphi][\text{a-cmt}]_i\psi) &&= [\text{a-cmt}]_i t(\varphi\to[\text{assert}_i\varphi]\psi)\\
&t([\text{assert}_i\varphi][\text{c-cmt}]_j\psi) &&= [\text{c-cmt}]_j t([\text{assert}_i\varphi]\psi) \quad (i\neq j)\\
&t([\text{assert}_i\varphi][\text{c-cmt}]_i\psi) &&= [\text{c-cmt}]_i t(\varphi\to[\text{assert}_i\varphi]\psi)\\
&t([\text{assert}_i\varphi][\text{assert}_j\psi]\chi) &&= t([\text{assert}_i\varphi]t([\text{assert}_j\psi]\chi))\\
&t([\text{assert}_i\varphi][\text{concede}_j\psi]\chi) &&= t([\text{assert}_i\varphi]t([\text{concede}_j\psi]\chi))\\
&t([\text{concede}_i\varphi]p) &&= p\\
&t([\text{concede}_i\varphi]\top) &&= \top\\
&t([\text{concede}_i\varphi]\neg\psi) &&= \neg t([\text{concede}_i\varphi]\psi)\\
&t([\text{concede}_i\varphi](\psi\wedge\chi)) &&= t([\text{concede}_i\varphi]\psi)\wedge t([\text{concede}_i\varphi]\chi)\\
&t([\text{concede}_i\varphi][\text{a-cmt}]_j\psi) &&= [\text{a-cmt}]_j t([\text{concede}_i\varphi]\psi)\\
&t([\text{concede}_i\varphi][\text{c-cmt}]_j\psi) &&= [\text{c-cmt}]_j t([\text{concede}_i\varphi]\psi) \quad (i\neq j)\\
&t([\text{concede}_i\varphi][\text{c-cmt}]_i\psi) &&= [\text{c-cmt}]_i t(\varphi\to[\text{concede}_i\varphi]\psi)\\
&t([\text{concede}_i\varphi][\text{assert}_j\psi]\chi) &&= t([\text{concede}_i\varphi]t([\text{assert}_j\psi]\chi))\\
&t([\text{concede}_i\varphi][\text{concede}_j\psi]\chi) &&= t([\text{concede}_i\varphi]t([\text{concede}_j\psi]\chi))\ .
\end{aligned}
$$

This translation enables us to derive the completeness of DMPCL from the completeness of $\mathscr{L}_{\mathsf{MPCL}}$.[7] Thus,

Theorem 2 (Completeness of DMPCL). *There is a complete axiomatization of* DMPCL.

4 A Further Extension DMPCL$^+$

Consider a formula of the form $[\text{assert}_i\chi][\text{assert}_j\xi][\text{assert}_i\eta]\varphi$. It means that φ holds after i asserts η after j asserts ξ after i asserts χ, and it is true at w in M if and only if φ is true at w in the updated model $((M_{\text{assert}_i\chi})_{\text{assert}_j\xi})_{\text{assert}_i\eta}$. Let an expression of the form $\circlearrowleft\text{assert}_i\chi$ and $\circlearrowleft\text{concede}_i\chi$ represent the type of i's acts of withdrawing i's own assertion that χ and i's acts of withdrawing i's own concession that χ respectively. Then what will we get if we update $((M_{\text{assert}_i\chi})_{\text{assert}_j\xi})_{\text{assert}_i\eta}$ with $\circlearrowleft\text{assert}_i\chi$, for example? We suggest that what we will get in that situation should be calculated by calculating what we would have in $(M_{\text{assert}_j\xi})_{\text{assert}_i\eta}$, as far as propositional commitments are concerned.

This is not meant to imply that the act of withdrawing could affect the past history. What we are proposing is that the set of propositional commitments an agent i will bear after withdrawing i's own act of asserting that χ in the situation $\langle((M_{\text{assert}_i\chi})_{\text{assert}_j\xi})_{\text{assert}_i\eta}, w\rangle$, for example, should be the same as the set of propositional commitments i would bear in the situation $\langle(M_{\text{assert}_j\xi})_{\text{assert}_i\eta}, w\rangle$. Intuitively, the set of propositional commitments an agent i will bear after withdrawing her own assertion that χ will be the same as the set of propositional commitments she would bear if she had not asserted that χ but had made all the other assertions and concessions she actually made. We develop a formal treatment of withdrawals which incorporates this intuitive idea as faithfully as possible in this section.

First, we extend the language.

Definition 9. Take the same countably infinite set *Aprop* of proposition letters and the same finite set I of agents as before, with p ranging over *Aprop*, and i over I. The language $\mathscr{L}_{\mathsf{DMPCL}^+}$ of dynamified multi-agent propositional commitment logic with withdrawals DMPCL$^+$ is given by:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid [\text{a-cmt}]_i\varphi \mid [\text{c-cmt}]_i\varphi \mid [\pi]\varphi$$
$$\pi ::= \text{assert}_i\varphi \mid \text{concede}_i\varphi \mid \circlearrowleft\text{assert}_i\varphi \mid \circlearrowleft\text{concede}_i\varphi\ .$$

Note that withdrawals are allowed only for assertions and concessions; we have not allowed withdrawals of withdrawals here. Instead of withdrawals of withdrawals, an

[7] The outline of the derivation is completely similar to that of the completeness of ECL given in [16].

agent can assert or concede again the same propositions which she once asserted or conceded but has subsequently withdrawn.[8]

Note also that we only allow agents to withdraw their own assertions or concessions. Although there may be agents who have the authority to withdraw certain speech acts performed by other agents in hierarchical organizations, we leave such complexity for further research.

In order to give a truth definition for the above language, we need to consider the effects of acts of withdrawing performed at the stages we will be in after going through various arbitrary sequences of relevant speech acts involving acts of withdrawing as well as acts of asserting and conceding. We call such sequences commitment affecting act sequences, or caa-sequences for short. Before examining the effects of acts of withdrawing with reference to arbitrary caa-sequences, however, we will consider their effects with reference to somewhat simpler sequences consisting of only acts of asserting and conceding. We call such a sequence a positive commitment act sequence, or a pca-sequence for short. Then the above example suggests that the effects of an act of withdrawing performed at the stage we will be in after going through an arbitrary pca-sequence σ starting from $\langle M, w\rangle$ can be captured by considering the model obtained from M by updating M with another pca-sequence obtained from σ by deleting from σ the assertion or the concession that was withdrawn. Thus, in the above example, we considered the model $(M_{\text{assert}_j\xi})_{\text{assert}_i\eta}$ in order to examine the effects of the act of withdrawing of the form $\circlearrowleft\text{assert}_i\chi$ performed in the situation $\langle((M_{\text{assert}_j\chi})_{\text{assert}_j\xi})_{\text{assert}_i\eta}, w\rangle$, and the pca-sequence $\langle\text{assert}_j\xi, \text{assert}_i\eta\rangle$ is exactly what we get by deleting the occurrence of $\text{assert}_i\chi$ from the pca-sequence $\langle\text{assert}_i\chi, \text{assert}_j\xi, \text{assert}_i\eta\rangle$.

Note that an arbitrary pca-sequence σ might include two or more occurrences of a given assertion or concession, or might include none of them. For the sake of generality, we will also talk of σ as a sequence even if σ is empty or σ consists of only one speech act. Thus we define:

Definition 10. Let $\sigma = \langle\pi_1, \cdots, \pi_n\rangle$ be a (possibly empty) pca-sequence. We define the reduced pca-sequences $\sigma\restriction\circlearrowleft\text{assert}_i\varphi$ and $\sigma\restriction\circlearrowleft\text{concede}_i\varphi$, to be obtained by deleting from σ every occurrence of the act of type $\text{assert}_i\varphi$ and every occurrence of the act of type $\text{concede}_i\varphi$ respectively, as follows:

[8] Strictly speaking, such acts of re-asserting and re-conceding do not always have the same effects as acts of withdrawals of withdrawals. The effect of the act of withdrawing the act of withdrawing of the form $\circlearrowleft\text{assert}_i\varphi$, for example, can be different from the effects of re-asserting φ. Consider a case where φ is asserted at stage s_i, withdrawn at a later stage s_j, and re-asserted at a still later stage s_k by an agent. What would happen if, instead of re-asserting φ at s_k, the agent withdrew at s_k her earlier withdrawal of φ at s_j? Her earlier assertion of φ at s_i would become effective again. Now, the effects of re-asserting φ at s_k can be different from the effects of asserting φ at s_i as the former depend on the things said during the discourse between s_i and s_k. To be sure the things said during the discourse between s_i and s_k would also affect the states after her "resurrected" assertion of φ at S_i. But there is no guarantee that they would "neutralize", so to speak, the difference in such a way that the state after the withdrawal at s_K of her earlier withdrawal would be exactly the same as the actual state after her re-asserting of φ at s_K.

$$\sigma \upharpoonright \circlearrowleft \text{assert}_i\varphi = \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \pi_1, \cdots, \pi_{n-1} \rangle \upharpoonright \circlearrowleft \text{assert}_i\varphi & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \text{assert}_i\varphi \\ \langle \langle \pi_1, \cdots, \pi_{n-1} \rangle \upharpoonright \circlearrowleft \text{assert}_i\varphi, \pi_n \rangle & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n \neq \text{assert}_i\varphi \end{cases}$$

and

$$\sigma \upharpoonright \circlearrowleft \text{concede}_i\varphi = \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \pi_1, \cdots, \pi_{n-1} \rangle \upharpoonright \circlearrowleft \text{concede}_i\varphi & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \text{concede}_i\varphi \\ \langle \langle \pi_1, \cdots, \pi_{n-1} \rangle \upharpoonright \circlearrowleft \text{concede}_i\varphi, \pi_n \rangle & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n \neq \text{concede}_i\varphi\ . \end{cases}$$

Note that $\sigma \upharpoonright \circlearrowleft \text{assert}_i\varphi$ and $\sigma \upharpoonright \circlearrowleft \text{concede}_i\varphi$ are pca-sequences. Note also that we allow acts of withdrawing to withdraw repeated assertions or repeated concessions in one go, so to speak. For example, we have:

$$\langle \text{assert}_i\chi, \text{assert}_j\xi, \text{assert}_k\eta, \text{assert}_i\chi \rangle \upharpoonright \circlearrowleft \text{assert}_i\chi = \langle \text{assert}_j\xi, \text{assert}_k\eta \rangle\ .$$

If an agent who insisted that φ by repeatedly asserting that φ comes to wish to withdraw her assertion that φ, it would be strange if she wished to withdraw only some of her acts of asserting φ while keeping others untouched. She would still be a-committed to φ.

We are now in a position to define a special pca-sequence σ^* that can be used to calculate the propositional commitments agents bear after going through an arbitrary caa-sequence σ. We get σ^* from σ by applying the procedures we have just introduced to the occurrences of withdrawals in σ according to the order they occur in σ.

Definition 11. Given an arbitrary caa-sequence σ possibly involving acts of withdrawing as well as acts of asserting and acts of conceding, we define its corresponding pca-sequence σ^* as follows:

$$\sigma^* = \begin{cases} \sigma & \text{if } \sigma \text{ is empty} \\ \langle \langle \pi_1, \cdots, \pi_{n-1} \rangle^*, \text{assert}_i\varphi \rangle & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \text{assert}_i\varphi \\ \langle \langle \pi_1, \cdots, \pi_{n-1} \rangle^*, \text{concede}_i\varphi \rangle & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \text{concede}_i\varphi \\ \langle \pi_1, \cdots, \pi_{n-1} \rangle^* \upharpoonright \circlearrowleft \text{assert}_i\varphi & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \circlearrowleft \text{assert}_i\varphi \\ \langle \pi_1, \cdots, \pi_{n-1} \rangle^* \upharpoonright \circlearrowleft \text{concede}_i\varphi & \text{if } \sigma = \langle \pi_1, \cdots, \pi_n \rangle\text{, and } \pi_n = \circlearrowleft \text{concede}_i\varphi\ . \end{cases}$$

This definition enables us to deal with the effects of an act of withdrawing performed at the stage we will be in after going through an arbitrary caa-sequence σ. We just have to work with the reduced pca-sequences $\sigma^* \upharpoonright \circlearrowleft \text{assert}_i\varphi$ and $\sigma^* \upharpoonright \circlearrowleft \text{concede}_i\varphi$.

In order to give a truth definition for DMPCL^+ with the help of these definitions, however, we have to exercise due care. In the notation used in the truth definition for DMPCL, $(\cdots((M_{\pi_1})_{\pi_2})\cdots)_{\pi_n}$ represents the model obtained from M by updating it successively with the sequence of speech acts $\langle\pi_1,\pi_2,\cdots,\pi_n\rangle$, which is a pca-sequence in our current terminology. If $\sigma = \langle\pi_1,\pi_2,\cdots,\pi_n\rangle$, we may wish to abbreviate $(\cdots((M_{\pi_1})_{\pi_2})\cdots)_{\pi_n}$ as M_σ, and talk of it as the model obtained from M by updating it with σ. Then the model world pair $\langle M_\sigma, w\rangle$ will represent the situation we will be in after going through the whole pca-sequence σ of assertions and concessions starting from $\langle M, w\rangle$. But there can be another $\mathscr{L}_{\mathsf{MPCL}}$-model N and another pca-sequence τ such that $M = N_\tau$. Thus σ can be considered as a partial representation of the whole discourse that leads to $\langle M_\sigma, w\rangle$.

This is unsurprising since agents involved may have non-trivial propositional commitments even in the situation $\langle M, w\rangle$; such commitments can be considered as the products of previous discourse that led to $\langle M, w\rangle$. If we only deal with acts of asserting and conceding, there is nothing problematic about this. But it can lead to a contradiction when we take acts of withdrawing into consideration. The result of updating M_σ with $\circlearrowleft\text{assert}_i\chi$ might not be identical with the result of updating $(N_\tau)_\sigma$ with $\circlearrowleft\text{assert}_i\chi$ since $\text{assert}_i\chi$ might occur in τ. Such a discrepancy is inadmissible since $M = N_\tau$. In order to avoid this problem, we will keep models and sequences of speech acts separate as you will see in the truth definition below.

Definition 12. Let M be an $\mathscr{L}_{\mathsf{MPCL}}$-model, σ an arbitrary caa-sequence, σ^* the corresponding pca-sequence of σ, and w a point in M. If $p \in Aprop$, and $i \in I$, then:

(a) $M,\sigma,w \models_{\mathsf{DMPCL}^+} p$ iff $w \in V^M(p)$

(b) $M,\sigma,w \models_{\mathsf{DMPCL}^+} \top$

(c) $M,\sigma,w \models_{\mathsf{DMPCL}^+} \neg\varphi$ iff it is not the case that $M,\sigma,w \models_{\mathsf{DMPCL}^+} \varphi$

(d) $M,\sigma,w \models_{\mathsf{DMPCL}^+} (\varphi \wedge \psi)$ iff $M,\sigma,w \models_{\mathsf{DMPCL}^+} \varphi$ and $M,\sigma,w \models_{\mathsf{DMPCL}^+} \psi$

(e) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\text{a-cmt}]_i\varphi$ iff for all v such that $\langle w,v\rangle \in \rhd_i^M \upharpoonright \sigma^*$, $M,\sigma^*,v \models_{\mathsf{DMPCL}^+} \varphi$

(f) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\text{c-cmt}]_i\varphi$ iff for all v such that $\langle w,v\rangle \in \blacktriangleright_i^M \upharpoonright \sigma^*$, $M,\sigma^*,v \models_{\mathsf{DMPCL}^+} \varphi$

(g) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\text{assert}_i\chi]\varphi$ iff $M,\langle\sigma, \text{assert}_i\chi\rangle,w \models_{\mathsf{DMPCL}^+} \varphi$

(h) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\text{concede}_i\chi]\varphi$ iff $M,\langle\sigma, \text{concede}_i\chi\rangle,w \models_{\mathsf{DMPCL}^+} \varphi$

(i) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\circlearrowleft\text{assert}_i\chi]\varphi$ iff $M,\sigma^* \upharpoonright \circlearrowleft\text{assert}_i\chi, w \models_{\mathsf{DMPCL}^+} \varphi$

(j) $M,\sigma,w \models_{\mathsf{DMPCL}^+} [\circlearrowleft\text{concede}_i\chi]\varphi$ iff $M,\sigma^* \upharpoonright \circlearrowleft\text{concede}_i\chi, w \models_{\mathsf{DMPCL}^+} \varphi$,

where

$$\rhd_i^M\upharpoonright\sigma^* = \begin{cases} \rhd_i^M & \text{if } \sigma^* \text{ is empty,} \\ \{\langle x,y\rangle \in \rhd_i^M\upharpoonright\langle\pi_1,\ldots,\pi_{n-1}\rangle | M,\langle\pi_1,\ldots,\pi_{n-1}\rangle, y \models_{\mathsf{DMPCL}^+} \psi\} & \\ \qquad\qquad \text{if } \sigma^* = \langle\pi_1,\ldots,\pi_n\rangle \text{ and } \pi_n = \text{assert}_i\psi, \\ \rhd_i^M\upharpoonright\langle\pi_1,\ldots,\pi_{n-1}\rangle \quad \text{if } \sigma^* = \langle\pi_1,\ldots,\pi_n\rangle \text{ and } \pi_n \neq \text{assert}_i\psi, \end{cases}$$

and

$$\blacktriangleright_i^M\upharpoonright\sigma^* = \begin{cases} \blacktriangleright_i^M & \text{if } \sigma^* \text{ is empty,} \\ \{\langle x,y\rangle \in \blacktriangleright_i^M\upharpoonright\langle\pi_1,\ldots,\pi_{n-1}\rangle | M,\langle\pi_1,\ldots,\pi_{n-1}\rangle, y \models_{\mathsf{DMPCL}^+} \psi\} & \\ \qquad \text{if } \sigma^* = \langle\pi_1,\ldots,\pi_n\rangle \text{ and } \pi_n = \text{assert}_i\psi \text{ or } \pi_n = \text{concede}_i\psi, \\ \blacktriangleright_i^M\upharpoonright\langle\pi_1,\ldots,\pi_{n-1}\rangle & \\ \qquad \text{if } \sigma^* = \langle\pi_1,\ldots,\pi_n\rangle, \pi_n \neq \text{assert}_i\psi \text{ and } \pi_n \neq \text{concede}_i\psi\,. \end{cases}$$

A formula φ is true in an $\mathscr{L}_{\mathsf{MPCL}}$-model M with respect to an arbitrary caa-sequence σ at a point w of M if $M,\sigma,w \models_{\mathsf{DMPCL}^+} \varphi$. The semantic consequence relation and the notion of validity can also be defined in the obvious way.

Note that acts of withdrawing behave very differently from what theorists of belief revision call “contraction”. Let $\mathscr{B}$ be the set of beliefs of an agent, say a. Then in the AGM approach, contraction $\ominus$ is supposed to satisfy the postulate that $\varphi \notin \mathscr{B}\ominus\varphi$ if $\not\vdash \varphi$, but we have $M,\sigma\upharpoonright\circlearrowleft\text{assert}_a p, w \models_{\mathsf{DMPCL}^+} [\text{a-cmt}]_a p$ if σ includes $\text{assert}_a q$ and $\text{assert}_a(q \to p)$, for example. Thus, even if a withdraws a’s own acts of the form $\text{assert}_a p$ (or of the form $\text{concede}_a p$), if there is a set of propositions jointly implying p in the set of propositions a has asserted (or conceded), a is still a-committed (or c-committed) to p. Acts of withdrawing do not directly nullify propositional commitments but do so only indirectly; we can only withdraw actually performed acts of asserting and conceding. We record this fact as a proposition.

Proposition 3. *Acts of withdrawing do not satisfy the AGM postulates for contraction.*

The AGM postulates, when considered as postulates for theory revision, characterize the desirable properties the revised theory has to have. But in order to have a theory which has such desirable properties, we have to restate the theory explicitly, and the task of restatement might not be so straightforward in some cases. DMPCL^+ seems to reflect this difficulty correctly.

5 Scorekeeping for Argumentation Games

In DMPCL and in DMPCL^+, we have characterized propositional commitments as products of various courses of discourse. This suggests an interesting possibility of applying DMPCL^+ to scorekeeping for debates or argumentation games. The notion of scorekeeping is introduced into the discussion of language by Lewis in [8], and used by Brandom in [3] in his attempt to develop a theory of meaning based on

Wittgenstein's notion of meaning as use. In Brandom's version, each agent is considered as a deontic scorekeeper, and "the significance of an assertion of p" is considered as "a mapping that associates with one social deontic score —characterizing the stage before that speech act is performed, according to some scorekeeper—the set of scores for the conversational stage that results from the assertion, according to the same scorekeeper" ([3], p.190). In this paper, however, we will only consider "the official score" kept by an idealized scorekeeper, and examine how DMPCL^+ can be applied to such official scorekeeping.

In order to do so, we need to take account of the fact that we may have various propositional commitments in $\langle M, \sigma, w\rangle$ even if σ is empty. Some of them are merely unavoidable commitments; for example, if φ is a tautology, we have $M, \sigma, w \models_{\mathsf{DMPCL}^+} [\text{a-cmt}]_i\varphi$ and $M, \sigma, w \models_{\mathsf{DMPCL}^+} [\text{c-cmt}]_i\varphi$. But there may be other contingent commitments in $\langle M, \sigma, w\rangle$ as well, and, as we have seen, we can think of them as products of the discourse that precedes $\langle M, \sigma, w\rangle$.

This means that only certain special $\mathscr{L}_{\mathsf{MPCL}}$-models can be used to represent the initial stage of a piece of discourse in which no speech acts have been made yet. In order to apply DMPCL^+ to scorekeeping for an argumentation game played by two players, for example, we have to define a special model that represents the initial stage of the game, where both players have neither a-commitments nor c-commitments other than the unavoidable ones. Thus we define:

Definition 13. Given a countably infinite set *Aprop* of proposition letters, and the set $I = \{a, b\}$ of players a and b, with p ranging over *Aprop*, and i over I. Then, the initial stage model is the tuple $M^0 = \langle W^0, \{\rhd_i^0 \mid i \in I\}, \{\blacktriangleright_i^0 \mid i \in I\}, V^0\rangle$, where:

(i) W^0 is the power set $\mathscr{P}(Aprop)$ of *Aprop*,

(ii) $\rhd_i^0 = W^0 \times W^0$ for each $i \in I$,

(iii) $\blacktriangleright_i^0 = W^0 \times W^0$ for each $i \in I$,

(iv) V^0 is the function that assigns a subset $V^0(p) = \{w \in W^0 \mid p \in w\}$ of W^0 to each proposition letter $p \in Aprop$.

Note that M^0 is an $\mathscr{L}_{\mathsf{MPCL}}$-model, and that, if σ is empty, for any proposition letter p, any world w, and any agent i, we have $(\neg[\text{a-cmt}]_i p \wedge \neg[\text{a-cmt}]_i \neg p)$ and $(\neg[\text{c-cmt}]_i p \wedge \neg[\text{c-cmt}]_i \neg p)$ in M^0 at w with respect to σ.

Thus, if σ is empty and w is the actual world, $\langle M^0, \sigma, w\rangle$ can be used to represent the initial stage of an argumentation game. The scores for subsequent stages can then be calculated according to the updating procedures defined for interpreting assertions, concessions, and their withdrawals, as far as propositional commitments are concerned.

Of course, there must be many other features that the scorekeeper has to record, such as penalties for withdrawing, for example.[9] It should be clear that DMPCL^+

[9] Such a feature may require very careful treatment. For example, if an agent a in an argumentation game has withdrawn her earlier assertion or concession after many things have said by her opponent

only gives us a partial characterization of the scorekeeping function. But the fact that it gives us a partial characterization shows that records of changing propositional commitments belong to the public score that characterizes conversational stages. Thus propositional commitments belong to the dynamic social reality.

6 Conclusion

We have shown that acts of asserting and acts of conceding can be modeled as updators of propositional commitments in DMPCL, and presented a complete set of reduction axioms for it. Since the acts of asserting and conceding are exactly the kind of acts with respect to which Austin's notion of conventional effect may seem most dubious, having a sound and complete logic that deals with their objective or public effects can be of considerable significance.

We have also given a truth definition for the language of DMPCL^+, Dynamified Multi-agent Propositional Commitment Logic With withdrawals. Having formulated the truth definition, the obvious next step is to examine what principles are valid, and whether there can be a complete axiomatization of it or not. Since the effects of an act of withdrawing depend not only on the conditions that hold in the directly preceding situation but also on the earlier updating history, no complete set of reduction axioms seems to be forthcoming. But even the mere truth definition for the formulas with modalities standing for acts of withdrawing assertions and concessions may be said to provide the notion of conventional effects of illocutionary acts of asserting and conceding with further support, as the possibility of withdrawal seems to be a distinguishing characteristic common to a wide range of acts whose effects are conventional or institutional. As we have seen, changing propositional commitments that agents bear are part of the public social reality. [10]

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b as well as by her, some of the things said by *b* may be the kind of things which *b* would not have said if *a* had not made the very assertion or concession *a* has just withdrawn. Should we allow *b* to withdraw some of his own assertions or concessions for free? And how about *a*'s further withdrawals motivated by *b*'s withdrawals?

[10] For more on Austin's notion of conventional effect, see [18] and [19].

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