Logical Dynamics of Commands and Obligations ⋆

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Abstract. In this paper, an eliminative command logic ECL introduced in Yamada [21] will be slightly refined into ECL II by allowing command terms and deontic operators to be indexed by a Cartesian Product of a given finite set of agents and a given finite set of command issuing authorities. Complete axiomatization and interesting validities will be presented, and a concrete example of a situation in which conflicting commands are given to the same agent by different authorities will be discussed extensively.

1 Introduction

Suppose your political guru commanded you to join an important political demonstration to be held in Tokyo next year. Unfortunately, it was to be held on the very same day on which an international one-day conference on logic was to be held in São Paulo, and the boss of your department had commanded you to attend that conference. It is possible for you to obey either command, but it is not possible for you to obey both as no available means of transportation is fast enough to enable you to join the demonstration in Tokyo and attend the conference in São Paulo on the same day. You have to decide which command to obey. But you are sure whichever command you may choose, you will regret not being able to obey the other.

In this paper, ECL (eliminative command logic) developed in Yamada [21] will be slightly refined in order to analyze adequately the situation you are supposed to be in after the issuance of your guru's command in the above example and the two commands that jointly brought about that situation. ECL is a variant of update logic, inspired by the development of DEL (dynamic epistemic logic) in Plaza [15], Groeneveld [9], Gerbrandy and Groeneveld [6], Gerbrandy [5], Baltag, Moss, & Solecki [2], and Kooi & van Benthem [11] among others. In DEL, the language of standard epistemic logic is utilized as the static base language on which its dynamic extensions are based. In the case of the logic of public announcements, for example, formulas of the static language of epistemic logic are used to describe situations before and after the announcements.

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Each situation is represented by an epistemic model, and public announcements are analyzed as events that update epistemic models. Thus in DEL, the dynamic extension of the epistemic logic, we have formulas of the form $[\phi!]K_i\psi$, which means that after every truthful public announcement to the effect that ϕ, an agent i knows that ψ.

The basic idea of ECL is to capture the workings of acts of commanding in terms of changes they bring about in the deontic status of possible courses of actions in the form of update logic by using multi-agent deontic logic instead of epistemic logic as a vehicle. Thus in ECL, we have formulas of the form $[!_i\phi]O_i\psi$, which means that after every successful acts of commanding an agent i to see to it that ϕ, it is obligatory upon i to see to it that ψ. Although ECL inherits various inadequacies from monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless. [1]

Moreover, since in ECL, effects of acts of commanding are captured in terms of changes in deontic aspects of the situation, it enables us to isolate the effects of illocutionary acts of commanding from the perlocutionary consequences utterances may have upon actions and attitudes of addressees. Since Grice [8], lots of philosophers, linguists and computer scientists have tried to characterize uses of sentences in terms of utterers' intentions to produce various effects in addressees. But utterers' intentions usually go beyond illocutionary acts by involving reference to perlocutionary consequences, while illocutionary acts can be effective even if they failed to produce intended perlocutionary consequences. Thus, in the above example, even if you refuse to go to Brazil, it will not make your boss's command void. Your refusal would not constitute disobedience if it could make her command void. Her command is effective even if she failed to get you to form the intention to attend the conference; she has changed the deontic status of your possible alternative courses of actions. [2] Thus, ECL can be seen as an interesting case study from the point of view of speech act theory.[3]

In this paper, ECL will be slightly refined by allowing deontic operators and command type terms to be indexed by the Cartesian product of a given set of agents and a given set of command issuing authorities. In the resulting logic ECL II, the situation you are supposed to be in after the issuance of your guru's command in the above example will be represented as an obligational dilemma, so to speak.

2 The static base logic MDL⁺II

In Yamada [21], the language of command logic $\mathcal{L}_{\mathsf{CL}}$ was defined by dynamifying a static base language, the language of monadic deontic logic with an alethic modality

[1] As is noted in [21], the use of monadic deontic language in ECL does not reflect any substantial theoretical commitment. It is used just to keep things as simple as possible at this early stage of the development of dynamic deontic logic. By dynamifying richer deontic languages interesting possibilities for further exploration into the logical dynamics of communicational acts will be opened up.

[2] There seems to be a growing recognition of the importance of such kind of institutional effects of speech acts among logicians and computer scientists recently. For example, more than one third of the authors explicitly talk about the *'count-as'* relation and/or institutional facts in their papers in DEON 2006 workshop. See Goble & Meyer [7].

[3] For more on the distinction between illocutionary and perlocutionary acts, see Lectures VIII–X of Austin [1].

$\mathcal{L}_{\mathsf{MDL}^+}$. $\mathcal{L}_{\mathsf{CL}}$ then was given a truth definition that incorporates an eliminative notion of acts of commanding, which leads to a version of eliminative command logic ECL.[4] In $\mathcal{L}_{\mathsf{MDL}^+}$, a formula of the form $O_i\varphi$ is used to represent the proposition that it is obligatory upon an agent i to see to it that φ. We refine this base language as follows:

Definition 1. *Take a countably infinite set* Aprop *of proposition letters, a finite set I of agents, and a finite set J of command issuing authorities, with p ranging over* Aprop*, i over I, and j over J. The refined multi-agent monadic deontic language $\mathcal{L}_{\mathsf{MDL}^{+\text{II}}}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_{(i,j)}\varphi$$

The set of all well formed formulas (sentences) of $\mathcal{L}_{\mathsf{MDL}^{+\text{II}}}$ is denoted by $S_{\mathsf{MDL}^{+\text{II}}}$ and operators of the form $O_{(i,j)}$ are called deontic operators. For each $i \in I$ and $j \in J$, we call a sentence (i, j)-free if no operators of the form $O_{(i,j)}$ occur in it. We call sentence alethic if no deontic operators occur in it, and boolean if no modal operators occur in it. For each $i \in I$ and $j \in J$, the set of all (i, j)-free sentences is denoted by $S_{(i,j)\text{-free}}$. The set of all alethic sentences and the set of all boolean sentences are denoted by S_{Aleth} and S_{Boole} respectively.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\Diamond$ are assumed to be introduced by standard definitions. We also abbreviate $\neg O_{(i,j)}\neg\varphi$ as $P_{(i,j)}\varphi$, and $O_{(i,j)}\neg\varphi$ as $F_{(i,j)}\varphi$. Note that Aprop $\subset S_{\text{Boole}} \subset S_{\text{Aleth}} \subset S_{(i,j)\text{-free}} \subset S_{\mathsf{MDL}^{+\text{II}}}$ for each $i \in I$ and $j \in J$.

A formula of the form $O_{(i,j)}\varphi$ is to be understood as meaning that it is obligatory upon an agent i with respect to an authority j to see to it that φ. In order to accommodate the distinction of authorities, we allow deontic accessibility relations to be indexed by $I \times J$. Thus we define:

Definition 2. *By an $\mathcal{L}_{\mathsf{MDL}^{+\text{II}}}$-model, I mean a quadruple $M = (W^M, R_A^M, R_D^M, V^M)$ where:*

(i) *W^M is a non-empty set (heuristically, of 'possible worlds')*

(ii) $R_A^M \subseteq W^M \times W^M$

(iii) *R_D^M is a function that assigns a subset $R_D^M(i, j)$ of R_A^M to each pair (i, j) of an agent $i \in I$ and an authority $j \in J$*

(iv) *V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in$* Aprop .

We usually abbreviate $R_D^M(i, j)$ as $R_{(i,j)}^M$. Note that for each $i \in I$ and $j \in J$, $R_{(i,j)}^M$ is required to be a subset of R_A^M. Thus we assume that whatever is permitted is possible.

The truth definition for the formulas of $\mathcal{L}_{\mathsf{MDL}^{+\text{II}}}$ can be given in a standard way by associating the alethic modal operator $\Box$ with R_A^M and each deontic operator $O_{(i,j)}$ with $R_{(i,j)}^M$ as follows:

[4] I refer to the language not as $\mathcal{L}_{\mathsf{ECL}}$ but as $\mathcal{L}_{\mathsf{CL}}$ as it might be given a truth definition that incorporates a non-eliminative notion of acts of commanding. For more on this, see Section 5.

Definition 3. *Let M be an $\mathcal{L}_{\mathsf{MDL^+II}}$-model and w a point in M. If $p \in Aprop$, $\varphi, \psi \in S_{\mathsf{MDL^+II}}$, and $(i, j) \in I \times J$, then:*

(a) $M, w \models_{\mathsf{MDL^+II}} p$ *iff* $w \in V^M(p)$
(b) $M, w \models_{\mathsf{MDL^+II}} \top$
(c) $M, w \models_{\mathsf{MDL^+II}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{MDL^+II}} \varphi$ (*i.e. it is not the case that* $M, w \models_{\mathsf{MDL^+II}} \varphi$)
(d) $M, w \models_{\mathsf{MDL^+II}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{MDL^+II}} \varphi$ *and* $M, w \models_{\mathsf{MDL^+II}} \psi$
(e) $M, w \models_{\mathsf{MDL^+II}} \Box\varphi$ *iff for every v such that* $(w, v) \in R^M_A$, $M, v \models_{\mathsf{MDL^+II}} \varphi$
(f) $M, w \models_{\mathsf{MDL^+II}} O_{(i,j)}\varphi$ *iff for every v such that* $(w, v) \in R^M_{(i,j)}$, $M, v \models_{\mathsf{MDL^+II}} \varphi$.

A formula φ is true in an $\mathcal{L}_{\mathsf{MDL^+II}}$-model M at a point w of M if $M, w \models_{\mathsf{MDL^+II}} \varphi$. We say that a set Σ of formulas of $\mathcal{L}_{\mathsf{MDL^+II}}$ is true in M at w, and write $M, w \models_{\mathsf{MDL^+II}} \Sigma$, if $M, w \models_{\mathsf{MDL^+II}} \psi$ for every $\psi \in \Sigma$. If $\Sigma \cup \{\varphi\}$ is a set of formulas of $\mathcal{L}_{\mathsf{MDL^+II}}$, we say that φ is a semantic consequence of Σ, and write $\Sigma \models_{\mathsf{MDL^+II}} \varphi$, if for every $\mathcal{L}_{\mathsf{MDL^+II}}$-model M and every point w such that $M, w \models_{\mathsf{MDL^+II}} \Sigma$, $M, w \models_{\mathsf{MDL^+II}} \varphi$. We say that a formula φ is valid, and write $\models_{\mathsf{MDL^+II}} \varphi$, if $\emptyset \models_{\mathsf{MDL^+II}} \varphi$.

The formulas of $\mathcal{L}_{\mathsf{MDL^+II}}$ can be used to talk about the situations before and after the issuance of a command. Consider again the previous example. Before the issuance of your boss's command, it was not obligatory upon you to attend the workshop in São Paulo, but after the issuance it became obligatory. Let p express the proposition that you will attend that workshop, and a and b represent you and your boss respectively. Furthermore, let (M, s) and (N, s) be the model world pairs that represent the situations before and after the issuance respectively. Then we have:

$$M, s \models_{\mathsf{MDL^+II}} \neg O_{(a,b)}p \tag{1}$$
$$N, s \models_{\mathsf{MDL^+II}} O_{(a,b)}p \ . \tag{2}$$

Thus the change brought about by your boss's command is captured in a sense by using formulas of $\mathcal{L}_{\mathsf{MDL^+II}}$.[5]

Now we define proof system for $\mathsf{MDL^+II}$:

Definition 4. *The proof system for $\mathsf{MDL^+II}$ contains the following axioms and rules:*

(Taut)	*all instantiations of propositional tautologies over the present language*
($\Box$-Dist)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
($O_{(i,j)}$-Dist)	$O_{(i,j)}(\varphi \to \psi) \to (O_{(i,j)}\varphi \to O_{(i,j)}\psi)$ *for each* $(i, j) \in I \times J$
(Mix)	$P_{(i,j)}\varphi \to \Diamond\varphi$ *for each* $(i, j) \in I \times J$
(MP)	$\dfrac{\varphi \quad \varphi \to \psi}{\psi}$

[5] Note, however, that the change is talked about as a change in the meta-language, and not in $\mathcal{L}_{\mathsf{MDL^+II}}$. We will return to this point in the next section.

$$(\Box\text{-Nec}) \quad \frac{\varphi}{\Box\varphi}$$

$$(O_{(i,j)}\text{-Nec}) \quad \frac{\varphi}{O_{(i,j)}\varphi} \quad \textit{for each } (i, j) \in I \times J\ .$$

An MDL$^+$II*-proof of a formula* φ *is a finite sequence of* $\mathcal{L}_{\mathsf{MDL^+II}}$*-formulas having* φ *as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{MDL^+II}} \varphi$*. If* $\Sigma \cup \{\varphi\}$ *is a set of* $\mathcal{L}_{\mathsf{MDL^+II}}$*-formulas, we say that* φ *is deducible in* MDL$^+$II *from* Σ *and write* $\Sigma \vdash_{\mathsf{MDL^+II}} \varphi$ *if* $\vdash_{\mathsf{MDL^+II}} \varphi$ *or there are formulas* $\psi_1, \ldots, \psi_n \in \Sigma$ *such that* $\vdash_{\mathsf{MDL^+II}} (\psi_1 \wedge \cdots \wedge \psi_n) \to \varphi$*.*

The above rules obviously preserve validity, and all the axioms are easily seen to be valid. Thus this proof system is sound. [6]

The completeness of this proof system can be proved in a completely standard way by building a canonical model. Thus we have:

Theorem 1 (Completeness of MDL$^+$II**).** *Let* $\Sigma \cup \{\varphi\}$ *be a set of* $\mathcal{L}_{\mathsf{MDL^+II}}$*-formulas. Then, if* $\Sigma \models_{\mathsf{MDL^+II}} \varphi$ *then* $\Sigma \vdash_{\mathsf{MDL^+II}} \varphi$*.*

3 A dynamic extension ECL II

In the previous section, we have seen that $\mathcal{L}_{\mathsf{MDL^+II}}$-formulas can be used to describe the situations before and after the issuance of your boss's command. But the change brought about by your boss's command was not talked about as a change in $\mathcal{L}_{\mathsf{MDL^+II}}$ but in the meta-language, and it is simply impossible to use $\mathcal{L}_{\mathsf{MDL^+II}}$ to talk about the act of commanding which changed M into N. In Yamada [21], $\mathcal{L}_{\mathsf{MDL^+}}$ was extended to $\mathcal{L}_{\mathsf{CL}}$ by introducing operators indexed by the terms of the form $!_i\varphi$ in order to talk about effects of acts of commanding. Now we introduce expressions of the form $!_{(i,j)}\varphi$ for each pair $(i, j) \in I \times J$ in order to denote the type of an act of commanding in which an authority j commands an agent i to see to it that φ. The static base language $\mathcal{L}_{\mathsf{MDL^+II}}$ shall be expanded by introducing new modalities indexed by expressions of this form. Then, in the resulting language, the language $\mathcal{L}_{\mathsf{CL\,II}}$, of Command Logic, we have formulas of the form $[!_{(i,j)}\varphi]\psi$, which is to mean that after every successful act of commanding of type $!_{(i,j)}\varphi$, ψ holds. Thus we define:

Definition 5. *Take the same countably infinite set* Aprop *of proposition letters, the same finite set* I *of agents, and the same finite set* J *of command issuing authorities as before, with* p *ranging over* Aprop*,* i *over* I*, and* j *over* J*. The refined language of command logic* $\mathcal{L}_{\mathsf{CL\,II}}$ *is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_{(i,j)}\varphi \mid [\pi]\varphi$$
$$\pi ::= !_{(i,j)}\varphi$$

[6] Strictly speaking, $O_{(i,j)}$-Nec is redundant since it is derivable. It is included here just to record the fact that MDL$^+$II is normal.

Terms of the form $!_{(i,j)}\varphi$ *and operators of the form* $[!_{(i,j)}\varphi]$ *are called command type terms and command operators, respectively. The set of all well formed formulas of* $\mathcal{L}_{\mathsf{CL\,II}}$ *is referred to as* $S_{\mathsf{CL\,II}}$*, and the set of all the well formed command type terms as* Com II.

$\bot, \vee, \rightarrow, \leftrightarrow, \Diamond, P_{(i,j)}, F_{(i,j)}$, and $\langle !_{(i,j)}\varphi \rangle$ are assumed to be introduced by definition in the obvious way. Note that $S_{\mathsf{MDL^+II}} \subset S_{\mathsf{CL\,II}}$.

Then, the truth definition for the sentences of $\mathcal{L}_{\mathsf{CL\,II}}$ can be given with reference to $\mathcal{L}_{\mathsf{MDL^+II}}$-models as follows:

Definition 6. *Let* $M = (W^M, R^M_A, R^M_D, V^M)$ *be an* $\mathcal{L}_{\mathsf{MDL^+II}}$*-model, and* $w \in W^M$*. If* $p \in$ Aprop, $\varphi, \psi, \chi \in S_{\mathsf{CL\,II}}$*, and* $(i, j) \in I \times J$*, then:*

(a) $M, w \models_{\mathsf{ECL\,II}} p$ *iff* $w \in V^M(p)$

(b) $M, w \models_{\mathsf{ECL\,II}} \top$

(c) $M, w \models_{\mathsf{ECL\,II}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{ECL\,II}} \varphi$

(d) $M, w \models_{\mathsf{ECL\,II}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{ECL\,II}} \varphi$ *and* $M, w \models_{\mathsf{ECL\,II}} \psi$

(e) $M, w \models_{\mathsf{ECL\,II}} \Box\varphi$ *iff* $M, v \models_{\mathsf{ECL\,II}} \varphi$ *for every* v *such that* $(w, v) \in R^M_A$

(f) $M, w \models_{\mathsf{ECL\,II}} O_{(i,j)}\varphi$ *iff* $M, v \models_{\mathsf{ECL\,II}} \varphi$ *for every* v *such that* $(w, v) \in R^M_D(i, j)$

(g) $M, w \models_{\mathsf{ECL\,II}} [!_{(i,j)}\chi]\varphi$ *iff* $M_{!_{(i,j)}\chi}, w \models_{\mathsf{ECL\,II}} \varphi$,

where $M_{!_{(i,j)}\chi}$ *is an* $\mathcal{L}_{\mathsf{MDL^+II}}$*-model obtained from* M *by replacing* R^M_D *with the function* $R^{M_{!_{(i,j)}\chi}}_D$ *such that:*

(i) $R^{M_{!_{(i,j)}\chi}}_D(k, l) = R^M_D(k, l)$, *for each* $(k, l) \in I \times J$ *such that* $(k, l) \neq (i, j)$

(ii) $R^{M_{!_{(i,j)}\chi}}_D(k, l) = \{(x, y) \in R^M_D(i, j) \mid M, y \models_{\mathsf{ECL\,II}} \chi\}$ *if* $(k, l) = (i, j)$.

We abbreviate $\{(x, y) \in R^M_D(i, j) \mid M, y \models_{\mathsf{ECL\,II}} \chi\}$ *as* $R^M_{(i,j)} \upharpoonright \chi^{\downarrow}$*. A formula* φ *is true in an* $\mathcal{L}_{\mathsf{MDL^+II}}$*-model* M *at a point* w *of* M *if* $M, w \models_{\mathsf{ECL\,II}} \varphi$*. We say that a set* Σ *of formulas of* $\mathcal{L}_{\mathsf{CL\,II}}$ *is true in* M *at* w*, and write* $M, w \models_{\mathsf{ECL\,II}} \Sigma$*, if* $M, w \models_{\mathsf{ECL\,II}} \psi$ *for every* $\psi \in \Sigma$*. If* $\Sigma \cup \{\varphi\}$ *is a set of formulas of* $\mathcal{L}_{\mathsf{CL\,II}}$*, we say that* φ *is a semantic consequence of* Σ*, and write* $\Sigma \models_{\mathsf{ECL\,II}} \varphi$*, if for every* $\mathcal{L}_{\mathsf{MDL^+II}}$*-model* M *and every point* w *of* M *such that* $M, w \models_{\mathsf{ECL\,II}} \Sigma$*,* $M, w \models_{\mathsf{ECL\,II}} \varphi$*. We say that a formula* φ *is valid, and write* $\models_{\mathsf{ECL\,II}} \varphi$*, if* $\emptyset \models_{\mathsf{ECL\,II}} \varphi$*.*

The crucial clause here is (g). The truth condition of $[!_{(i,j)}\chi]\varphi$ at w in M is defined in terms of the truth condition of φ at w in the updated model $M_{!_{(i,j)}\chi}$. Let a pair (w, v) of points be referred to as the R-arrow from w to v if it is in an accessibility relation R. Then the workings of an act of commanding of the form $!_{(a,b)}\varphi$ can be captured by saying that it eliminate every $R^M_{(a,b)}$-arrow (w, v) such that $M, v \not\models_{\mathsf{ECL\,II}} \varphi$ from $R^M_{(a,b)}$ if it is performed at some world in M. Note that the only possible difference between $M_{!_{(i,j)}\chi}$ and M consists in the possible difference between $R^M_{(i,j)} \upharpoonright \chi^{\downarrow}$ and $R^M_{(i,j)} (= R^M_D(i, j))$. All the other constituents are common to them. Since we always have $R^M_{(i,j)} \upharpoonright \chi^{\downarrow} \subseteq R^M_{(i,j)}$, we also have $R^M_{(i,j)} \restriction \chi^{\downarrow} \subseteq R^M_A$ as required in the clause (iii) of Definition 2. Thus $M_{!_{(i,j)}\chi}$ is guaranteed to be an $\mathcal{L}_{\mathsf{MDL^+II}}$-model. [7]

Note also that each of the remaining clauses in the above definition reproduces the corresponding clause in the truth definition for $\mathcal{L}_{\mathsf{MDL^+II}}$. Obviously, we have:

Corollary 1. *Let M be an $\mathcal{L}_{\mathsf{MDL^+II}}$-model and w a point of M. Then for any $\varphi \in S_{\mathsf{MDL^+II}}$, $M, w \models_{\mathsf{ECL\,II}} \varphi$ iff $M, w \models_{\mathsf{MDL^+II}} \varphi$.*

The following corollary can be proved by induction on the length of ψ:

Corollary 2. *Let ψ be an (i, j)-free formula. Then, for any $\varphi \in S_{\mathsf{CL\,II}}$, $M, w \models_{\mathsf{ECL\,II}} \psi$ iff $M_{!_{(i,j)}\varphi}, w \models_{\mathsf{ECL\,II}} \psi$.*

One of the things this corollary means is that acts of commanding do not affect so-called brute facts and alethic possibilities in any direct way.

Consider the previous example again. Let p, a and b be understood as before. Let c represent your political guru, and let q express the proposition that you will attend the political demonstration c mentioned. In the situation before the issuance of your boss's command, it was not obligatory upon you to see to it that p, nor was it so to see to it that $\neg p$. Let (M, s) represent that situation as before. Then we have:

$$M, s \models_{\mathsf{ECL\,II}} \neg O_{(a,b)} p \wedge \neg O_{(a,b)} \neg p \quad . \tag{3}$$

This means that we have:

$$M, s \models_{\mathsf{ECL\,II}} P_{(a,b)} \neg p \wedge P_{(a,b)} p \quad . \tag{4}$$

As we have assumed that whatever is permitted is possible, we have:

$$M, s \models_{\mathsf{ECL\,II}} \Diamond \neg p \wedge \Diamond p \quad . \tag{5}$$

In this situation, we also have:

$$M, s \models_{\mathsf{ECL\,II}} \neg O_{(a,c)} q \wedge \neg O_{(a,c)} \neg q \quad . \tag{6}$$

Hence we have:

$$M, s \models_{\mathsf{ECL\,II}} \Diamond p \wedge \Diamond q \quad . \tag{7}$$

[7] If we impose additional frame conditions on models by adding extra axioms to the proof system of MDL$^+$II, however, the above model updating operation may yield models which violate these conditions. Thus we will have to impose matching constraints upon updating operation, but it might not always be possible. For example, see the discussion on Dead End principle in Section 5. Model updating operations are used and studied in dynamic epistemic logics and a more general discussion can be found in van Benthem & Liu [3].

Since $\Diamond p$ and $\Diamond q$ are (a, b)-free and (a, c)-free, Corollary 2 guarantees:

$$(M_{!(a,b)p})_{!(a,c)q}, s \models_{\mathsf{ECL\,II}} \Diamond p \wedge \Diamond q \ . \quad (8)$$

Thus, Corollary 2 enables us to capture, at least partially, unchanging aspects of the changing situations.

As regards the changing aspects, the semantics defined above validates:

$$M, s \models_{\mathsf{ECL\,II}} [!_{(a,b)}p]O_{(a,b)}p \ . \quad (9)$$

Your boss's command eliminates all the $R^M_{(a,b)}$-arrows (w, v) such that $M, v \not\models_{\mathsf{ECL\,II}} p$, and consequently we have:

$$M_{!_{(a,b)}p}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \ . \quad (10)$$

In fact this is an instantiation of the following principle:

Proposition 1 (CUGO Principle). *If* $\varphi \in S_{(i,j)\text{-free}}$, *then* $\models_{\mathsf{ECL\,II}} [!_{(i,j)}\varphi]O_{(i,j)}\varphi$.

CUGO Principle here characterizes, at least partially, the workings of acts of commanding; though not without exceptions, commands usually generate obligations. The restriction on φ here is motivated by the fact that the truth of φ at a point v in M does not guarantee the truth of φ at v in $M_{!_{(i,j)}\varphi}$ if φ involves deontic modalities for the pair (i, j). For example, $[!_{(i,j)}P_{(i,j)}q]O_{(i,j)}P_{(i,j)}q$ is not valid. [8]

Let's go back to the example. As $O_{(a,b)}p$ is (a, c)-free, Corollary 2 guarantees:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \ . \quad (11)$$

As another instantiation of CUGO Principle, we have:

$$M_{!_{(a,b)}p}, s \models_{\mathsf{ECL\,II}} [!_{(a,c)}q]O_{(a,c)}q \ . \quad (12)$$

By definition, this is equivalent to:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,c)}q \ . \quad (13)$$

Hence we have:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \wedge O_{(a,c)}q \ . \quad (14)$$

[8] For more on this point, see Yamada [21].

Thus, it is obligatory upon you to see to it that p with respect to your boss while it is obligatory upon you to see to it that q with respect to your guru.

Unfortunately, however, as we have supposed earlier, no means of transportation that is fast enough to enable you to join the demonstration in Tokyo and attend the conference in São Paulo on the same day happened to be available. It is not possible for you to obey both commands. One possible way of expressing this supposition is to assume:

$$M, s \models_{\mathsf{ECL\,II}} \neg\Diamond(p \wedge q) \ . \tag{15}$$

Then, as $\neg\Diamond(p \wedge q)$ is (a, b)-free and (a, c)-free, Corollary 2 guarantees:

$$(M_{!(a,b)p})_{!(a,c)q}, s \models_{\mathsf{ECL\,II}} \neg\Diamond(p \wedge q) \ . \tag{16}$$

Thus, if we accept (15), we will have:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \wedge O_{(a,c)}q \wedge \neg\Diamond(p \wedge q) \ . \tag{17}$$

If you obey your boss's command you will disobey your guru's command; if you obey your guru's command you will disobey your boss's command. You are in an obligational dilemma. As $p \wedge q$ is not a logical contradiction, there may be a possible situation in which you could obey both commands, but unfortunately it is not the situation you are in.

Whether this is really a good way of representing the situation you are in, however, doesn't seem to be obvious, since the impossibility involved in this situation is not an alethic (*i.e.* metaphysical) impossibility. If a sufficiently fast means of transportation were available, it would be possible for you to obey both commands. I will return to this point after looking at an obligational dilemma of a different kind.

4 Proof system for ECL II

Now we define proof system for ECL II.

Definition 7. *The proof system for* ECL II *contains all the axioms and all the rules of the proof system for* $\mathsf{MDL}^{+}\mathsf{II}$*, and in addition the following reduction axioms and rules:*

(RAt)	$[!_{(i,j)}\varphi]p \leftrightarrow p$ *where* $p \in Aprop$	(Reduction to Atoms)
(RVer)	$[!_{(i,j)}\varphi]\top \leftrightarrow \top$	(Reduction to Verum)
(FUNC)	$[!_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[!_{(i,j)}\varphi]\psi$	(Functionality)
($[!_{(i,j)}\varphi]$-Dist)	$[!_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow ([!_{(i,j)}\varphi]\psi \wedge [!_{(i,j)}\varphi]\chi)$	($[!_{(i,j)}\varphi]$-Distribution)
(RAleth)	$[!_{(i,j)}\varphi]\Box\psi \leftrightarrow \Box[!_{(i,j)}\varphi]\psi$	(Reduction for Alethic Modality)

(RObl) $[!_{(i,j)}\varphi]O_{(i,j)}\psi \leftrightarrow O_{(i,j)}(\varphi \to [!_{(i,j)}\varphi]\psi)$ (Reduction for Obligation)

(RInd) $[!_{(i,j)}\varphi]O_{(k,l)}\psi \leftrightarrow O_{(k,l)}[!_{(i,j)}\varphi]\psi$ *where* $(i,j) \neq (k,l)$ (Independence)

($[!_{(i,j)}\varphi]$-Nec) $\dfrac{\psi}{[!_{(i,j)}\varphi]\psi}$ *for each* $(i,j) \in I \times J$. ($[!_{(i,j)}\varphi]$-necessitation)

An ECL II*-proof of a formula* φ *is a finite sequence of* $\mathcal{L}_{\mathsf{CL\,II}}$*-formulas having* φ *as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{ECL\,II}} \varphi$*. If* $\Sigma \cup \{\varphi\}$ *is a set of* $\mathcal{L}_{\mathsf{CL\,II}}$*-formulas, we say that* φ *is deducible in* ECL II *from* Σ *and write* $\Sigma \vdash_{\mathsf{ECL\,II}} \varphi$ *if* $\vdash_{\mathsf{ECL\,II}} \varphi$ *or there are formulas* $\psi_1, \ldots, \psi_n \in \Sigma$ *such that* $\vdash_{\mathsf{ECL\,II}} (\psi_1 \wedge \cdots \wedge \psi_n) \to \varphi$*.*

It is easy to verify that all these axioms are valid and the rules preserve validity. Hence the proof system for ECL II is sound. Obviously the following condition holds:

Corollary 3. *Let* $\Sigma \cup \{\varphi\} \subseteq S_{\mathsf{MDL^{+}II}}$*. Then, if* $\Sigma \vdash_{\mathsf{MDL^{+}II}} \varphi$*, then* $\Sigma \vdash_{\mathsf{ECL\,II}} \varphi$*.*

RAt and Rver axioms allow us to eliminate command operators prefixed to a proposition letter and $\top$ respectively, and other axioms enable us to reduce the length of sub-formulas to which command operators are prefixed. Consequently, any sentence of $\mathcal{L}_{\mathsf{ECL\,II}}$ can be translated into a sentence of $\mathcal{L}_{\mathsf{MDL^{+}II}}$ that is provably equivalent to it. Thus:

Definition 8 (Translation). *The translation function t that takes a formula from* $\mathcal{L}_{\mathsf{CL\,II}}$ *and yields a formula in* $\mathcal{L}_{\mathsf{MDL^{+}II}}$ *is defined as follows:*

$$
\begin{array}{lll lll}
t(p) & = & p & t([!_{(i,j)}\varphi]p) & = & p \\
t(\top) & = & \top & t([!_{(i,j)}\varphi]\top) & = & \top \\
t(\neg\varphi) & = & \neg t(\varphi) & t([!_{(i,j)}\varphi]\neg\psi) & = & \neg t([!_{(i,j)}\varphi]\psi) \\
t(\varphi \wedge \psi) & = & t(\varphi) \wedge t(\psi) & t([!_{(i,j)}\varphi](\psi \wedge \chi)) & = & t([!_{(i,j)}\varphi]\psi) \wedge t([!_{(i,j)}\varphi]\chi) \\
t(\Box\varphi) & = & \Box t(\varphi) & t([!_{(i,j)}\varphi]\Box\psi) & = & \Box t([!_{(i,j)}\varphi]\psi) \\
t(O_{(i,j)}\varphi) & = & O_{(i,j)}t(\varphi) & t([!_{(i,j)}\varphi]O_{(i,j)}\psi) & = & O_{(i,j)}(t(\varphi) \to t([!_{(i,j)}\varphi]\psi)) \\
 & & & t([!_{(i,j)}\varphi]O_{(k,l)}\psi) & = & O_{(k,l)}t([!_{(i,j)}\varphi]\psi) \text{ where } (i,j) \neq (k,l) \\
 & & & t([!_{(i,j)}\varphi][!_{(k,l)}\psi]\chi) & = & t([!_{(i,j)}\varphi]t([!_{(k,l)}\psi]\chi)) \\
 & & & & & \text{for any } (k,l) \in I \times J\ .
\end{array}
$$

It is easy, though sometimes tedious, to prove that this translation has the properties stated by the following corollaries and lemmas:

Corollary 4 (Translation Effectiveness). *For every formula* $\eta \in S_{\mathsf{CL\,II}}$*,* $t(\eta) \in S_{\mathsf{MDL^{+}II}}$*.*

Lemma 1 (Translation Correctness). *Let* M *be an* $\mathcal{L}_{\mathsf{MDL^{+}II}}$*-model, and* w *a point of* M*. Then for any formula* η *of* $\mathcal{L}_{\mathsf{CL\,II}}$*,* $M, w \models_{\mathsf{ECL\,II}} \eta$ *iff* $M, w \models_{\mathsf{ECL\,II}} t(\eta)$*.*

Corollary 5. *Let* M *be an* $\mathcal{L}_{\mathsf{MDL^{+}II}}$*-model, and* w *a point of* M*. Then for any formula* η *of* $\mathcal{L}_{\mathsf{CL\,II}}$*,* $M, w \models_{\mathsf{ECL\,II}} \eta$ *iff* $M, w \models_{\mathsf{MDL^{+}II}} t(\eta)$*.*

Lemma 2. *For any formula $\eta \in S_{\mathsf{ECL\,II}}$, $\vdash_{\mathsf{ECL\,II}} \eta \leftrightarrow t(\eta)$.*

These properties enable us to derive the completeness of ECL II from the known completeness of MDL$^+$II. The use of translation based on reduction axioms has been a standard method in the development of the logic of public announcements.[9] The proof of the completeness of ECL II is exactly similar to that of the completeness of ECL given in Yamada [20]. Here we only state the result.

Theorem 2 (Completeness of ECL II). *For any set $\Sigma \cup \{\varphi\}$ of formulas of $\mathcal{L}_{\mathsf{CL\,II}}$, if $\Sigma \models_{\mathsf{ECL\,II}} \varphi$, then $\Sigma \vdash_{\mathsf{ECL\,II}} \varphi$.*

5 Built-in assumptions and interesting validities and non-validities

As $I \times J$ is a finite set, from a purely formal point of view, all instances of $\mathcal{L}_{\mathsf{MDL^+II}}$ and $\mathcal{L}_{\mathsf{CL\,II}}$ are instances of $\mathcal{L}_{\mathsf{MDL^+}}$ and $\mathcal{L}_{\mathsf{CL}}$ respectively, and all $\mathcal{L}_{\mathsf{MDL^+II}}$-models are $\mathcal{L}_{\mathsf{MDL^+}}$-models. As the truth definition for $\mathcal{L}_{\mathsf{CL\,II}}$ exactly parallels that for $\mathcal{L}_{\mathsf{CL}}$, ECL II inherit all three built-in assumptions from ECL; (1) acts of commanding are assumed to be always eliminative so that we always have $R^M_{(i,j)}\upharpoonright\chi^{\downarrow} \subseteq R^M_{(i,j)}$; (2) acts of commanding of the form $!(i, j)\varphi$ performed at some world in an model M are assumed to have no effects on the deontic accessibility relation other than $R^M_{(i,j)}$; and (3) commands are assumed to have no preconditions for their issuance.[10] Moreover, all the validities are inherited mutatis mutandis.

But in concrete applications, the distinction between command issuing authorities provide us with a finer grained treatment of examples. Suppose, for example, your boss were so stupid that he gave you a command of the form $!_{(a,b)}\neg p$ on the same day he had commanded you to see to it that p. Now, ECL II inherits the following principles from ECL:

(DE) $[!_{(i,j)}(\varphi \wedge \neg\varphi)]O_{(i,j)}\psi$ (Dead End)

(RSC) $[!_{(i,j)}\varphi][!_{(i,j)}\psi]\chi \leftrightarrow [!_{(i,j)}(\varphi \wedge \psi)]\chi$ where $\varphi, \psi \in S_{(i,j)\text{-free}}$
(Restricted Sequential Conjunction)

(ROI) $[!_{(i,j)}\varphi][!_{(i,j)}\psi]\chi \leftrightarrow [!_{(i,j)}\psi][!_{(i,j)}\varphi]\chi$ where $\varphi, \psi \in S_{(i,j)\text{-free}}$.
(Restricted Order Invariance)

As $\neg p$ is (a, b)-free, by Restricted Sequential Conjunction Principle, we have:

$$[!_{(a,b)}p][!_{(a,b)}\neg p]\chi \leftrightarrow [!_{(a,b)}(p \wedge \neg p)]\chi \ . \tag{18}$$

By Dead End Principle, we have:

[9] Van Benthem & Liu [3] proved that every relation changing operation that is definable in PDL without iteration has a complete set of reduction axioms in dynamic epistemic logic.
[10] For a detailed discussion of these assumptions, see [21].

$$M, s \models_{\mathsf{ECL\,II}} [!_{(a,b)}(p \wedge \neg p)]O_{(a,b)}\psi \ . \tag{19}$$

Hence:

$$M, s \models_{\mathsf{ECL\,II}} [!_{(a,b)}p][!_{(a,b)}\neg p]O_{(a,b)}\psi \ . \tag{20}$$

This is equivalent to:

$$(M_{!_{(a,b)}p})_{!_{(a,b)}\neg p}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}\psi \ . \tag{21}$$

As $(R^M_{(a,b)} \upharpoonright p^{\downarrow}) \upharpoonright \neg p^{\downarrow}$ is empty, no world that is compatible with the obligations with respect to your boss is accessible from s for you; you are in an absurd state. [11]

But if it is not your boss but your guru that commanded you to see to it that $\neg p$, you will be in a slightly different situation. We then have:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}\neg p}, s \models_{\mathsf{ECL\,II}} (O_{(a,b)}p \wedge O_{(a,c)}\neg p) \wedge \neg\Diamond(p \wedge \neg p) \ . \tag{22}$$

As $p \wedge \neg p$ is a contradiction, it is logically impossible for you to obey both your boss's command and your guru's command. But there might still be worlds $R_{(a,b)}$-accessible from s and worlds $R_{(a,c)}$-accessible from s in $(M_{!_{(a,b)}p})_{!_{(a,c)}\neg p}$. And so, you are not in an obligational dead end but in an obligational dilemma.

Now let's go back to the first example, in which your guru commanded you to see to it that q after your boss commanded you to see to it that p. We have considered one possible way of representing the situation you are supposed to be in after the issuance of your guru's command in this example, namely (17). I reproduce it here as (23).

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \wedge O_{(a,c)}q \wedge \neg\Diamond(p \wedge q) \ . \tag{23}$$

The most important difference between (22) and (23) consists in the fact that $p \wedge q$ is not a contradiction while $p \wedge \neg p$ is. So there might be a world t, even in M, for which the following condition holds:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, t \models_{\mathsf{ECL\,II}} p \wedge q \ . \tag{24}$$

So, the fact that the impossibility involved in this situation is not a logical impossibility can be said to be reflected in a sense even if we accept (23).

Accepting (23) as a way of representing the situation here, however, still seems to be a bit problematic. As I remarked earlier, we may say that if a sufficiently fast means of transportation were available, it would be possible for you to obey both commands.

[11] Note that Dead End Principle precludes the possibility of adding multi-agent variant of the so-called D axiom, $O_{(a,b)}\phi \rightarrow P_{(a,b)}\phi$, to ECL II. For more on this point, see Yamada [21].

Thus, if $((M_{!_{(a,b)}p})_{!_{(a,c)}q}, s)$ is to represent the situation you are supposed to be in, it seems that we ought to have:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} \Diamond(p \wedge q) \ . \tag{25}$$

But then, (23) cannot be correct. Thus, the only remaining way of representing the sort of impossibility involved here in $\mathcal{L}_{\mathsf{CLII}}$ seems to be to say:

$$(M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} O_{(a,b)}p \wedge O_{(a,c)}q \wedge \neg(p \wedge q) \ . \tag{26}$$

Thus, if you obey your boss's command, then you will disobey your guru's command $((M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} p \rightarrow \neg q)$, and if you obey your guru's command, then you will disobey your boss's command $((M_{!_{(a,b)}p})_{!_{(a,c)}q}, s \models_{\mathsf{ECL\,II}} q \rightarrow \neg p)$, in the real world you are in. Even if a sufficiently fast means of transportaion were available to you in any other possible worlds, it would not be of much help to you. In this example, you are in an obligational dilemma in the real world just because of a contingent fact about the present state of the system of transportation in it. The situation looks very closely similar to those situations in which you are in moral dilemmas. [12]

6 Conclusion

In this paper, an eliminative command logic ECL is slightly refined into ECL II by allowing command terms and deontic operators to be indexed by a Cartesian Product of a given finite set of agents and a given finite set of command issuing authorities. Complete axiomatization and interesting validities are presented, and a concrete example of a situation in which conflicting commands are given to one and the same agent by different authorities is discussed extensively.

In ECL and ECLII, model updating operations are used to model effects of acts of commanding. This idea is imported from dynamic epistemic logics developed in Plaza [15], Groeneveld [9], Gerbrandy and Groeneveld [6], Gerbrandy [5], Baltag, Moss, & Solecki [2], and Kooi & van Benthem [11] among others. In these logics, model updating operations are used to model effects of various forms of information transmissions. In the field of deontic reasoning, van der Torre & Tan [18] and Žarnić [22] extended update semantics of Veltman [19] and uses model updating operations to interpret normative sentences and natural language imperatives respectively. As is noted in Yamada [21], the relation between their semantics on the one hand and ECL and ECLII on the other is analogous to that between Veltman's update semantics and various epistemic logics. In this respects, DLP_{dyn} of Pucella & Weissman [16] and DLP^{+}_{dyn} of Demri [4] are closer to ECL and ECLII in spirit. They use model updating operations to model changes in legal policies and thereby dynamified DLP, a logic of permission, of van der Meyden [14]. And more recently, van Benthem & Liu [3] proposed "preference upgrade" as a counter part to information update. According to them, my "command

[12] An illuminating discussions on moral dilemmas can be found in Marcus [13].

operator for proposition A can be modeled exactly as an upgrade sending R to $R; ?A$" in their system, and their paper "provides a much more general treatment of possible upgrade instructions" ([3]). Their preference upgrade really has a much wider application than the deontic update of the present paper. But, as is noted in [21], the notion of preference upgrade seems to be connected with perlocutionary consequences of various utterances, while the deontic update is used to capture effects of acts of commanding as a specific kind of illocutionary acts. They can be seen as mutually complementary.

With regards to the possibilities of further research, there is an apparent need of dynamifying richer deontic languages. The dynamified languages $\mathcal{L}_{\mathsf{CL}}$ and $\mathcal{L}_{\mathsf{CLII}}$ inherit various inadequacies from the static base languages $\mathcal{L}_{\mathsf{MDL}^+}$ and $\mathcal{L}_{\mathsf{MDL}^+\mathsf{II}}$. [13] Moreover, the possibilities of update logics of various other kinds of illocutionary acts suggest themselves. For example, an act of promising can be considered as another deontic updator, and an act of asserting as an updator of propositional commitments. Here I only mention one interesting immediate application. A command type term of the form $!_{(i,j)}\varphi$ can be reinterpreted as a term for a type of an act of promising with a promisor i and an promisee j to the effect that i will see to it that φ. Then the analogue of CUGO principle will state that acts of promising usually generates obligations. Comparing this with Searle's discussion on the relation between acts of promising and obligations in Searle [17] will be a task for another paper.

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[13] Kooi & Tamminga [12] introduced a formula of the form $\odot^{\mathcal{F}}_{\mathcal{G}}\varphi$ in order to deal with conflicting obligations. Intuitively, a formula of this form is supposed to mean that group $\mathcal{G}$ of agents ought to see to it that φ in the interest of group $\mathcal{F}$. When $\mathcal{G}$ and $\mathcal{F}$ are unit sets, say, $\{i\}$ and $\{j\}$ respectively, we get a formula of the form $\odot^{j}_{i}\varphi$. A formula of this form can be used to express what we express by using a formula of the form $O_{(i,j)}\varphi$ in $\mathcal{L}_{\mathsf{MDL}^+\mathsf{II}}$. As their logic extends a simplified version of Horty's multi-agent deontic logic based on stit theory developed in [10], it suggests interesting possibilities of extending $\mathsf{MDL}^+\mathsf{II}$, though such extended logics will obviously be more difficult to dynamify than $\mathsf{MDL}^+\mathsf{II}$.

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