Acts of Promising in Dynamified Deontic Logic

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Abstract. In this paper, the logic of acts of commanding ECL II introduced in Yamada (2007b) will be extended in order to model acts of promising together with acts of commanding. Effects of both kinds of acts are captured in terms, not of changes they bring about on propositional attitudes of their addressees, but of changes they bring about on deontic status of relevant action alternatives; they are modeled as deontic updators. This enables us to see how an act of promising performed by an agent and an act of commanding performed by another agent can jointly bring about a conflict of obligations. Complete axiomatization will be presented, and a comparison with Searle's treatment of acts of promising in his argument for the derivability of "ought" from "is" will be made.

1 Introduction

In the last two decades, systems of dynamic epistemic logic (DEL) have been developed to deal with dynamic changes brought about by various kinds of information transmissions including public announcements as well as private communications in Plaza (1989), Groeneveld (1995), Gerbrandy & Groeneveld (1997), Baltag, Moss, & Solecki (1999), and Kooi & van Benthem (2004) among others. Model updating operations are introduced to interpret these communicative acts as what update epistemic states of agents involved, and dynamic epistemic logics are obtained as dynamic extensions of static epistemic logics. In the logics of acts of commanding, ECL of Yamada (2007a) and ECL II of Yamada (2007b) , similar model updating operations are introduced to interpret acts of commanding as updators of deontic aspects of the situations in which agents are involved.

ECL and ECL II are dynamic extensions of multi-agent variants of static monadic deontic logic MDL^+ and $\mathsf{MDL}^+\mathsf{II}$ respectively. In MDL^+, deontic operators are allowed to be indexed by a given finite set of agents in order to distinguish agents to whom commands are given from other agents, and in $\mathsf{MDL}^+\mathsf{II}$, they are allowed to be indexed by the Cartesian product of a given finite set of agents and a given finite set of command issuing authorities in order to deal with (possibly conflicting) obligations generated by commands given to the same agents by different authorities. The purpose of the present paper is to extend $\mathsf{MDL}^+\mathsf{II}$ and ECL II in order to deal with changes brought about by acts of promising along with those brought about by acts of commanding.

Consider the following example:

Example 1. Suppose you have received a letter from your political guru, in which he commanded you to join an important political demonstration to be held in Tokyo next month. Unfortunately, it is to be held on the very same day on which an international one-day conference on logic is to be held in Sào Paulo, and you had already promised your former student who organizes that conference that you would give an invited lecture there. It is possible for you to join the demonstration in Tokyo, but if you choose to do so, you will fail to keep your promise. It is also possible for you to give a lecture at the conference in Sào Paulo, but if you choose to do so, you will fail to obey your guru's command. No available means of transportaion are fast enough to enable you to join both events on the same day even though the time in Sào Paulo is 12 hours behind the time in Tokyo. You have to decide which alternative to choose. But you are sure whichever alternative you may choose, you will regret not being able to choose the other.

In this example, your guru's command is in conflict with your earlier promise. In this paper we will show how such a conflict could be brought about by your act of promising and your guru's act of commanding by developing a logic in which both kinds of acts are modeled as deontic updators. For this purpose, ECL II will be extended by introducing terms standing for types of acts of promising. In order to do so, however, we also have to reconsider MDL$^+$II.

In Section 2, we reconsider MDL$^+$II, and extend it by allowing deontic operators to be indexed by a triad of agents. Then in section 3, we develop a dynamic extension of this extended Multi-agent Deontic Logic by introducing modalities that model various acts of commanding and promising. In Section 4, we discuss some interesting notions and interplay between acts of commanding and acts of promising expressible in the extended language. Then in section 5, complete axiomatization will be given to the dynamified logic, and finally in section 6, we compare our treatment of acts of promising with John Searle's treatment in his argument for the derivability of 'ought' from 'is', and make a brief comment on the difference between Searle's treatment of illocutionary acts and Austin's.

2 The Static Base Logic MDL$^+$III

In MDL$^+$II (and in ECLII), we have a formula of the form $O_{(i,j)}\varphi$. Intuitively, it means that it is obligatory upon an agent i with respect to an authority j to see to it that φ. Let p and q denote the proposition that you will give a lecture at the workshop in Sào Paulo and the proposition that you will join the demonstration in Tokyo respectively, and let a, b, and c represent you, your former student, and your guru respectively. Then in ECLII, the type of your guru's command is represented by the expression of the form $!_{(a,c)}q$, where a is the commandee and c is the commander, and the following formula is shown to be valid:

$$[!_{(a,c)}q]O_{(a,c)}q \quad . \tag{1}$$

Intuitively, this formula means that after c's successful act of commanding a to see to it that q, it is obligatory upon a with respect to c to see to it that q. The validity of this formula guarantees that in the updated model, which represents the situation after the issuance of your guru's command, the following formula holds at the current world:

$$O_{(a,c)}q \ . \tag{2}$$

Thus the conventional effect of your guru's act of commanding is captured in MDL$^+$II (and in ECLII, too, since ECLII extends MDL$^+$II).

Now consider your earlier promise. Let the expression of the form $\mathrm{Prom}_{(i,j)}\varphi$ denote the type of acts of promising in which an agent i promises an agent j that she will see to it that φ. Then the type of your act of promising can be denoted by $\mathrm{Prom}_{(a,b)}p$, and it's not difficult to define semantics which validates the following formula:

$$[\mathrm{Prom}_{(a,b)}p]O_{(a,b)}p \ . \tag{3}$$

This means that the following formula holds at the current world in the updated model:

$$O_{(a,b)}p \ . \tag{4}$$

Notice, however, that this is not exactly what we want, since your former student need not be among the command issuing authorities. In order to accommodate obligation generated by acts of promising, we have to reinterpret the second constituent of the indexing pair. If we reinterpret it as the agent whose act generates the obligation, we will have:

$$[\mathrm{Prom}_{(a,b)}p]O_{(a,a)}p \ . \tag{5}$$

But then, the promisee, towards whom the promiser has obligation, will be left unmentioned in the formula characterizing the obligation generated. It doesn't seem quite right as the obligations generated by acts of promising are among the kind of obligations sometimes referred to as "special obligations" in the literature.[1] They are owed to some subset of persons but not to all the persons. Although there are disputes on whether we have genuinely special obligations, common sense morality seems to understand us as having special obligations. It seems desirable, thus, not to ignore the agent to whom the obligations generated by acts of promising are owed. Moreover, the distinction between the agents to whom the obligations are owed was crucial to the adequate treatment of the obligational dilemma generated by the conflicting commands coming from different authorities in Yamada (2007b).

Our working hypothesis in this paper, then, is that an obligation can be considered to be related to a triad of agents (i, j, k) and a proposition φ such that it is obligatory upon i with respect to j in the name of k to see to it that φ. Thus, we define:

[1] A detailed discussion of special obligation can be found in Jeske (2002).

Definition 1. *Take a countably infinite set* **Aprop** *of proposition letters, and a finite set I of agents, with p ranging over* **Aprop**, *and i, j, k over I. The refined multi-agent monadic deontic language $\mathcal{L}_{\mathsf{MDL^+III}}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_{(i,j,k)}\varphi$$

The set of all well formed formulas (sentences) of $\mathcal{L}_{\mathsf{MDL^+III}}$ is denoted by $S_{\mathsf{MDL^+III}}$ and operators of the form $O_{(i,j,k)}$ are called deontic operators. For each $i, j, k \in I$, we call a sentence (i, j, k)-free if no $O_{(i,j,k)}$'s occur in it. We call sentence alethic if no deontic operators occur in it, and boolean if no modal operators occur in it. For each $i, j, k \in I$, the set of all (i, j, k)-free sentences is denoted by $S_{(i,j,k)\text{-free}}$. The set of all alethic sentences and the set of all boolean sentences are denoted by S_{Aleth} and S_{Boole} respectively.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\Diamond$ are assumed to be introduced by standard definitions. We also abbreviate $\neg O_{(i,j,k)}\neg\varphi$ as $P_{(i,j,k)}\varphi$, and $O_{(i,j,k)}\neg\varphi$ as $F_{(i,j,k)}\varphi$. Note that **Aprop** $\subset S_{\text{Boole}} \subset S_{\text{Aleth}} \subset S_{(i,j,k)\text{-free}} \subset S_{\mathsf{MDL^+III}}$ for each $i, j, k \in I$.

A formula of the form $O_{(i,j,k)}\varphi$ is to be understood as meaning that it is obligatory upon an agent i with respect to an agent j in the name of k to see to it that φ. In order to accommodate this fine grained notion of obligation, we allow deontic accessibility relations to be indexed by $I \times I \times I$. Thus we define:

Definition 2. *By an $\mathcal{L}_{\mathsf{MDL^+III}}$-model, we mean a quadruple $M = (W^M, R_A^M, R_D^M, V^M)$ where:*

(i) *W^M is a non-empty set (heuristically, of 'possible worlds'),*
(ii) *$R_A^M \subseteq W^M \times W^M$,*
(iii) *R_D^M is a function that assigns a subset $R_D^M(i, j, k)$ of R_A^M to each triad (i, j, k) of agents $i, j, k \in I$,*
(iv) *V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in$* **Aprop**.

We usually abbreviate $R_D^M(i, j, k)$ as $R_{(i,j,k)}^M$.

The truth definition for the formulas of $\mathcal{L}_{\mathsf{MDL^+III}}$ can be given in a standard way by associating the alethic modal operator $\Box$ with R_A^M and each deontic operator $O_{(i,j,k)}$ with $R_{(i,j,k)}^M$.

Definition 3. *Let M be an $\mathcal{L}_{\mathsf{MDL^+III}}$-model and w a point in M. If $p \in Aprop$, $\varphi, \psi \in S_{\mathsf{MDL^+III}}$, and $i, j, k \in I$, then:*

(a) $M, w \models_{\mathsf{MDL^+III}} p$ *iff* $w \in V^M(p)$,
(b) $M, w \models_{\mathsf{MDL^+III}} \top$,
(c) $M, w \models_{\mathsf{MDL^+III}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{MDL^+III}} \varphi$,
(d) $M, w \models_{\mathsf{MDL^+III}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{MDL^+III}} \varphi$ *and* $M, w \models_{\mathsf{MDL^+III}} \psi$,
(e) $M, w \models_{\mathsf{MDL^+III}} \Box\varphi$ *iff for every v such that* $(w, v) \in R_A^M$, $M, v \models_{\mathsf{MDL^+III}} \varphi$,
(f) $M, w \models_{\mathsf{MDL^+III}} O_{(i,j,k)}\varphi$ *iff for every v such that* $(w, v) \in R_{(i,j,k)}^M$, $M, v \models_{\mathsf{MDL^+III}} \varphi$.

A formula φ is true in an $\mathcal{L}_{\mathsf{MDL^+III}}$-model M at a point w of M if $M, w \models_{\mathsf{MDL^+III}} \varphi$. We say that a set Σ of formulas of $\mathcal{L}_{\mathsf{MDL^+III}}$ is true in M at w, and write $M, w \models_{\mathsf{MDL^+III}} \Sigma$, if $M, w \models_{\mathsf{MDL^+III}} \psi$ for every $\psi \in \Sigma$. If $\Sigma \cup \{\varphi\}$ is a set of formulas of $\mathcal{L}_{\mathsf{MDL^+III}}$, we say that φ is a semantic consequence of Σ, and write $\Sigma \models_{\mathsf{MDL^+III}} \varphi$, if for every $\mathcal{L}_{\mathsf{MDL^+III}}$-model M and every point w such that $M, w \models_{\mathsf{MDL^+III}} \Sigma$, $M, w \models_{\mathsf{MDL^+III}} \varphi$. We say that a formula φ is valid, and write $\models_{\mathsf{MDL^+III}} \varphi$, if $\emptyset \models_{\mathsf{MDL^+III}} \varphi$.

From a purely formal point of view, every $\mathcal{L}_{\mathsf{MDL^+III}}$-model is also an $\mathcal{L}_{\mathsf{MDL^+}}$-model, since $I \times I \times I$ is a finite set. This guarantees that the completeness of $\mathsf{MDL^+III}$ can be derived from that of $\mathsf{MDL^+}$.

Definition 4. *The proof system for* $\mathsf{MDL^+III}$ *contains the following axioms and rules:*

(Taut)	*all instantiations of propositional tautologies over the present language*	
($\Box$-Dist)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	
(O-Dist)	$O_{(i,j,k)}(\varphi \to \psi) \to (O_{(i,j,k)}\varphi \to O_{(i,j,k)}\psi)$	*for each* $(i, j, k) \in I \times I \times I$
(Mix)	$P_{(i,j,k)}\varphi \to \Diamond\varphi$	*for each* $(i, j, k) \in I \times I \times I$
(MP)	$\dfrac{\varphi \quad \varphi \to \psi}{\psi}$	
($\Box$-Nec)	$\dfrac{\varphi}{\Box\varphi}$	
(O-Nec)	$\dfrac{\varphi}{O_{(i,j,k)}\varphi}$	*for each* $(i, j, k) \in I \times I \times I$.

An $\mathsf{MDL^+III}$*-proof of a formula φ is a finite sequence of $\mathcal{L}_{\mathsf{MDL^+III}}$-formulas having φ as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of φ, we write $\vdash_{\mathsf{MDL^+III}} \varphi$. If $\Sigma \cup \{\varphi\}$ is a set of $\mathcal{L}_{\mathsf{MDL^+III}}$-formulas, we say that φ is deducible in* $\mathsf{MDL^+III}$ *from Σ and write $\Sigma \vdash_{\mathsf{MDL^+III}} \varphi$ if $\vdash_{\mathsf{MDL^+III}} \varphi$ or there are formulas $\psi_1, \ldots, \psi_n \in \Sigma$ such that $\vdash_{\mathsf{MDL^+III}} (\psi_1 \wedge \cdots \wedge \psi_n) \to \varphi$.*

The above rules obviously preserve validity, and all the axioms are easily seen to be valid. Thus this proof system is sound. [2] Moreover, as is said above, the completeness of this proof system is guaranteed by the completeness of $\mathsf{MDL^+}$.

Theorem 1 (Completeness of $\mathsf{MDL^+III}$). *Let $\Sigma \cup \{\varphi\}$ be a set of $\mathcal{L}_{\mathsf{MDL^+III}}$-formulas. Then, if $\Sigma \models_{\mathsf{MDL^+III}} \varphi$ then $\Sigma \vdash_{\mathsf{MDL^+III}} \varphi$.*

3 The Dynamified Multi-Agent Deontic Logic DMDL⁺III

The formulas of $\mathcal{L}_{\mathsf{MDL^+III}}$ can be used to talk about the situations before and after the issuance of a promise or a command. In the previous example, before the issuance of

[2] Strictly speaking, O-Nec is redundant since it is derivable. It is included here just to record the fact that $\mathsf{MDL^+III}$ is normal.

your promise, it was not obligatory upon you to give a lecture at the workshop in São Paulo, but since the issuance it has become obligatory. Let p, a and b be understood as before, and (L, s) and (M, s) be the model world pairs that represent the situations before and after the issuance respectively. Then we should have:

$$L, s \models_{\mathsf{MDL^+III}} \neg O_{(a,b,a)} p \tag{6}$$
$$M, s \models_{\mathsf{MDL^+III}} O_{(a,b,a)} p \quad . \tag{7}$$

Note that we have assumed that in the case of an obligation created by an act of promising, the agent who is obligated is identical with the agent in whose name obligation is created. In the case of an obligation created by an act of commanding, in contrast, they are usually distinct. Let (N, s) represent the situation after the issuance of your guru's command, and let q and c be understood as before. Then we should have:

$$M, s \models_{\mathsf{MDL^+III}} \neg O_{(a,c,c)} q \tag{8}$$
$$N, s \models_{\mathsf{MDL^+III}} O_{(a,c,c)} q \quad . \tag{9}$$

In order to have a way of talking about changes of this kind and the acts that bring them about in the object language, we dynamify $\mathsf{MDL^+III}$. Thus we define:

Definition 5. *Take the same countably infinite set* **Aprop** *of proposition letters and the same finite set I of agents as before, with p ranging over* **Aprop**, *and i, j, k over I. The refined language of dynamified multi-agent deontic logic $\mathcal{L}_{\mathsf{DMDL^+III}}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_{(i,j,k)}\varphi \mid [\pi]\varphi$$
$$\pi ::= \mathrm{Com}_{(i,j)}\varphi \mid \mathrm{Prom}_{(i,j)}\varphi$$

Terms of the form $\mathrm{Com}_{(i,j)}\varphi$ and $\mathrm{Prom}_{(i,j)}\varphi$ are called command type terms and promise type terms respectively, and operators of the form $[\mathrm{Com}_{(i,j)}\varphi]$ and $[\mathrm{Prom}_{(i,j)}\varphi]$ are called command operators and promise operators respectively. The set of all well formed formulas of $\mathcal{L}_{\mathsf{DMDL^+III}}$ is referred to as $S_{\mathsf{DMDL^+III}}$.

Here the command type term of the form $!_{(j,i)}\varphi$ of $\mathcal{L}_{\mathsf{ECL\,II}}$ is replaced by the term of the form $\mathrm{Com}_{(i,j)}\varphi$ for mnemonic convenience. Note the inversion of the order of the constituents of the indexing pair; the term of the form $\mathrm{Com}_{(i,j)}\varphi$ stands for the type of acts of commanding to the effect that j should see to it that φ of which the commander is i and the commandee is j . Note also that $S_{\mathsf{MDL^+III}} \subset S_{\mathsf{DMDL^+III}}$.

The truth definition for this language can be given with reference to $\mathcal{L}_{\mathsf{MDL^+III}}$-models.

Definition 6. *Let M be an $\mathcal{L}_{\mathsf{MDL^+III}}$-model and w a point in M. If $p \in Aprop$, φ, $\psi \in S_{\mathsf{DMDL^+III}}$, and $i, j, k \in I$, then:*

(a) $M, w \models_{\mathsf{DMDL^+III}} p$ *iff* $w \in V^M(p)$,
(b) $M, w \models_{\mathsf{DMDL^+III}} \top$,

(c) $M, w \models_{\mathsf{DMDL^+III}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{DMDL^+III}} \varphi$,
(d) $M, w \models_{\mathsf{DMDL^+III}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{DMDL^+III}} \varphi$ *and* $M, w \models_{\mathsf{DMDL^+III}} \psi$,
(e) $M, w \models_{\mathsf{DMDL^+III}} \Box\varphi$ *iff for every* v *such that* $(w, v) \in R^M_A$, $M, v \models_{\mathsf{DMDL^+III}} \varphi$,
(f) $M, w \models_{\mathsf{DMDL^+III}} O_{(i,j,k)}\varphi$ *iff for every* v *such that* $(w, v) \in R^M_{(i,j,k)}$, $M, v \models_{\mathsf{DMDL^+III}} \varphi$,
(g) $M, w \models_{\mathsf{DMDL^+III}} [\mathrm{Com}_{(i,j)}\chi]\varphi$ *iff* $M_{\mathrm{Com}_{(i,j)}\chi}, w \models_{\mathsf{DMDL^+III}} \varphi$,
(h) $M, w \models_{\mathsf{DMDL^+III}} [\mathrm{Prom}_{(i,j)}\chi]\varphi$ *iff* $M_{\mathrm{Prom}_{(i,j)}\chi}, w \models_{\mathsf{DMDL^+III}} \varphi$,

where

(i) $M_{\mathrm{Com}_{(i,j)}\chi}$ *is the* $\mathcal{L}_{\mathsf{MDL^+III}}$*-model obtained from* M *by replacing* $R^M_D(j, i, i)$ *with* $\{(x, y) \in R^M_D(j, i, i) \mid M, y \models_{\mathsf{DMDL^+III}} \chi\}$, *and*
(ii) $M_{\mathrm{Prom}_{(i,j)}\chi}$ *is the* $\mathcal{L}_{\mathsf{MDL^+III}}$*-model obtained from* M *by replacing* $R^M_D(i, j, i)$ *with* $\{(x, y) \in R^M_D(i, j, i) \mid M, y \models_{\mathsf{DMDL^+III}} \chi\}$.

A formula φ *is true in an* $\mathcal{L}_{\mathsf{MDL^+III}}$*-model* M *at a point* w *of* M *if* $M, w \models_{\mathsf{DMDL^+III}} \varphi$. *We say that a set* Σ *of formulas of* $\mathcal{L}_{\mathsf{DMDL^+III}}$ *is true in* M *at* w, *and write* $M, w \models_{\mathsf{DMDL^+III}} \Sigma$, *if* $M, w \models_{\mathsf{DMDL^+III}} \psi$ *for every* $\psi \in \Sigma$. *If* $\Sigma \cup \{\varphi\}$ *is a set of formulas of* $\mathcal{L}_{\mathsf{DMDL^+III}}$, *we say that* φ *is a semantic consequence of* Σ, *and write* $\Sigma \models_{\mathsf{DMDL^+III}} \varphi$, *if for every* $\mathcal{L}_{\mathsf{DMDL^+III}}$*-model* M *and every point* w *such that* $M, w \models_{\mathsf{DMDL^+III}} \Sigma$, $M, w \models_{\mathsf{DMDL^+III}} \varphi$. *We say that a formula* φ *is valid, and write* $\models_{\mathsf{DMDL^+III}} \varphi$, *if* $\emptyset \models_{\mathsf{DMDL^+III}} \varphi$.

The clause (g) here is a restatement of the clause for the formulas of the form $[!_{(j,i)}\chi]\varphi$ of the truth definition for $\mathcal{L}_{\mathsf{ECL\,II}}$. As the truth of $[\mathrm{Com}_{(i,j)}\chi]\varphi$ at w in M is defined in terms of the truth of φ at w in the updated model $M_{\mathrm{Com}_{(i,j)}\chi}$ in (g), the truth of $[\mathrm{Prom}_{(i,j)}\chi]\varphi$ at w in M is defined in terms of the truth of φ at w in the updated model $M_{\mathrm{Prom}_{(i,j)}\chi}$ in (h).

As we have $\{(x, y) \in R^M_D(j, i, i) \mid M, y \models_{\mathsf{DMDL^+III}} \chi\} \subseteq R^M_D(j, i, i) \subseteq R^M_A$ and $\{(x, y) \in R^M_D(i, j, i) \mid M, y \models_{\mathsf{DMDL^+III}} \chi\} \subseteq R^M_D(i, j, i) \subseteq R^M_A$, the updated models $M_{\mathrm{Com}_{(i,j)}\chi}$ and $M_{\mathrm{Prom}_{(i,j)}\chi}$ are guaranteed to be $\mathcal{L}_{\mathsf{MDL^+III}}$-models. Moreover, as the remaining clauses faithfully reproduce the clauses of the truth definition for $\mathcal{L}_{\mathsf{MDL^+III}}$, we obviously have:

Corollary 1. *Let* M *be an* $\mathcal{L}_{\mathsf{MDL^+III}}$*-model and* w *a point of* M. *Then for any* $\varphi \in S_{\mathsf{MDL^+III}}$, $M, w \models_{\mathsf{DMDL^+III}} \varphi$ *iff* $M, w \models_{\mathsf{MDL^+III}} \varphi$.

The following corollary can be proved by induction on the length of ψ:

Corollary 2. *Let* ψ *and* χ *be an* (j, i, i)*-free formula and an* (i, j, i)*-free formula respectively. Then, for any* $\varphi \in S_{\mathsf{DMDL^+III}}$, *the following two equivalences hold:*

(i) $M, w \models_{\mathsf{DMDL^+III}} \psi$ *iff* $M_{Com_{(i,j)}\varphi}, w \models_{\mathsf{DMDL^+III}} \psi$
(ii) $M, w \models_{\mathsf{DMDL^+III}} \chi$ *iff* $M_{Prom_{(i,j)}\varphi}, w \models_{\mathsf{DMDL^+III}} \chi$.

One of the things this corollary means is that acts of commanding and acts of promising do not affect so-called brute facts and alethic possibilities in any direct way. But it means more, as we will see below.

DMDL$^+$III inherits the following principle from ECL II:

Proposition 1 (CUGO Principle). *If* $\varphi \in S_{(j,i,i)\text{-free}}$, *then* $\models_{\mathsf{DMDL^+III}} [Com_{(i,j)}\varphi]O_{(j,i,i)}\varphi$.

As is noted in Yamada(2007a), CUGO Principle characterizes, at least partially, the workings of acts of commanding; though not without exceptions, commands usually generate obligations.[3] Our semantics also validates the following principle for $\mathsf{DMDL}^{+}\mathsf{III}$:

Proposition 2 (PUGO Principle). *If* $\varphi \in S_{(i,j,i)}\text{-free}$, *then* $\models_{\mathsf{DMDL^{+}III}} [Prom_{(i,j)}\varphi]O_{(i,j,i)}\varphi$.

PUGO Principle means that, though not without exceptions, promises usually generate obligations.[4]

Now let's go back to our discussion of Example 1 above. Since p is (a,b,a)-free, PUGO Principle guarantees that we have:

$$L, s \models_{\mathsf{DMDL^{+}III}} [Prom_{(a,b)}p]O_{(a,b,a)}p \ . \tag{10}$$

This is equivalent to:

$$L_{Prom_{(a,b)}p}, s \models_{\mathsf{DMDL^{+}III}} O_{(a,b,a)}p \ . \tag{11}$$

Since q is (a,c,c)-free, CUGO Principle guarantees that we have:

$$L_{Prom_{(a,b)}p}, s \models_{\mathsf{DMDL^{+}III}} [Com(c,a)q]O_{(a,c,c)}q \ . \tag{12}$$

This is equivalent to:

$$(L_{Prom_{(a,b)}p})_{Com(c,a)q}, s \models_{\mathsf{DMDL^{+}III}} O_{(a,c,c)}q \ . \tag{13}$$

Moreover, since $O_{(a,b,a)}p$ is (a,c,c)-free, Corollary 2 and (11) jointly imply:

$$(L_{Prom_{(a,b)}p})_{Com(c,a)q}, s \models_{\mathsf{DMDL^{+}III}} O_{(a,b,a)}p \ . \tag{14}$$

Hence we have

$$(L_{Prom_{(a,b)}p})_{Com(c,a)q}, s \models_{\mathsf{DMDL^{+}III}} (O_{(a,b,a)}p \wedge O_{(a,c,c)}q) \ . \tag{15}$$

The model world pair $((L_{Prom_{(a,b)}p})_{Com(c,a)q}, s)$ here represents the situation you are in after the issuance of your guru's command. In that situation, it is obligatory upon you to see to it that q with respect to your guru in the name of your guru, but it is also obligatory upon you to see to it that p with respect to your former student in your name. Your guru's command added a new obligation without removing your earlier commitment. They are independent from each other as Corollary 2 indicates.

[3] The restriction on φ here is motivated by the fact that the truth of φ at a point v in M does not guarantee the truth of φ at v in $M_{Com_{(i,j)}\varphi}$ if φ is not (j,i,i)-free. For example, $[Com_{(i,j)}P_{(j,i,i)}q]O_{(j,i,i)}P_{(j,i,i)}q$ is not valid. For more on CUGO Principle, see Yamada(2007a).

[4] The motivation for the restriction on φ here is similar to that for CUGO Principle.

Now, given that we have $L, s \models_{\mathsf{DMDL^{+}III}} \neg(p \wedge q)$, Corollary 2 again enables us to derive:

$$(L_{Prom_{(a,b)}p})_{Com(c,a)q}, s \models_{\mathsf{DMDL^{+}III}} (O_{(a,b,a)}p \wedge O_{(a,c,c)}q) \wedge \neg(p \wedge q) \ . \tag{16}$$

Thus we have captured how the contingent conflict of obligations in our example is brought about jointly by your act of promising and your guru's act of commanding.

4 Some Interesting Things Expressible in $\mathcal{L}_{\mathsf{MDL^{+}III}}$ and $\mathcal{L}_{\mathsf{DMDL^{+}III}}$

Our definitions of $\mathcal{L}_{\mathsf{MDL^{+}III}}$ and of $\mathcal{L}_{\mathsf{DMDL^{+}III}}$ leaves room for interesting possibilities. For example, we may ask if there can be an obligation of the form $O_{(i,j,k)}\varphi$ such that $i \neq j \neq k \neq i$. Suppose, for example, the director of your research center utters the following sentence seriously and sincerely to someone over the phone:

My secretary will call you back right away. (17)

Let a, b, c be the director, his secretary, and the addressee respectively, and let p represent the proposition that b will call c back right away. Does his utterance create an obligation of the form $O_{(b,c,a)}p$?

Notice that no combinations of acts of promising and commanding will generate such an obligation in $\mathsf{DMDL^{+}III}$. Although the director's utterance can be taken as an act of promising, it can only be represented as an act of the form $Prom_{(a,c)}p$ in $\mathcal{L}_{\mathsf{MDL^{+}III}}$. Let (M, s) represent the situation before his utterance. Then we have:

$$M, s \models_{\mathsf{DMDL^{+}III}} [Prom_{(a,c)}p]O_{(a,c,a)}p \ . \tag{18}$$

This is equivalent to:

$$M_{Prom_{(a,c)}p}, s \models_{\mathsf{DMDL^{+}III}} O_{(a,c,a)}p \ . \tag{19}$$

Thus it is now obligatory upon the director to see to it that p. Suppose the director commanded his secretary to call c back right away in order to live up to his obligation. Then we have:

$$(M_{Prom_{(a,c)}p})_{Com_{(a,b)}p}, s \models_{\mathsf{DMDL^{+}III}} O_{(b,a,a)}p \ . \tag{20}$$

Thus, if we are to take the director's utterance as generating an obligation of the form $O_{(b,c,a)}p$, we will have to introduce a new program term of the form, say, $Prom^{*}_{(i,j,k)}\varphi$, and let it represent the new type of acts of promising to the effect that k will see to it that φ with i the promisor, j the promisee, and k the agent who owes the obligation to j in the name of i. Although we will not pursue this possibility further in this paper, we just mention that, if we do so, it will become possible to define usual acts of promising of the form $Prom_{(i,j)}\varphi$ as an abbreviation for $Prom^{*}_{(i,j,i)}\varphi$.

Another interesting possibility is an obligation of the form $O_{(i,i,i)}\varphi$. Such an obligation will be generated by an act of commanding of the type $Com_{(i,i)}\varphi$ or an act of promising of the type $Prom_{(i,i)}\varphi$, that is, an act of commanding oneself to see to it that φ or an act of promising oneself that (s)he will see to it that φ. If φ is (i, i, i)-free, the following formulas are instances of CUGO and PUGO Principles respectively:

$$[Com_{(i,i)}\varphi]O_{(i,i,i)}\varphi \tag{21}$$

$$[Prom_{(i,i)}\varphi]O_{(i,i,i)}\varphi \ . \tag{22}$$

Note that there is no difference between the effect of an act of commanding oneself to see to it that φ and that of an act of promising oneself to see to it that φ in $\mathsf{DMDL}^{+}\mathsf{III}$. We have $M_{Prom_{(i,i)}\varphi} = M_{Com_{(i,i)}\varphi}$.

In ordinary cases where different agents are involved, however, we can capture an interesting interplay between acts of commanding and acts of promising in terms of the different effects they have. Consider the contingent obligational dilemma above again. Suppose you have decided to obey your guru and write to your guru that you will join the demonstration in Tokyo. What effects will your letter have?

One obvious effect will be a change in your guru's epistemic states. He now knows that you have received and understood his command. But there is another more interesting effect. Let p, q, a, b, and c be understood as in our earlier discussion of this example. We now have:

$$((M_{Prom_{(a,b)}p})_{Com_{(c,a)}q})_{Prom_{(a,c)}q}, s \models_{\mathsf{DMDL}^{+}\mathsf{III}} O_{(a,c,a)}q \ . \tag{23}$$

Thus we have:

$$((M_{Prom_{(a,b)}p})_{Com_{(c,a)}q})_{Prom_{(a,c)}q}, s \models_{\mathsf{DMDL}^{+}\mathsf{III}} O_{(a,c,c)}q \wedge O_{(a,c,a)}q \ . \tag{24}$$

Now it is obligatory upon you to see to it that q not only in your guru's name but also in your own name. You have explicitly committed yourself.

One tempting step here is to see an obligation of the form $O_{(i,j,k)}\varphi$ as representing commitment of the agent i if $k = i$. As we have seen, however, our semantics validates (21). In the extreme case where the commander and the commandee is identical, an act of commanding can generate an obligation of this form. Whether or not this result shows that commitment should not be considered as special kind of obligation but as something distinct from obligation seems to be a very interesting problem. Although we will not discuss this problem in this paper, we note that it is not difficult to develop a propositional modal logic which has operators standing for commitments as well as operators for obligations as far as we keep them independent from each other.

5 The Proof System for $\mathsf{DMDL^+III}$

The proof system for $\mathsf{DMDL^+III}$ can be obtained by adding so-called reduction axioms and necessitation rules for each command operator and each promise operator to the proof system of $\mathsf{MDL^+III}$ as follows:

Definition 7. *The proof system for* $\mathsf{DMDL^+III}$ *contains all the axioms and rules of the proof system for* $\mathsf{MDL^+III}$*, and in addition, the following axioms and rules:*

(C1) $[Com_{(i,j)}\varphi]p \leftrightarrow p$

(C2) $[Com_{(i,j)}\varphi]\top \leftrightarrow \top$

(C3) $[Com_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[Com_{(i,j)}\varphi]\psi$

(C4) $[Com_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow [Com_{(i,j)}\varphi]\psi \wedge [Com_{(i,j)}\varphi]\chi$

(C5) $[Com_{(i,j)}\varphi]\Box\psi \leftrightarrow \Box[Com_{(i,j)}\varphi]\psi$

(C6) $[Com_{(i,j)}\varphi]O_{(l,m,n)}\psi \leftrightarrow O_{(l,m,n)}[Com_{(i,j)}\varphi]\psi$ *if* $(l,m,n) \neq (j,i,i)$

(C7) $[Com_{(i,j)}\varphi]O_{(j,i,i)}\psi \leftrightarrow O_{(j,i,i)}(\varphi \rightarrow [Com_{(i,j)}\varphi]\psi)$

(C8) $[Com_{(i,j)}\varphi][Prom_{(l,m)}\psi]\chi \leftrightarrow [Prom_{(l,m)}\psi][Com_{(i,j)}\varphi]\chi$ *if* $(l,m,l) \neq (j,i,i)$

(C9) $[Com_{(i,j)}\varphi][Prom_{(l,m)}\psi]\chi \leftrightarrow [Prom_{(l,m)}\varphi][Com_{(i,j)}\psi]\chi$
if $(l,m,l) = (j,i,i)$*, i.e.* $i = j = l = m$

(P1) $[Prom_{(i,j)}\varphi]p \leftrightarrow p$

(P2) $[Prom_{(i,j)}\varphi]\bot \leftrightarrow \bot$

(P3) $[Prom_{(i,j)}\varphi]\neg\psi \leftrightarrow \neg[Prom_{(i,j)}\varphi]\psi$

(P4) $[Prom_{(i,j)}\varphi](\psi \wedge \chi) \leftrightarrow [Prom_{(i,j)}\varphi]\psi \wedge [Prom_{(i,j)}\varphi]\chi$

(P5) $[Prom_{(i,j)}\varphi]\Box\psi \leftrightarrow \Box[Prom_{(i,j)}\varphi]\psi$

(P6) $[Prom_{(i,j)}\varphi]O_{(l,m,n)}\psi \leftrightarrow O_{l,m,n)}[Prom_{(i,j)}\varphi]\psi$ *if* $(l,m,n) \neq (i,j,i)$

(P7) $[Prom_{(i,j)}\varphi]O_{(i,j,i)}\psi \leftrightarrow O_{(i,j,i)}(\varphi \rightarrow [Prom_{(i,j)}\varphi]\psi)$

([Com]-Nec) $\dfrac{\psi}{[Com_{(i,j)}\varphi]\psi}$

([Prom]-Nec) $\dfrac{\psi}{[Prom_{(i,j)}\varphi]\psi}$

The notion of $\mathsf{DMDL^+III}$*-proof and the notion of logical consequence* $\vdash_{\mathsf{DMDL^+III}}$ *can be defined in the obvious way.*

The above axioms can easily be seen to be valid, and the above rules obviously preserve validity. Thus this proof system is sound.

Moreover, the axioms (C1), (C2), (P1), and (P2) allow us to eliminate command operators and promise operators prefixed to propositional letters and $\top$ respectively, and other axioms enable us to reduce the length of sub-formulas to which command operators and promise operators are prefixed. Thus, these axioms enables us to define translation function that translate any formula of $\mathcal{L}_{\mathsf{DMDL^+III}}$ into a formula of $\mathcal{L}_{\mathsf{MDL^+III}}$

that is provably equivalent to it. Then the completeness of $\mathsf{DMDL}^{+}\mathsf{III}$ is derived from that of $\mathsf{MDL}^{+}\mathsf{III}$.

Theorem 2 (Completeness of $\mathsf{DMDL}^{+}\mathsf{III}$**).** *Let* $\Sigma \cup \{\varphi\}$ *be a set of* $\mathcal{L}_{\mathsf{DMDL}^{+}\mathsf{III}}$*-formulas. Then, if* $\Sigma \models_{\mathsf{DMDL}^{+}\mathsf{III}} \varphi$ *then* $\Sigma \vdash_{\mathsf{DMDL}^{+}\mathsf{III}} \varphi$*.*

6 The Comparison with Searle's Treatment of Acts of Promising

In this section we will compare our treatment of acts of promising with Searle's treatment in his argument for the derivability of "ought" from "is" (1964, and 1969). Searle's argument can be considered as consisting of three parts. In the first part, he derives the statement about an institutional fact (ii) below from the factual premise (i) with the help of the constitutive rule (ia) and the empirical assumption (ib) (1969, pp.177–178.):

(i) Jones uttered the words "I hereby promise to pay you, Smith, five dollars".
(ia) Under certain conditions *C* anyone who utters the words (sentence) "I hereby promise to pay you, Smith, five dollars" promises to pay Smith five dollars.
(ib) Conditions *C* obtain.
(ii) Jones promised to pay Smith five dollars.

In the second part, he derives the "evaluative statement" (iii) below from (ii) and (iia), and then derive (iv) from (iii) and (iiia) (pp.178–180.):

(ii) Jones promised to pay Smith five dollars.
(iia) All promises are acts of placing oneself under (undertaking) an obligation to do the thing promised.
(iii) Jones placed himself under (undertook) an obligation to pay Smith five dollars.
(iiia) All those who place themselves under an obligation are (at the time when they so place themselves) under an obligation.
(iv) Jones is under an obligation to pay Smith five dollars.

And finally, in the third part, he derives (v) from (iv) and (iva) (pp.180-181):

(iv) Jones is under an obligation to pay Smith five dollars.
(iva) If one is under an obligation to do something, then as regards that obligation one ought to do what one is under an obligation to do.
(v) As regards his obligation to pay Smith five dollars, Jones ought to pay Smith five dollars.

As (iii) is evaluative, the first two parts of this argument, if sound, has done the substantial work of deriving the evaluative from the factual.

The first part of this argument derives the institutional fact (ii) from the factual premise (i). According to Searle,

> Every institutional fact is underlain by a (system of) rule(s) of the form "*X* counts as *Y* in context *C*". (1969, pp.51-52.)

In the second part, (iia) unfolds what is involved in the institutional fact (ii).[5]

Broadly speaking, PUGO Principle correspond to the premise (iia), and can vindicate the second part of this argument. Let j, s, r, and (M, t) represent Jones, Smith, the proposition that Jones will pay Smith five dollars, and the situation before Jones uttered the sentence in question respectively. Then we have:

$$M, t \models_{\mathsf{DMDL^{+}III}} [Prom_{(j,s)} r] O_{(j,s,j)} r \quad . \tag{25}$$

This is equivalent to:

$$M_{Prom_{(j,s)} r}, t \models_{\mathsf{DMDL^{+}III}} O_{(j,s,j)} r \quad . \tag{26}$$

Since $(M_{Prom_{(j,s)} r}, t)$ represents the situation after Jones made his promise, this confirms (iv).

This is not a fortuitous correspondence, as $\mathsf{DMDL^{+}III}$ is designed to incorporate Austin's notion of illocutionary acts as acts producing conventional effects (Austin, 1955, pp.103-104). Searle's treatment of acts of promising inherits much from Austin's. Searle finds Austin's definition of commissives unexceptionable, and includes the category of commissives in his taxonomy as the class of "those illocutionary acts whose point is to commit the speaker $\cdots$ to some future course of action "(Searle 1979, p.14). Thus Searle's characterization of commissives refers to conventional effects of commissives.

The category of commissives, however, is the only category of Austin's classification that survives in Searle's taxonomy. As CUGO Principle indicates, $\mathsf{DMDL^{+}III}$ treats acts of commanding also as acts producing conventional effects. But according to Searle, the illocutionary point of the category of directives, to which acts of commanding belong, "consists in the fact that they are attempts (of varying degrees, $\cdots$) by the speaker to get the hearer to do something"(Searle 1979, p.13). It doesn't seem to refer to conventional effects at all. Notice that to get the hearer to do something is a perlocutionary effect. Thus Searle characterizes directive illocutionary acts as attempts to produce certain perlocutionary effects. In this respect, $\mathsf{DMDL^{+}III}$ is more Austinian than Searle's standard theory.[6]

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[5] Searle refers to "the constitutive rule that to make a promise is to undertake an obligation" in his argument (1969, p.185). Although this "rule" looks like (iia), (ia) seems to instantiate the general form of a constitutive rule. We may rewrite it as follows: uttering the sentence "I hereby promise to pay you, Smith, five dollars" counts as promising to pay Smith five dollars in context where conditions C hold. The notion of institution and "count-as" relation have come to be the topics of lively discussions recently.

[6] For more on Austin's distinction between illocutionary acts and perlocutionary acts, see Yamada (2002), Yamada (2007c), or Sbisà (2005). In Yamada (2007c), deontic updates of Yamada (2007b) is combined with preference upgrades of van Benthem & Liu (to appear) in order to differentiate illocutionary acts of commanding from perlocutionary acts that affect preferences of addressees.

7 Conclusion

We have extended $\mathsf{MDL}^+\mathsf{II}$ into $\mathsf{MDL}^+\mathsf{III}$, and developed a dynamified multi-agent deontic logic $\mathsf{DMDL}^+\mathsf{III}$ as a dynamic extension of $\mathsf{MDL}^+\mathsf{III}$. In $\mathsf{DMDL}^+\mathsf{III}$, acts of promising as well as acts of commanding are characterized as acts producing conventional effects. By modeling them as deontic updators, we have incorporated Austin's notion of illocutionary acts in the limited domain consisting of acts of promising and acts of commanding. $\mathsf{DMDL}^+\mathsf{III}$ enables us to analyze how a conflict of obligations can be generated jointly by an act of commanding and an act of promising, as well as what effects an act of promising to do what is commanded can have. Moreover, PUGO principle can be used to vindicate part of Searle's argument for derivability of "ought" from "is". As $\mathsf{DMDL}^+\mathsf{III}$ is proven to be sound, having such a vindication is of considerable significance.

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