Acts of Commanding and Changing Obligations ⋆

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Abstract. If we are to take the notion of speech act seriously, we must be able to treat speech acts as acts. In this paper, we will try to model changes brought about by various acts of commanding in terms of a variant of update logic. We will combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations. Although the resulting logic inherits various inadequacies from monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless. A complete axiomatization and some interesting valid principles together with concrete examples will be presented, and suggestions for further research will be made.

1 Introduction

Consider the following example:

Example 1. Suppose you are reading an article on logic in the office you share with your boss and a few other colleagues. While you are reading, the temperature of the room rises, and it is now above 30 degrees Celsius. There is a window and an air conditioner. You can open the window, or turn on the air conditioner. You can also concentrate on the article and ignore the heat. Then, suddenly, you hear your boss's voice. She commanded you to open the window. What effects does her command have on the current situation?

Your boss's act of commanding does not affect the state of the window directly. Nor does it affect the number of alternatives you have. It is still possible for you to turn on the air conditioner, to ignore the heat, or to open the window. But it has now become impossible for you to choose alternatives other than that of opening the window without going against your obligation. It is now obligatory upon you to open the window, although it was not so before.

If the notion of speech acts, or more specifically that of illocutionary acts, is to be taken seriously, it must be possible to see utterances not only as acts of uttering words but also as acts of doing something more. But speech acts do not seem to affect so called

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brute facts directly, except for those various physical and physiological conditions involved in the production and perception of sounds or written symbols. What differences can they bring about in our life?

In attempting to answer this question, it is important to be careful not to blur the distinction between illocutionary acts and perlocutionary acts. Since Grice [10], many philosophers, linguists, and computer scientists have talked about utterers' intentions to produce various changes in the attitudes of addressees in their theories of communication. But utterers' intentions usually go beyond illocutionary acts by involving perlocutionary consequences, while illocutionary acts can be effective even if they do not produce intended perlocutionary consequences. Thus, in the above example, even if you refuse to open the window in question, that will not make her command void. Your refusal would not constitute disobedience if it could make her command void. Her command is effective in a sense even if she has failed to get you to form the intention to open the window. In order to characterize effects of illocutionary acts adequately, we need to be able to isolate them from perlocutionary consequences of utterances.

It is interesting to note, in this connection, that some illocutionary acts such as commanding, forbidding, permitting, and promising seem to affect our social life by bringing about changes in the deontic status of various alternative courses of actions. Thus, in the above example, before the issuance of your boss's command, none of your three alternatives were obligatory upon you, but after the issuance, one of them has become obligatory. In what follows, we will model changes acts of commanding bring about in terms of a new update logic. We will combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations. Although the resulting language inherits various inadequacies from the language of monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless.

The idea of update logic of acts of commanding is inspired by the update logics of public announcements and private information transmissions developed in Plaza [16], Groeneveld [11], Gerbrandy & Groeneveld [9], Gerbrandy [8], Baltag, Moss, & Solecki [2], and Kooi & van Benthem [13] among others. In van Benthem [4], the logics of such epistemic actions are presented as exemplars of a view of logic as "the analysis of general informational processes: knowledge representation, giving or receiving information, argumentation, communication", and used to show "how using a 'well-known' system as a vehicle, viz. standard epistemic logic, leads to totally *new issues* right from the start"(p.33). The basic idea of the update logic of acts of commanding is to capture the workings of acts of commanding by using deontic logic instead of epistemic logic as a vehicle. This may lead to a significant extension of the range of the kind of logical analysis advocated in van Benthem [4], since acts of commanding exemplify a kind of speech acts radically different from those discussed in the logics of epistemic actions.

2 A Static Base Language $\mathcal{L}_{\mathsf{MDL}^+}$ and a Static Logic MDL^+

Let's go back to Example 1. In the situation before the command is given, it was neither obligatory upon you to open the window, nor was it so not to open it. But in the situation after your boss's act of commanding, it has become obligatory upon you to open it. In order to describe these situations, we use a language $\mathcal{L}_{\mathsf{MDL}^+}$, the Language of Multi-agent monadic Deontic Logic With an alethic modal operator, MDL^+. We represent the two situations by two models M and N with a world s for $\mathcal{L}_{\mathsf{MDL}^+}$. Thus, we can describe the difference between these situations as follows:

$$M, s \models_{\mathsf{MDL}^+} \neg O_a p \wedge \neg O_a \neg p \tag{1}$$
$$N, s \models_{\mathsf{MDL}^+} O_a p \ , \tag{2}$$

where the proposition letter p stands for the proposition that the window is open at such and such a time, say t_1. The operator O_a here is indexed by a given finite set $I = \{a, b, c, \ldots, n\}$ of agents, and the index a represents you. Intuitively, a formula of form $O_i\varphi$ means that it is obligatory upon the agent i to see to it that φ. Thus:

Definition 1. *Take a countably infinite set* Aprop *of proposition letters and a finite set I of agents, with p ranging over* Aprop *and i over I. The multi-agent monadic deontic language $\mathcal{L}_{\mathsf{MDL}^+}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi$$

The set of all well formed formulas (sentences) of $\mathcal{L}_{\mathsf{MDL}^+}$ is denoted by S_{MDL^+} and operators of the form O_i are called deontic operators. For each $i \in I$, we call a sentence i-free if no O_i's occur in it. We call sentence alethic if no deontic operators occur in it, and boolean if no modal operators occur in it. For each $i \in I$, the set of all i-free sentences is denoted by $S_{i\text{-free}}$. The set of all alethic sentences and the set of all boolean sentences are denoted by S_{Aleth} and S_{Boole} respectively.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, and $\Diamond$ are assumed to be introduced by standard definitions. We also abbreviate $\neg O_i \neg\varphi$ as $P_i\varphi$, and $O_i\neg\varphi$ as $F_i\varphi$. Note that $\text{Aprop} \subset S_{\text{Boole}} \subset S_{\text{Aleth}} \subset S_{i\text{-free}} \subset S_{\mathsf{MDL}^+}$ for each $i \in I$.[1]

Definition 2. *By an $\mathcal{L}_{\mathsf{MDL}^+}$-model, we mean a quadruple $M = \langle W^M, R_A^M, R_I^M, V^M \rangle$ where:*

(i) *W^M is a non-empty set (heuristically, of 'possible worlds')*

(ii) $R_A^M \subseteq W^M \times W^M$

(iii) *R_I^M is a function that assigns a subset $R_I^M(i)$ of R_A^M to each agent $i \in I$*

(iv) *V^M is a function that assigns a subset $V^M(p)$ of W^M to each proposition letter $p \in \text{Aprop}$.*

We usually abbreviate $R_I^M(i)$ as R_i^M.

[1] Formally there is no difference between S_{MDL^+} and $\mathcal{L}_{\mathsf{MDL}^+}$ since a formal language can be identified with the set of its sentences. Thus we have two names for the same thing here.

Note that for any $i \in I$, R_i^M is required to be a subset of R_A^M. Thus we assume that whatever is permitted is possible.

Definition 3. *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model and w a point in M. If $p \in Aprop$, $\varphi, \psi \in S_{\mathsf{MDL}^+}$, and $i \in I$, then:*

(a) $M, w \models_{\mathsf{MDL}^+} p$ *iff* $w \in V^M(p)$

(b) $M, w \models_{\mathsf{MDL}^+} \top$

(c) $M, w \models_{\mathsf{MDL}^+} \neg\varphi$ *iff it is not the case that* $M, w \models_{\mathsf{MDL}^+} \varphi$ (*hereafter,* $M, w \not\models_{\mathsf{MDL}^+} \varphi$)

(d) $M, w \models_{\mathsf{MDL}^+} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{MDL}^+} \varphi$ *and* $M, w \models_{\mathsf{MDL}^+} \psi$

(e) $M, w \models_{\mathsf{MDL}^+} \Box\varphi$ *iff for every* v *such that* $\langle w, v\rangle \in R_A^M$, $M, v \models_{\mathsf{MDL}^+} \varphi$

(f) $M, w \models_{\mathsf{MDL}^+} O_i\varphi$ *iff for every* v *such that* $\langle w, v\rangle \in R_i^M$, $M, v \models_{\mathsf{MDL}^+} \varphi$.

A formula φ is true in an $\mathcal{L}_{\mathsf{MDL}^+}$-model M at a point w of M if $M, w \models_{\mathsf{MDL}^+} \varphi$. We say that a set Σ of formulas of $\mathcal{L}_{\mathsf{MDL}^+}$ is true in M at w, and write $M, w \models_{\mathsf{MDL}^+} \Sigma$, if $M, w \models_{\mathsf{MDL}^+} \psi$ for every $\psi \in \Sigma$. If $\Sigma \cup \{\varphi\}$ is a set of formulas of $\mathcal{L}_{\mathsf{MDL}^+}$, we say that φ is a semantic consequence of Σ, and write $\Sigma \models_{\mathsf{MDL}^+} \varphi$, if for every $\mathcal{L}_{\mathsf{MDL}^+}$-model M and every point w such that $M, w \models_{\mathsf{MDL}^+} \Sigma$, $M, w \models_{\mathsf{MDL}^+} \varphi$. We say that a formula φ is valid, and write $\models_{\mathsf{MDL}^+} \varphi$, if $\emptyset \models_{\mathsf{MDL}^+} \varphi$.

Intuitively, $\langle w, v\rangle \in R_i^M$ means that the world v is compatible with i's obligations at w in M. Thus, according to this semantics, it is obligatory upon i to see to it that φ at w in M iff φ holds at every world compatible with i's obligations at w in M.

Note that it is not standard to relativize obligation to agents. In dealing with moral or legal obligations, for example, it is natural to work with un-relativized obligations. But we are here trying to capture the effects of acts of commanding, and commands can be, and usually are, given to some specific addressees. In order to describe how such commands work in a situation where their addressees and non-addressees are present, it is necessary to work with a collection of accessibility relations relativized to various agents. In such multi-agent settings, we may have to talk about commands given to every individual agent in a specified group, as distinct not only from commands given to a single agent but also from commands meant for every agent, e.g. "Thou shalt not kill". And even among commands given to a group of agents, we may have to distinguish commands to be executed jointly by all the members of the group from commands to be executed individually by each of them. Although we will only consider commands given to a single agent in this paper, it doesn't seem impossible to extend our analysis to commands given to more than one agents.

A word about the use of monadic deontic operators here may be in order. Monadic deontic logics are known to be inadequate to deal with conditional obligations and R. M. Chisholm's contrary-to-duty imperative paradox; dyadic deontic logics are better in this respect. But there are still other problems which are unsolved even by dyadic deontic logics, and Åqvist [1], for example, stresses the importance of temporal and quantificational machinery to viable deontic logics. The use of the language of monadic deontic logic here does not reflect any substantial theoretical commitment. It is used to keep things as simple as possible as we are in such an early stage of the development.

We will discuss some shortcomings resulting from the static nature of this language and the possibility of using different languages as vehicles later.

A word about the use of alethic modal operator may also be in order. It can be used to describe unchanging aspects of the changing situations. As we have seen in the above example, even after your boss's act of commanding, it was still possible for you to turn on the air conditioner or to ignore the heat. Thus we have:

$$M, s \models_{\mathsf{MDL}^+} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q) \tag{3}$$

$$N, s \models_{\mathsf{MDL}^+} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q) \ , \tag{4}$$

where p is to be understood as before, and q as meaning that the air conditioner is running at t_1. Note that the notion of possibility here is that of alethic (or metaphysical) possibility, and not that of epistemic possibility. Suppose, for example, you obeyed your boss's command by opening the window by t_1. Then we have $N, s \models_{\mathsf{MDL}^+} p$. But we may still have, for some world w alethically accessible from s, $N, w \models_{\mathsf{MDL}^+} \neg p$. Thus, even after all the people in the office came to know that you had opened it, some of your colleagues, without noticing that you had been commanded to do so, might complain that if you hadn't opened it, they wouldn't have been disturbed by the outside noises. [2]

Now we define proof system for MDL^+.

Definition 4. *The proof system for* MDL^+ *contains the following axioms and rules:*

(Taut)	*all instantiations of propositional tautologies over the present language*	
($\Box$-Dist)	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	($\Box$-distribution)
(O_i-Dist)	$O_i(\varphi \to \psi) \to (O_i\varphi \to O_i\psi)$ *for each* $i \in I$	(O_i-distribution)
(Mix)	$P_i\varphi \to \Diamond\varphi$ *for each* $i \in I$	(Mix Axiom)
(MP)	$\dfrac{\varphi \quad \varphi \to \psi}{\psi}$	(Modus Ponens)
($\Box$-Nec)	$\dfrac{\varphi}{\Box\varphi}$	($\Box$-necessitation)
(O_i-Nec)	$\dfrac{\varphi}{O_i\varphi}$ *for each* $i \in I$.	(O_i-necessitation)

An MDL^+*-proof of a formula* φ *is a finite sequence of* $\mathcal{L}_{\mathsf{MDL}^+}$*-formulas having* φ *as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{MDL}^+} \varphi$*. If* $\Sigma \cup \{\varphi\}$ *is a set of* $\mathcal{L}_{\mathsf{MDL}^+}$*-formulas, we say that* φ *is deducible in* MDL^+ *from* Σ *and write* $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$ *if* $\vdash_{\mathsf{MDL}^+} \varphi$ *or there are formulas* $\psi_1, \ldots, \psi_n \in \Sigma$ *such that* $\vdash_{\mathsf{MDL}^+} (\psi_1 \wedge \cdots \wedge \psi_n) \to \varphi$*.*

[2] The notion of alethic possibility may be said to be too weak to capture the kind of possibility involved in the notion of possible alternative courses of actions. Although the possibility of interpreting $\Diamond$ and $\Box$ in terms of notions of possibility and necessity stronger than those of alethic ones is tempting, we will not pursue it in this paper.

The above rules obviously preserve validity, and all the axioms are easily seen to be valid. Thus this proof system is sound.[3]

The completeness of this proof system can be proved in a completely standard way by building a canonical model. Thus we have:

Theorem 1 (Completeness of MDL$^+$). *Let $\Sigma \cup \{\varphi\} \subseteq S_{\mathsf{MDL}^+}$. Then, if $\Sigma \models_{\mathsf{MDL}^+} \varphi$ then $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$.*

3 A Dynamic Language $\mathcal{L}_{\mathsf{CL}}$ and a Dynamic Logic ECL

As is clear from the above example, formulas of $\mathcal{L}_{\mathsf{MDL}^+}$ can be used to describe the situations before and after the issuance of your boss's command. But note that your boss's act of commanding, which change M into N, is talked about not in $\mathcal{L}_{\mathsf{MDL}^+}$ but in the meta-language. In order to have an object language in which we can talk about acts of commanding, we introduce expressions of the form $!_i\varphi$ for each $i \in I$. An expression of this form denotes the type of an act of commanding in which someone commands an agent i to see to it that φ. Let a and p be understood as before. Then your boss's act of commanding was of type $!_a p$, where a represents not your boss but you. The static base language $\mathcal{L}_{\mathsf{MDL}^+}$ shall be expanded by introducing new modalities indexed by expressions of this form. Then, in the resulting language, the language $\mathcal{L}_{\mathsf{CL}}$, of Command Logic, we have formulas of the form $[!_i\varphi]\psi$, which is to mean that after every successful act of commanding of type $!_i\varphi$, ψ holds. Thus we define:

Definition 5. *Take the same countably infinite set* Aprop *of proposition letters and the same finite set I of agents as before, with p ranging over* Aprop *and i over I. The language of command logic $\mathcal{L}_{\mathsf{CL}}$ is given by:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid O_i\varphi \mid [\pi]\varphi$$
$$\pi ::= !_i\varphi$$

Terms of the form $!_i\varphi$ and operators of the form $[!_i\varphi]$ are called command type terms and command operators, respectively. The set of all well formed formulas of $\mathcal{L}_{\mathsf{CL}}$ is referred to as S_{CL}, and the set of all the well formed command type terms as Com.

$\bot$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\Diamond$, P_i, F_i , and $\langle !_i\varphi\rangle$ are assumed to be introduced by definition in the obvious way. Note that $S_{\mathsf{MDL}^+} \subset S_{\mathsf{CL}}$.

Now, in order to give truth definition for this language, we have to specify how acts of commanding change models. As we have observed earlier, in the situation before the issuance of your boss's command, we have $\neg O_a p$ at s. This means that in M, at some point v such that $\langle s, v\rangle \in R_a^M$, $\neg p$ holds. Let t be such a point. Now in the updated situation N we have $O_a p$ at s, and this means that in N, there is no point w such that $\langle s, w\rangle \in R_a^N$ and $N, w \models_{\mathsf{MDL}^+} \neg p$. But since we have $M, t \models_{\mathsf{MDL}^+} \neg p$, we also have

[3] Strictly speaking, O_i-necessitation is redundant since it is derivable. It is included here just to record the fact that MDL$^+$ is normal.

$N, t \models_{\mathsf{MDL}^+} \neg p$. As we have remarked, her command does not affect the state of the window directly. This means that in N, $\langle s, t\rangle$ is not in R_a^N.

A bit of terminology is of some help here. If a pair of points $\langle w, v\rangle$ is in some accessibility relation R, the pair will be referred to as the R-arrow from w to v. Thus we will talk about R_A^M-arrows, R_i^N-arrows, and so on. We will sometimes omit superscripts for models when there is no danger of confusion. Then the above consideration suggests that an act of commanding of the form $!_i\varphi$, if performed at w in M, eliminates from R_i^M every R_i^M-arrow that terminates in a world where φ doesn't hold. Thus, the updated model N differs from the original only in that it has $R_a^M - \{\langle w, v\rangle \in R_a^M \mid M, v \not\models_{\mathsf{MDL}^+} \varphi\}$, or equivalently $\{\langle w, v\rangle \in R_a^M \mid M, v \models_{\mathsf{MDL}^+} \varphi\}$, in place of R_a^M as the deontic accessibility relation for the agent a. [4]

A command is said to be eliminative if it always works in this way — that is, never adds arrows. Then, the truth definition for the sentences of $\mathcal{L}_{\mathsf{CL}}$ that incorporates this conception of an eliminative command can be given with reference to $\mathcal{L}_{\mathsf{MDL}^+}$-models. Note that the subscript "ECL" in the following definition is different from the subscript "CL" used in the name of the language. We use "ECL" instead of "CL" just to indicate that the logic to be studied below is based on this conception of an eliminative command.[5]

Definition 6. *Let* $M = \langle W^M, R_A^M, R_I^M, V^M\rangle$ *be an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model, and* $w \in W^M$*. If* $p \in$ Aprop*,* $\varphi, \psi, \chi \in S_{\mathsf{CL}}$*, and* $i \in I$*, then:*

(a) $M, w \models_{\mathsf{ECL}} p$ *iff* $w \in V^M(p)$

(b) $M, w \models_{\mathsf{ECL}} \top$

(c) $M, w \models_{\mathsf{ECL}} \neg\varphi$ *iff* $M, w \not\models_{\mathsf{ECL}} \varphi$

(d) $M, w \models_{\mathsf{ECL}} (\varphi \wedge \psi)$ *iff* $M, w \models_{\mathsf{ECL}} \varphi$ *and* $M, w \models_{\mathsf{ECL}} \psi$

(e) $M, w \models_{\mathsf{ECL}} \Box\varphi$ *iff* $M, v \models_{\mathsf{ECL}} \varphi$ *for every* v *such that* $\langle w, v\rangle \in R_A^M$

(f) $M, w \models_{\mathsf{ECL}} O_i\varphi$ *iff* $M, v \models_{\mathsf{ECL}} \varphi$ *for every* v *such that* $\langle w, v\rangle \in R_i^M$

(g) $M, w \models_{\mathsf{ECL}} [!_i\chi]\varphi$ *iff* $M_{!_i\chi}, w \models_{\mathsf{ECL}} \varphi$,

where $M_{!_i\chi}$ *is an* $\mathcal{L}_{\mathsf{MDL}^+}$*-model obtained from* M *by replacing* R_I^M *with the function* $R_I^{M_{!_i\chi}}$ *such that:*

[4] We can think of a more restricted, or local, variant of update operation, namely, that of replacing R_i^M with $R_i^M - \{\langle w, v\rangle \in R_i^M \mid w = s \text{ and } M, v \not\models_{\mathsf{MDL}^+} \varphi\}$ when an act of commanding of the form $!_i\varphi$ is performed at s in M. It is much harder to work with this operation than with the one we use in this paper, though.

[5] The subscript "CL" in "$\mathcal{L}_{\mathsf{CL}}$", on the other hand, is used to emphasize the fact that the definition of $\mathcal{L}_{\mathsf{CL}}$ does not by itself preclude the possibility of giving truth definition based on some non-eliminative operation. In personal communications, some people have shown interest in using some operation which sometimes adds arrows to interprete command operators. Some arrow adding operation may well be necessary when we deal with acts of permitting. But whether any arrow adding operation is necessary for interpreting command operators is not so clear as it may seem. For more on this, see Section 5.

(i) $R_I^{M_{!_i\chi}}(j) = R_I^M(j)$, *for each* $j \in I$ *such that* $j \neq i$

(ii) $R_I^{M_{!_i\chi}}(i) = \{\langle x, y\rangle \in R_i^M \mid M, y \models_{\mathsf{ECL}} \chi\}$.

We abbreviate $\{\langle x, y\rangle \in R_i^M \mid M, y \models_{\mathsf{ECL}} \chi\}$ *as* $R_i^M \upharpoonright \chi^{\downarrow}$. *A formula* φ *is true in an* $\mathcal{L}_{\mathsf{MDL}^+}$-*model* M *at a point* w *of* M *if* $M, w \models_{\mathsf{ECL}} \varphi$. *We say that a set* Σ *of formulas of* $\mathcal{L}_{\mathsf{CL}}$ *is true in* M *at* w, *and write* $M, w \models_{\mathsf{ECL}} \Sigma$, *if* $M, w \models_{\mathsf{ECL}} \psi$ *for every* $\psi \in \Sigma$. *If* $\Sigma \cup \{\varphi\}$ *is a set of formulas of* $\mathcal{L}_{\mathsf{CL}}$, *we say that* φ *is a semantic consequence of* Σ, *and write* $\Sigma \models_{\mathsf{ECL}} \varphi$, *if for every* $\mathcal{L}_{\mathsf{MDL}^+}$-*model* M *and every point* w *of* M *such that* $M, w \models_{\mathsf{ECL}} \Sigma$, $M, w \models_{\mathsf{ECL}} \varphi$. *We say that a formula* φ *is valid, and write* $\models_{\mathsf{ECL}} \varphi$, *if* $\emptyset \models_{\mathsf{ECL}} \varphi$.

The crucial clause here is (g). The truth value of $[!_i\chi]\varphi$ at w in M is defined in terms of the truth value of φ at w in the updated model $M_{!_i\chi}$.[6] Note that $M_{!_i\chi}$ has the same domain (the set of the worlds), the same alethic accessibility relation, and the same valuation as M. Since we always have $R_i^M \upharpoonright \chi^{\downarrow} \subseteq R_i^M$, we also have $R_i^M \upharpoonright \chi^{\downarrow} \subseteq R_A^M$ as required in the clause (iii) of Definition 2. Thus $M_{!_i\chi}$ is guaranteed to be an $\mathcal{L}_{\mathsf{MDL}^+}$-model. [7]

Also note that the remaining clauses in the definition reproduce the corresponding clauses in the truth definition for $\mathcal{L}_{\mathsf{MDL}^+}$. Obviously, we have:

Corollary 1. *Let* M *be an* $\mathcal{L}_{\mathsf{MDL}^+}$-*model and* w *a point of* M. *Then for any* $\varphi \in S_{\mathsf{MDL}^+}$, $M, w \models_{\mathsf{ECL}} \varphi$ *iff* $M, w \models_{\mathsf{MDL}^+} \varphi$.

The following corollary can be proved by induction on the length of ψ:

Corollary 2. *Let* $\psi \in S_{i\text{-}free}$. *Then, for any* $\varphi \in S_{\mathsf{CL}}$, $M, w \models_{\mathsf{ECL}} \psi$ *iff* $M_{!_i\varphi}, w \models_{\mathsf{ECL}} \psi$.

This means that acts of commanding will not affect deontic status of possible courses of actions of agents other than the addressee. This may be said to be a simplification. We will return to this point later.

Another thing the above corollary means is that acts of commanding will not affect brute facts and alethic possibilities in any direct way. Thus, in our example, if s in M is the actual world before the issuance of your boss's command, then s in $M_{!_ip}$ is the actual world after the issuance, and we have:

[6] This notation for updated models is derived from the notation of van Benthem & Liu [5], in which the symbol of the form $\mathcal{M}_{\varphi!}$ denotes the model obtained by updating $\mathcal{M}$ with a public announcement of the form $\varphi!$ and that of the form $\mathcal{M}_{\sharp\varphi}$ denotes the model obtained by "upgrading" $\mathcal{M}$ by a suggestion of the form $\sharp\varphi$. This notation is adapted for deontic updates here in order to avoid the rather baroque notation used in Yamada [20], in which the model $M_{!_i\varphi}$ of the present article was denoted by the symbol of the form $[R_i^M / R_i^M \upharpoonright \varphi^{\downarrow}]M$.

[7] If we impose additional restrictions on deontic accessibility relations by adding extra axioms to the proof system of MDL^+, however, the above model updating operation may yield models which violate these conditions. Thus we will have to impose matching constraints upon updating operation, but it might not always be possible. For example, the so-called D Axiom cannot be added to MDL^+ as will be observed in the discussion on Dead End Principle in Section 6. Model updating operations has been used and studied in dynamic epistemic logics, and a useful general discussion can be found in van Benthem & Liu [5].

$$M_{!_i p}, s \models_{\mathsf{ECL}} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q) \ , \tag{5}$$

since we have $M, s \models_{\mathsf{ECL}} \Diamond p \wedge \Diamond q \wedge \Diamond(\neg p \wedge \neg q)$. But note that we also have:

$$M, s \models_{\mathsf{ECL}} [!_i p] O_i p \ . \tag{6}$$

Your boss's command eliminates all the R_i^M-arrows $\langle w, v \rangle$ such that $M, v \not\models_{\mathsf{ECL}} p$, and consequently we have $M_{!_i p}, s \models_{\mathsf{ECL}} O_i p$.

In fact this is an instantiation of the following principle:

Proposition 1 (CUGO Principle). *If* $\varphi \in S_{i\text{-free}}$, *then* $\models_{\mathsf{ECL}} [!_i \varphi] O_i \varphi$.

The restriction on φ here is motivated by the fact that the truth of φ at a point v in M does not guarantee the truth of φ at v in $M_{!_i \varphi}$ if φ involves deontic modalities for the agent i. Thus, $[!_i P_i q] O_i P_i q$ is not valid, as is seen in the following example:

Example 2. Let $I = \{i\}$, and $M = \langle W^M, R_A^M, R_I^M, V^M \rangle$ where $W^M = \{s, t, u\}$, $R_A^M = \{\langle s, t \rangle, \langle t, u \rangle\}$, $R_I^M(i) = \{\langle s, t \rangle, \langle t, u \rangle\}$, and $V^M(q) = \{u\}$. Then we have $M, u \models_{\mathsf{ECL}} q$. Hence we have $M, t \models_{\mathsf{ECL}} P_i q$ but not $M, u \models_{\mathsf{ECL}} P_i q$. This in turn means that $\langle s, t \rangle$ is, but $\langle t, u \rangle$ is not, in $R_i^M \upharpoonright P_i q^{\downarrow}$. Thus we have $M_{!_i P_i q}, t \not\models_{\mathsf{ECL}} P_i q$. This in turn means that we have $M_{!_i P_i q}, s \not\models_{\mathsf{ECL}} O_i P_i q$. Therefore we have $M, s \not\models_{\mathsf{ECL}} [!_i P_i q] O_i P_i q$.

As this example shows, the model updating operation used to interpret $[!_i \varphi]$ may eliminate R_i^M-arrows on which the truth of φ at a world accessible from the current world in M depends. [8]

Now, CUGO principle characterizes (at least partially) the effect of an act of commanding; though not without exceptions, commands usually generate obligations. The workings of an act of commanding of the form $!_i \varphi$ can be visualized by imagining $R_{!_i \varphi}$-arrows, so to speak. If an act of commanding $!_i \varphi$ is performed in M at a point w, it will take us to w in $M_{!_i \varphi}$ along an $R_{!_i \varphi}$-arrow. Thus $R_{!_i \varphi}$-arrows could be used to interpret acts of commanding. R_A^M-arrows, in contrast, only take us to points within M since they only connect points in M. While ordinary actions affect brute facts, acts of commanding affect deontic aspects of situations in our life. This difference is reflected in the difference between R_A-arrows and $R_{!_i \varphi}$-arrows. Different choices of different alternative actions are represented by different worlds within one and the same $\mathcal{L}_{\mathsf{MDL}^+}$-model and these worlds are connected by R_A-arrows of that model. In contrast, different $\mathcal{L}_{\mathsf{MDL}^+}$-models are used to represent situations differing from each other in deontic aspects, and only $R_{!_i \varphi}$-arrows connect those situations. Thus it seems that the difference between R_A-arrows and $R_{!_i \varphi}$-arrows exemplifies the difference between usual acts and illocutionary acts. Illocutionary acts affects institutional facts while usual acts affect brute facts.[9]

[8] Let S_{CGO} be the set of sentences φ such that $\models_{\mathsf{ECL}} [!_i \varphi] O_i \varphi$. Since $O_i \psi \to O_i \psi \in S_{\mathrm{CGO}}$, we have $S_{i\text{-free}} \subset S_{\mathrm{CGO}} \subset S_{\mathrm{CL}}$. But exactly how large S_{CGO} is is an interesting open question.

[9] CUGO principle may raise some worry. If p is an immoral proposition, for example, do we still have $[!_i p] O_i p$? For more on the conditions of successful issuance of commands, see Section 5.

4 Proof system for ECL

Now we define proof system for ECL.

Definition 7. *The proof system for* ECL *contains all the axioms and all the rules of the proof system for* MDL^+*, and in addition the following reduction axioms and rules:*

(RAt)	$[!_i\varphi]p \leftrightarrow p$ *where* $p \in Aprop$	(Reduction to Atoms)
(RVer)	$[!_i\varphi]\top \leftrightarrow \top$	(Reduction to Verum)
(FUNC)	$[!_i\varphi]\neg\psi \leftrightarrow \neg[!_i\varphi]\psi$	(Functionality)
($[!_i\varphi]$-Dist)	$[!_i\varphi](\psi \wedge \chi) \leftrightarrow ([!_i\varphi]\psi \wedge [!_i\varphi]\chi)$	($[!_i\varphi]$-Distribution)
(RAleth)	$[!_i\varphi]\Box\psi \leftrightarrow \Box[!_i\varphi]\psi$	(Reduction for Alethic Modality)
(RObl)*)*	$[!_i\varphi]O_i\psi \leftrightarrow O_i(\varphi \rightarrow [!_i\varphi]\psi)$	(Reduction for Obligation)
(RInd)	$[!_i\varphi]O_j\psi \leftrightarrow O_j[!_i\varphi]\psi$ *where* $i \neq j$	(Independence)
($[!_i\varphi]$-Nec)	$\dfrac{\psi}{[!_i\varphi]\psi}$ *for each* $i \in I$.	($[!_i\varphi]$-necessitation)

An ECL*-proof of a formula* φ *is a finite sequence of* $\mathcal{L}_{\mathsf{CL}}$*-formulas having* φ *as the last formula such that each formula is either an instance of an axiom, or it can be obtained from formulas that appear earlier in the sequence by applying a rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{ECL}} \varphi$*. If* $\Sigma \cup \{\varphi\}$ *is a set of* $\mathcal{L}_{\mathsf{CL}}$*-formulas, we say that* ϕ *is deducible in* ECL *from* Σ *and write* $\Sigma \vdash_{\mathsf{ECL}} \varphi$ *if* $\vdash_{\mathsf{ECL}} \varphi$ *or there are formulas* $\psi_1, \ldots, \psi_n \in \Sigma$ *such that* $\vdash_{\mathsf{ECL}} (\psi_1 \wedge \cdots \wedge \psi_n) \rightarrow \varphi$*.*

It is easy to verify that all these axioms are valid and the rules preserve validity. Hence the proof system for ECL is sound. Obviously the following condition holds:

Corollary 3. *Let* $\Sigma \cup \{\varphi\} \subseteq S_{\mathsf{MDL}^+}$*. Then, if* $\Sigma \vdash_{\mathsf{MDL}^+} \varphi$*, then* $\Sigma \vdash_{\mathsf{ECL}} \varphi$*.*

Note that the form of RObl axiom is very closely similar to, though not identical with, that of the following axiom of the logic of public announcements:

$$[\varphi!]K_i\psi \leftrightarrow (\varphi \rightarrow K_i\,[\varphi!]\psi) \ .$$

The similarity of the forms reflects the similarity of updating mechanisms; both of them are eliminative. The difference between the forms reflects the difference between the preconditions. Since a public announcement that φ is supposed to produce mostly the knowledge that φ, φ has to be true.[10] But in the case of an act of commanding of form $!_i\varphi$, φ need not be true in order for the command to be effective.

[10] The epistemic analogue of the unrestricted form of CUGO principle, namely $[\varphi!]K_i\varphi$, is not valid as is seen in the puzzle of the muddy children. See, for example, Gerbrandy & Groeneveld [9], p.163.

RAt and RVer axioms enable us to eliminate any command operator prefixed to a propositional letter and $\top$ respectively, and other reduction axioms enable us to reduce the length of any sub-formula to which a command operator is prefixed step by step. Thus these axioms enable us to translate any sentence of $\mathcal{L}_{\mathsf{CL}}$ into a sentence of $\mathcal{L}_{\mathsf{MDL}^+}$ that is provably equivalent to it. This means that the completeness of the dynamic logic of eliminative commands ECL is derivable from the completeness of the static deontic logic MDL^+.

The use of translation based on reduction axioms has been a standard method in the development of the dynamic logics of public announcements. Where a complete set of reduction axioms is available, it enables us to have an easy proof of the completeness.[11] We now present the outline of the proof of the completeness of ECL here.

First, we define translation from $\mathcal{L}_{\mathsf{CL}}$ to $\mathcal{L}_{\mathsf{MDL}^+}$.

Definition 8 (Translation). *The translation function t that takes a formula from $\mathcal{L}_{\mathsf{CL}}$ and yields a formula in $\mathcal{L}_{\mathsf{MDL}^+}$ is defined as follows:*

$$\begin{array}{lll@{\qquad}lll}
t(p) & = & p & t([!_i\varphi]p) & = & p \\
t(\top) & = & \top & t([!_i\varphi]\top) & = & \top \\
t(\neg\varphi) & = & \neg t(\varphi) & t([!_i\varphi]\neg\psi) & = & \neg t([!_i\varphi]\psi) \\
t(\varphi\wedge\psi) & = & t(\varphi)\wedge t(\psi) & t([!_i\varphi](\psi\wedge\chi)) & = & t([!_i\varphi]\psi)\wedge t([!_i\varphi]\chi) \\
t(\Box\varphi) & = & \Box t(\varphi) & t([!_i\varphi]\Box\psi) & = & \Box t([!_i\varphi]\psi) \\
t(O_i\varphi) & = & O_i t(\varphi) & t([!_i\varphi]O_i\psi) & = & O_i(t(\varphi)\rightarrow t([!_i\varphi]\psi)) \\
 & & & t([!_i\varphi]O_j\psi) & = & O_j t([!_i\varphi]\psi) \quad \text{where } i\neq j \\
 & & & t([!_i\varphi][!_j\psi]\chi) & = & t([!_i\varphi]t([!_j\psi]\chi)) \quad \text{for any } j\in I\ .
\end{array}$$

The following corollary can be proved by induction on the length of η:

Corollary 4 (Translation Effectiveness). *For any formula $\eta\in S_{\mathsf{CL}}$, $t(\eta)\in S_{\mathsf{MDL}^+}$.*

With the help of Corollary 4 and reduction axioms, the following lemma is proved by induction on the length of η:

Lemma 1 (Translation Correctness). *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model, and w a point of M. Then for any formula $\eta\in S_{\mathsf{CL}}$, $M,w\models_{\mathsf{ECL}}\eta$ iff $M,w\models_{\mathsf{ECL}} t(\eta)$.*

The following corollary is an immediate consequence of this lemma and Corollary 1:

Corollary 5. *Let M be an $\mathcal{L}_{\mathsf{MDL}^+}$-model, and w a point of M. Then for any formula $\eta\in S_{\mathsf{CL}}$, $M,w\models_{\mathsf{ECL}}\eta$ iff $M,w\models_{\mathsf{MDL}^+} t(\eta)$.*

Reduction axioms and Corollary 4 enable us to prove the following lemma by induction on the length of η:

[11] Van Benthem & Liu [5] proved that every relation changing operation that is definable in PDL without iteration has a complete set of reduction axioms in dynamic epistemic logic.

Lemma 2. *For any formula* $\eta \in S_{\mathsf{CL}}$, $\vdash_{\mathsf{ECL}} \eta \leftrightarrow t(\eta)$.

Finally, the completeness of ECL can be proved with the help of Corollary 3, Corollary 5 and Lemma 2.

Theorem 2 (Completeness of ECL). *Let* $\Sigma \cup \{\varphi\} \subseteq S_{\mathsf{CL}}$. *Then, if* $\Sigma \models_{\mathsf{ECL}} \varphi$, *then* $\Sigma \vdash_{\mathsf{ECL}} \varphi$.

5 Three Built-In Assumptions

The semantics of $\mathcal{L}_{\mathsf{CL}}$ defined in this paper incorporates a few assumptions. Firstly, as is mentioned earlier, it incorporates the conception of an eliminative command. Thus commands are assumed to be eliminative; we have $R_i^M \upharpoonright \varphi^{\downarrow} \subseteq R_i^M$ for any model M, any agent $i \in I$ and any formula φ. This might be said to be a simplification on the ground that some acts of commanding seem to add arrows. Consider the following example:

Example 3. Suppose you are in a combat troop and now waiting for your captain's command to fire. Then you hear the command, and it has become obligatory upon you to fire. But before that, you were not permitted to fire. This forbiddance is now no longer in force. Thus it seems that after his command, you are permitted to fire, at least in the sense of lack of forbiddance.

Does your captain's command in this example add arrows? Note that the forbiddance in force before the issuance of your captain's command is not an absolute one; although you were forbidden to fire without his command, it was not forbidden that you should fire at his command. Unfortunately, we have no systematic way of expressing these facts in $\mathcal{L}_{\mathsf{CL}}$. Since a command type term of the form $!_i\varphi$ is not a sentence, it cannot be used to state the fact that you are commanded to see to it that φ, and no sentence we can build with $!_i\varphi$ and other expressions can be used to do so, either. Furthermore, a world in which you fire at his command is not simply a world in which he has commanded you to fire and you fire, but a world in which you fire because he has commanded you to do so. Thus even if we postulate that p and q express the proposition that your captain has commanded you to fire and the proposition that you fire, respectively, $p \wedge q$ doesn't fully characterize a world to be a world in which you fire at his command. We can say, however, that a world in which you fire at his command is also a world in which you fire. Thus at least one world in which you fire is among permissible possible worlds with respect to you in the initial situation. This fact can be expressed in $\mathcal{L}_{\mathsf{CL}}$. Let M be your initial situation, and t the current world in that situation. Then we have $M, t \models_{\mathsf{ECL}} P_i q$. We also have $M, t \models_{\mathsf{ECL}} F_i(\neg p \wedge q) \wedge [!_i q]P_i q$. This formula can be read as saying that you are forbidden to fire without your captain's command and that you are permitted to fire after someone successfully command you to fire. Moreover, we have:

Proposition 2. *For any* $\varphi \in S_{\mathsf{CL}}$, $\models_{\mathsf{ECL}} [!_i\varphi]P_i\varphi \rightarrow P_i\varphi$.

This principle closely parallels the above discussion. This consideration suggests that whether an arrow-adding operation is necessary or not is not so clear as it may seem.

Secondly, as is noted in Corollary 2, commands of the form $!_i\varphi$ are assumed to have no effect on the deontic accessibility relations for any agents other than i. This might also be said to be a simplification. For example, suppose one of your colleague b was in your office in Example 1. Let p and q be understood as in earlier discussions of this example. We have $M, s \models_{\mathsf{ECL}} P_b\, p \wedge P_b\, q \wedge P_b\, (\neg p \wedge \neg q)$. Then by our semantics, we have $M_{!_a p}, s \models_{\mathsf{ECL}} P_b\, p \wedge P_b\, q \wedge P_b\, (\neg p \wedge \neg q)$. But if b turns on the air conditioner just after your boss's command, he would go against your boss's intention in a sense; his doing so will undermine the condition under which your opening the window would contribute to your boss's plan, and thereby prevent your boss's goal from being achieved as intended. Moreover, even b's opening the window could possibly be problematic in that it will preclude the possibility of your opening it.

One possible way of dealing with phenomena of this kind is to interpret your boss's command as meant to be heard by all the people in the office, and to obligate them to see to it that you see to it that p. But again, we have no systematic way of expressing this in $\mathcal{L}_{\mathsf{CL}}$. In order to do so, we need to extend our language by allowing deontic operators and actions terms to be indexed by groups of agents, and by introducing construction that enables us to have a formula which can expresses that you see to it that p. Although such an extension will be of much interest, it is beyond the scope of the present paper.

Thirdly, commands we talk about in ECL are assumed to be issued successfully. Although this assumption may be said to be unrealistic, it is not harmful. One obvious condition for successful issuance of a command is the condition that the commanding agent has authority to do so. Such conditions will be of central importance, for example, when we try to decide, given an particular utterance of an imperative sentence by an agent in a particular context, whether a command is successfully issued in that utterance or not. But it is important to notice that there is another more fundamental question to ask, namely that of what a successfully issued command accomplishes. This question requires us to say what an act of commanding is. It is this question that ECL is developed to address, and when we use ECL to answer it, we can safely assume that the commands we are talking about are issued by suitable authorities.

Another natural candidate for the precondition for the act of commanding i to do A is the requirement that it should be possible for i to do A. But we have no direct way of requiring this, since we have no way of talking about actions other than commanding. Thus, the best we could do might be to require that $\Diamond\varphi$ holds, for example, as the precondition for the successful issuance of a command of the form $!_i\varphi$. But even if we do so, there remains a real possibility of conflicting commands coming from different authorities. We will return to this point in the next section.

6 Some Interesting Validities and Non-validities

Here are a few more interesting principles our semantics validates.

Proposition 3. *The following principles are valid:*

(DE) $[!_i(\varphi \wedge \neg\varphi)]O_i\psi$ (Dead End)

(RSC) $[!_i\varphi][!_i\psi]\chi \leftrightarrow [!_i(\varphi \wedge \psi)]\chi$ *where* $\varphi, \psi \in S_{\text{i-free}}$
(Restricted Sequential Conjunction)
(ROI) $[!_i\varphi][!_i\psi]\chi \leftrightarrow [!_i\psi][!_i\varphi]\chi$ *where* $\varphi, \psi \in S_{\text{i-free}}$. (Restricted Order Invariance)

Dead End Principle states that a self-contradictory command leads to a situation where everything is obligatory. Such a situation is an obligational dead end. Whatever choice you may make, you will go against some of your obligations. The absurdity of such a situation is nicely reflected in the updated model. Since $\varphi \wedge \neg\varphi$ is not true at any world in any model, $R_i^M \upharpoonright (\varphi \wedge \neg\varphi)^{\downarrow}$ is empty for any model M. Thus, if a command of the form $!_i(\varphi \wedge \neg\varphi)$ is given to an agent i at some world w in M, every world will become $R_i^{M_{!_i(\varphi\wedge\neg\varphi)}}$- inaccessible from any world in the updated model $M_{!_i(\varphi\wedge\neg\varphi)}$. Hence every world in $M_{!_i(\varphi\wedge\neg\varphi)}$ will be a dead end with respect to $R_i^{M_{!_i(\varphi\wedge\neg\varphi)}}$- accessibility.

Restricted Sequential Conjunction Principle states that commands given in a sequence usually, though not always, add up to a command with a conjunctive content. Unrestricted form of sequential conjunction principle is not valid because $(R_i^M \upharpoonright \varphi^{\downarrow}) \upharpoonright \psi^{\downarrow}$ can be distinct from $R_i^M \upharpoonright (\varphi \wedge \psi)^{\downarrow}$. Similarly, Restricted Order Invariance Principle states that the order of issuance usually doesn't matter. Unrestricted form of this principle is not valid because $(R_i^M \upharpoonright \varphi^{\downarrow}) \upharpoonright \psi^{\downarrow}$ can be distinct from $(R_i^M \upharpoonright \psi^{\downarrow}) \upharpoonright \varphi^{\downarrow}$. [12]

As a consequence of Dead End Principle, right-unboundedness is not generally preserved with respect to deontic accessibility relations. Hence it is not possible for us to add the so-called D Axiom, i.e. $O_i\varphi \to P_i\varphi$, to our proof system. For example, for any M and w, we have:

$$M_{!_i(p\wedge\neg p)}, w \models_{\mathsf{ECL}} O_i(p \wedge \neg p) \wedge \neg P_i(p \wedge \neg p) \ . \tag{7}$$

Moreover, as an instance of Restricted Sequential Conjunction Principle, we have:

$$[!_i p][!_i \neg p]\chi \leftrightarrow [!_i(p \wedge \neg p)]\chi \ . \tag{8}$$

Hence, by Dead End Principle, we have:

$$[!_i p][!_i \neg p]O_i\varphi \ . \tag{9}$$

Although no boss might be silly enough to give you a command to see to it that $p \wedge \neg p$, you might have two bosses and after one of them gives you a command to see to it

[12] Note that our notation for models and deontic accessibility relations involves a record of updates. For example, $(M_{!_i\varphi})_{!_i\psi}$ is the model obtained by updating $M_{!_i\varphi}$ with a command of form $!_i\psi$, and the model $M_{!_i\varphi}$ in turn is the model obtained by updating M with a command of form $!_i\varphi$. Such records might be utilized to answer the interesting question raised by Ken Satoh at CLIMA VII workshop. He asked whether authorities can change their minds in ECL. Although we haven't incorporated action types for acts of canceling in ECL, it seems possible to extend it to include them. For example, if the earlier act of commanding of form $!_i\varphi$ performed at w in M is canceled at w in $(M_{!_i\varphi})_{!_i\psi}$, then, it seems, the resulting situation will be represented by $M_{!_i\psi}$. Whether this strategy turns out to be fruitful or not is yet to be seen.

that p, the other one might give you a command to see to it that $\neg p$. Unless both of them belong to the same hierarchy, neither command might be overridden by the other. Whichever command you may choose to obey, you will have to disobey the other.

If we require $\Diamond\varphi$ to hold as the precondition for successful issuance of a command of the form $!_i\varphi$, every command of the form $!_i(\psi \wedge \neg\psi)$ will be precluded. But even if we do this, there may be a situation M such that $\Diamond p \wedge \Diamond\neg p$ holds at w in M. In such a situation, a command of the form $!_i p$ can be issued. In the resulting situation, $\Diamond p \wedge \Diamond\neg p$ still holds at w, and hence it remains possible to issue a command of the form $!_i\neg p$.

One way of avoiding obligational dead end of this kind could be to require φ in $!_i\varphi$ to be in Aprop. But it would not be a real solution, and even if we do this, you might still find yourself in a contingent analogue of an obligational dead end. Consider the following example:

Example 4. Suppose the boss of your department commanded you to attend an international one-day conference on logic to be held in São Paulo next month. Also suppose that soon after that your political guru commanded you to join an important political demonstration to be held on the very same day in Tokyo. It is possible for you to obey either command, but it is transportationally impossible for you to obey both. Even after you decide which command to obey, you might still regret not being able to obey the other command.

As no logical inconsistency is involved in the combination of obligations generated by these commands, we may say, for example, that it would be possible for you to obey both command if a sufficiently fast means of transportation were available. But the metaphysical possibility of such a fast means of transportation is of no help to you in the real world. You are in a situation very closely similar to those in which you are said to be in real moral dilemmas. As Marcus [14] has argued, they can be real even if the moral rules involved are logically consistent.

In this example, each of your boss and your guru can be assumed to have suitable authority for the issuance of his or her command. Moreover, it is really possible for you to obey one or the other of the two commands. Still, their commands are in conflict with each other. Such conflicts can be sometimes extremely difficult to avoid in real life as conflicts can arise due to some unforeseen contingencies of the real world.

If we allow deontic accessibility relations, deontic operators, and command type terms to be indexed by the Cartesian product of a given set of agents and a given set of command issuing authorities, then your situation can be represented as a situation which may be suitably called an obligational dilemma. In the extended language, we can use expressions of the form $!_{(i,j)}\varphi$ to denote the type of an act of commanding in which an authority j commands an agent i that he or she should see to it that φ. Let a, b and c represent you, your boss and your guru, respectively. Let the model-world pair (M, s) represent the situation before the issuance of your boss's command, and p represent the proposition that you will attend the conference in São Paulo. Then, after your boss's command you are in $(M_{!_{(a,b)}p}, s)$. Now, let q represent the proposition that you will join the the demonstration in Tokyo. Then, after the issuance of your guru's command, you are in $((M_{!_{(a,b)}p})_{!_{(a,c)}q}, s)$. In this situation, from the real world s, only the worlds in which you attend the conference in São Paulo will be $R_{(a,b)}$-accessible, only the worlds in which you join the demonstration in Tokyo will be $R_{(a,c)}$-accessible, and s can be either among the $R_{(a,b)}$-accessible worlds or among the $R_{(a,c)}$-accessible worlds, though it cannot be among the worlds that are both $R_{(a,b)}$-accessible and $R_{(a,c)}$-accessible. Contingent facts about the present state of our system of transportation prevent it from being both $R_{(a,b)}$-accessible and $R_{(a,c)}$-accessible. It is obligatory upon you with respect to your boss that you attend the the conference in São Paulo, and it is obligatory upon you with respect to your guru that you join the demonstration in Tokyo. You can respect one or the other of these obligations, but you are not able to respect both. If you obey your boss's command, you will disobey your guru's command, and if you obey your guru's command, you will disobey your boss's command.

This refinement will also enable us to represent the situation you will be in if a command of the form $!_{(a,c)}\neg p$ is issued after a command of the form $!_{(a,b)}p$ is issued as an obligational dilemma. It is not difficult to incorporate this refinement into $\mathcal{L}_{\mathsf{MDL}^+}$ and $\mathcal{L}_{\mathsf{CL}}$. [13]

7 Related Works and Further Directions

As is noted in the introduction, the idea of ECL is inspired by the logics of epistemic actions developed in Plaza [16], Groeneveld [11], Gerbrandy & Groeneveld [9], Gerbrandy [8], Baltag, Moss, & Solecki [2], and Kooi & van Benthem [13] among others. In the field of deontic reasoning, van der Torre & Tan [18] and Žarnić [22] extended the update semantics of Veltman [19] so as to cover normative sentences and natural language imperatives, respectively. Apart from the fact that the languages they used are stronger than $\mathcal{L}_{\mathsf{MDL}^+}$, the main difference between their systems and ECL consists in that the former deal with the interpretation of sentences while the latter deals with the dynamics of acts of commanding. Broadly speaking, the relation between their systems and ECL is analogous to that between Veltman's update semantics and the logics of epistemic actions.

In this respect, PDL based systems of Pucella & Weissman [17], and Demri [6] are closer to the present work in spirit. Their systems dynamified DLP of van der Meyden [15]. DLP is obtained from test-free PDL by introducing operators which have semantics that distinguish permitted (green) transitions from forbidden (red) ones. The set of green transitions of each model is the so-called policy set. In Pucella & Weissman [17], DLP is dynamified so that in the resulting system DLP_{dyn} the policy set can be updated by adding or deleting transitions, and in Demri [6], DLP_{dyn} is extended to DLP^+_{dyn} by adding test operator "?" and allowing the operators for updating policy sets to be parameterized by the current policy set. One important difference between these PDL-based systems and ECL lies in the fact that in these PDL-based systems, we can talk about permitted or forbidden actions as well as obligatory state of affairs whereas we can only talk about permitted, forbidden or obligatory state of affairs in ECL.

Another interesting related work is stit theory developed in Belnap, Perloff, & Xu [3] and Horty [12]. As the wording in this paper might have already suggested, agentive sentences can be utilized to capture the contents of commands. But the language

[13] This refinement is incorporated in Yamada [21].

of monadic deontic logic lacks the resource for distinguishing agentives from non-agentives. This defect can be removed by using a language of stit theory. In order to do so, however, we have to rethink our update operation, as we talk about "moments" in stead of possible worlds in stit theory. Since moments are partially ordered in a tree like branching temporal structure, we have to take their temporal order into account. But the update operation of this paper is not sensitive to temporal order. Thus when we think of the points in our model not as possible worlds but as stages of some language game, for example, it might look a bit problematic, since it can eliminate deontic arrows that connect stages earlier than the stage at which the command is issued. Although this does not mean that the update operation of this paper could affect the past state of affairs, it means that deontic status of the past state of affairs can be affected afterward. As it is possible to define different update operations even with respect to $\mathcal{L}_{\mathsf{MDL}^+}$-models, one immediate task for us is to examine the logics obtained by replacing the update operation.

Finally, the most closely related work in this field is that of van Benthem & Liu [5]. They proposed what they call "preference upgrade" as a counter part to information update. According to them, my "command operator for propositions A can be modeled exactly as an upgrade sending R to $R;?A$ " in their system, and their paper "provides a much more general treatment of possible upgrade instructions"([5]). Although their preference upgrade clearly has much wider application than the deontic update of the present paper, the notion of preference upgrade seems to be connected with perlocutionary consequences, while the notion of deontic update is meant to be used to capture a differential feature of an act of commanding as a specific kind of illocutionary acts. They can be seen as mutually complementary.

8 Conclusion

We have shown that commands can be considered as deontic updators. Since the base language $\mathcal{L}_{\mathsf{MDL}^+}$ we dynamified is a variant of monadic deontic logic, our extended language $\mathcal{L}_{\mathsf{CL}}$ inherits various inadequacy of the language of monadic deontic logic. But the fact that even such a simple language can be used to capture some interesting principles may be said to suggest the possibility of further research, including dynamifying stronger deontic languages. Moreover, the possibilities of update logics of various other kinds of illocutionary acts suggest themselves. For example, an act of promising can be considered as another updator of obligations, and an act of asserting as an updator of propositional commitments. Logics of such acts may provide us with a fairly fine-grained picture of social interactions when combined not only with each other but also with logics of perlocutionary acts that update systems of knowledge, beliefs and preferences of agents.

In this paper, logic of acts of commanding is not yet combined with logics of other speech acts. There are many things yet to be done before it becomes possible to address various interesting issues relating to interactions among agents. But in order to combine logics, we need logics to combine. And even already within ECL, we can talk about the effects of a sequence of acts of commanding as Restricted Sequential Conjunction Principle and Restricted Order Invariance Principle exemplify. Furthermore, the refinement suggested in the last part of Section 6 will enable us to distinguish between the effects of a sequence of commands issued exclusively by one and the same authority and the effects of a sequence of commands involving commands issued by different authorities, and thereby raise interesting questions of preference management for agents who have roles to play in more than one organizations. The same kind of question can also arise when a command is issued which conflicts with a promise already made. Thus one of our immediate tasks is to extend ECL to deal with the interactions among agents involved in such a situation. Although the development of ECL is a small step towards the development of richer logics of acts of commanding, it can be part of the beginning of the explorations into the vast area of the logical dynamics of social interactions.

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