Logic of Agent Communication

Satoshi Tojo (Japan Advanced Institute of Science and Technology)

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1 Who knows what at which time?

Let us consider communication among multi-agents.

George de la Tour: Le Tricheur à l'as de carreau

In the above picture, we can see many aspects of belief change of agents triggered by an informing action by others.

- Public announcement
- Liar
- Belief revision
- Reliability of news source
- Mutual belief
- Method of communication, *i.e.*, channel

How can we formalize this (or more troublesome) situation in logic, in an efficient, scalable and reliable computation system?

2 Doxastic Logic with Channel

Let G be a set of agents and Prop be a set of propositional variables. The syntax of PDL-extension of dynamic doxastic logic with channel is defined by simultaneous induction on a program term π and a formula φ:

$$\pi ::= \mathsf{R}_a \mid (\pi \cup \pi) \mid (\pi;\pi) \mid \varphi? \mid \varphi\downarrow^a_b \quad (a \in G)$$
$$\varphi ::= p \mid \mathsf{c}_{ab} \mid \neg\varphi \mid \varphi \vee \varphi \mid [\pi]\varphi \quad (p \in \mathsf{Prop},\ a, b \in G)$$

- c_{ab}: 'there is a channel from agent a to b.'
- $[\mathsf{R}_a]\varphi$: 'agent a believes that φ holds.'
- $(\pi \cup \pi')$: 'do π or π', non-deterministically.'
- $(\pi;\pi')$: 'do π followed by π'.'
- $\varphi?$: 'proceed if φ true, else fail.'
- $[\varphi\downarrow^a_b]\psi$: 'after agent a informs agent b of message φ via channel, ψ.'

Given a model $\mathfrak{M} = (W, (R_a)_{a\in G}, (C_{ab})_{a,b\in G}, V)$, the semantics is defined as follows:

$$\begin{array}{lcl}
[\![\mathsf{R}_a]\!]_{\mathfrak{M}} & := & R_a, \\
[\![\pi \cup \pi']\!]_{\mathfrak{M}} & := & [\![\pi]\!]_{\mathfrak{M}} \cup [\![\pi']\!]_{\mathfrak{M}}, \\
[\![\pi;\pi']\!]_{\mathfrak{M}} & := & [\![\pi]\!]_{\mathfrak{M}} \circ [\![\pi']\!]_{\mathfrak{M}}, \\
[\![\varphi?]\!]_{\mathfrak{M}} & := & \{\, (w,v) \mid w = v \text{ and } w \in [\![\varphi]\!]_{\mathfrak{M}} \,\}, \\
[\![p]\!]_{\mathfrak{M}} & := & V(p), \\
[\![\mathsf{c}_{ab}]\!]_{\mathfrak{M}} & := & C_{ab}, \\
[\![[\pi]\varphi]\!]_{\mathfrak{M}} & := & \{\, w \in W \mid [\![\pi]\!]_{\mathfrak{M}}(w) \subseteq [\![\varphi]\!]_{\mathfrak{M}} \,\},
\end{array}$$

where $[\![[\mathsf{R}_a]\varphi]\!]_{\mathfrak{M}}$ is the truth set $\{\, w \in W \mid \mathfrak{M}, w \models \varphi \,\}$.

As for the semantics of $[\varphi\downarrow^a_b]\psi$, if agent a resides in $[\![\varphi]\!]$ and when there is a channel from a to b, b believes that ψ, otherwise b does not change his/her beliefs. That is, if $a \neq b$,

$$R'_b := (R_b \cap ([\![\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi]\!] \times [\![\varphi]\!])) \cup (R_b \cap ([\![\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)]\!] \times W)),$$

and otherwise $R'_b := R_b$. In PDL-format,

$$\pi_b := ((\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)?; \mathsf{R}_b; \varphi?) \cup (\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)?; \mathsf{R}_b).$$

3 Matrix Representation

Based on [Fitting, 2003], a matrix representation of accessibility relation is defined by $R^M(i,j) = 1$ if $(w_i, w_j) \in R$, otherwise 0. The valuation is $V(p)^M(i) = 1$ if $w_i \in V(p)$, otherwise 0. Thus, we can reformulate the above PDL-format of the semantics into a matrix representation as follows: If $a = b$, ${R'_b}^M = R_b^M$. If $a \neq b$, we obtain:

$${R'_b}^M = [\![(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)?]\!]^M R_b^M [\![\varphi?]\!]^M + [\![\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)?]\!]^M R_b^M.$$

Example When agent a sends a piece of information p to b, the first part of a matrix calculation of R_b becomes:

$$[\![(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,p)?]\!]^M R_b^M [\![p?]\!]^M = [\![\mathsf{c}_{ab}?]\!]^M [\![\mathsf{B}_a\,p?]\!]^M R_b^M [\![p?]\!]^M$$

$$= \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix} = \begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix}.$$

The result shows that b comes to believe that p. In order to get the final solution, we also need to calculate the rest of the part, *i.e.*, $[\![\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\,\varphi)?]\!]^M R_b^M$, and to combine the both results.