

$$\begin{aligned}x^5 + x^4 + x^3 + x^2 + x + 1 &= x^2(x^3 + x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}) \\&= x^2\{(x^3 + 1) + (x^2 + \frac{1}{x}) + (x + \frac{1}{x^2})\} \\&= x^2\{(x^3 + 1) + (\frac{x^3 + 1}{x}) + (\frac{x^3 + 1}{x^2})\} \\&= x^2(x^3 + 1) + x(x^3 + 1) + (x^3 + 1) \\&= (x^3 + 1)(x^2 + x + 1) \\&= (x + 1)(x^2 - x + 1)(x^2 + x + 1)\end{aligned}$$