Proposal for the Universal Unit System*

Takashi SUGA†

January 1st, 2002

1 Introduction

A unit of measure is "a quantity that is used as the basis for expressing a given quantity, and is of the same type[1] as the quantity that is to be expressed".(1) As evidenced by the term "weights and measures", the history of units began from the simple stage of using familiar quantities such as the weight of grain or the lengths of the human hand and foot as units to express the quantities that we deal with in daily life such as length, volume, and weight.

A unit that is used in exchanges between people must be guaranteed to have a constant magnitude within the scope of that exchange. Quantities that can, by consensus, serve as common standard over a broad scope were sought and selected to serve as units. The ultimate such quantity is an entity common to all of humankind, the earth itself, which was selected as the foundation for the metric system. Specifically, $1/86400^{\mathrm{th}}$ of the period of the earth's rotation is defined as one second, $1/40,000,000^{\mathrm{th}}$ of the total length of the earth's meridian is defined as one meter, and the mass of a cubic $1/10^{\mathrm{th}}$ meter of water is defined as one kilogram. [2]

The history of units of measure, on the other hand, is the history of the establishment of new concepts that have accompanied the development of natural science. The laws of nature describe the 'relationship' between 'a given quantity' and 'another quantity' specified as mathematical expressions. The 'given quantity' and 'another quantity' referred to here are often quantities that correspond to 'newly established or greatly transformed concepts' that are born of new discoveries, as occurred with mass, energy, and electrical charge. As this process goes on, the need arises to deal with quantities of a new concept and a quantity is selected as a standard for that purpose. That quantity becomes a new unit.

A system of units is a set of multiple units that are related on the basis of these kinds of laws and systematically organized. Consider, for example, the units for length and volume. It is, of course, possible to define virtually unrelated units for length and volume, such as we have with the units foot and gallon. However, by making use of the law which states that "the volume of a cube is proportional to the third power of the length of its side" to relate these two units, we can say that "the volume of a one-meter cube is the unit 1 meter^3 " which is a more systematic approach. In this way, a number of base units and a relationship formula that describes a natural law can be used to define all other units (which is referred to as "deriving" units in the terminology of units) and so obtain what is called a coherent unit system. In a coherent unit system, there is only one unit for one type of quantity. Thus,

*The original japanese version of this paper was released May 1984.

†E-Mail:suchowan@box.email.ne.jp, URL-http://www.hosi.org/.

[1] For more information, see Appendix A, "Basic approach to units".

[2] Currently, this basis is being replaced by definition methods in which the magnitudes of the units are virtually invariable, providing better reproducibility.

in a coherent unit system, the relationship formula on which the system rests is expressed in the most succinct way (specifically, in such a way that the coefficients of the formula are in the simplest form) .

So then, what comes next?

Of course, we can consider going beyond the framework of the earth and defining units with concepts for which agreement can be reached within a broader scope. The quantities that then become available to serve as the standards for defining units include the quantities of the 'fundamental physical constant' category, quantities such as 'the speed of light in a vacuum', 'the quantum of action', 'the Boltzmann constant', and so on. These quantities are believed to have values that remain constant everywhere in the universe. When trying to construct a coherent unit system, however, it is not reasonably possible to use all of the fundamental physical constants in the definitions of units. Then, wouldn't we expect the fundamental physical constants that were not used in defining units have fractional magnitudes of unit quantities of the same dimension?

By a surprising coincidence,[3] however, if the dozenal number system is used to express 'the speed of light in a vacuum' and 'the quantum of action' as the defining constants such that these constants are strictly multiples of integer powers of 12 of the unit quantities, it is possible to construct a coherent unit system in which not only the constant that was used in the definition, but the Rydberg constant (R_∞), the atomic mass unit (u), the Bohr radius (a_B), and half the value of the Planck length ($l_P = (1/2)\sqrt{G\hbar/c_0^3\alpha}$) as well, can be approximated to within an error of 1% by a multiple of integer powers of 12 of the unit quantities. In that case, many other physical constants, including the charge of an electron, the mass of an electron, the fine structure constant, the molar volume of an ideal gas under standard conditions, the black-body radiation at the ice point, the density of water, and others, can be approximated by multiples of integer powers of 12 of the unit quantities. Moreover, by adding the Boltzmann constant and using it in the definition of thermodynamic temperature, the gas constant of an ideal gas can be approximated by a multiple of an integer power of 12 of the unit quantity and the Stephan-Boltzmann constant and the specific heat of water can be approximated by multiples of integer powers of 12 of the unit quantities with a factor 2 remaining.

We define the Universal Unit System as "the unit system that is constructed by using the dozenal system and using 'the speed of light in a vacuum', 'the quantum of action', and 'the Boltzmann constant' as the defining constants in such a way that these constants become strict multiples of integer powers of 12 of the unit quantities and 'the Rydberg constant', 'the atomic mass unit', 'the Bohr radius', and 'half the value of the Planck length' can be approximated by multiples of integer powers of 12 of the unit quantities". This Universal Unit System is described in the remainder of this paper.

[3] To prevent any misunderstanding, let me emphasize that these are simply accidental coincidences as far as physical science is concerned. Also, please understand that the author has no intention to promote the use of the "Universal Unit System" in the real world.

2 Why the dozenal system?

First, we consider the physical and mathematical advantages of the dozenal system.

2.1 Dimensionless quantities that can be constructed of combinations of fundamental physical constants

To eliminate the influence of the unit system, let's try to list some of the dimensionless quantities that can be made up from combinations of fundamental physical constants. For putting these coincidences to use, the dozenal system is the only choice.

2.1.1 The fine structure constant and the elementary electrical quantity

The fine structure constant, α, a dimensionless quantity, was originally introduced for the purpose of explaining of the fine structure spectral emission lines.

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c_0 \hbar} \tag{1}$$

By multiplying both sides of Eq. (1) by $\frac{c_0\hbar}{r^2}$, we get

$$\alpha \frac{c_0\hbar}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \tag{2}$$

The right side of Eq. (2) expresses the Coulomb force acting between two elementary electrical quantities (i.e. the electrical charge of an electron) separated by a distance of r . The left side indicates that this force is proportional to $\frac{c_0\hbar}{r^2}$ by a factor of α. For this reason, α can be interpreted as a dimensionless quantity that represents the strength of electromagnetic interaction.

The value of the fine structure constant, α, is close to 12^{-2}.

$$\alpha = \frac{1}{137.03599} = 1.0508188 \times 12^{-2}_{(10)} \tag{3}$$

Therefore, the ratio of the elementary electrical quantity ,e, and "the dimensioned quantity of charge, which is derived from the speed of light in a vacuum, c_0, and the quantum of action, $\hbar$", is

$$\alpha^{1/2} = \frac{e}{\sqrt{4\pi\epsilon_0 c_0 \hbar}} = 1.0250946 \times 12^{-1}_{(10)} \qquad (!) \tag{4}$$

2.1.2 The Rydberg constant and the Bohr radius

The deviation of the fine structure constant, α, from an integer power of 12 is nearly the same as the deviation of 4π from 12.

$$4\pi = 1.0471976 \times 12^{1}_{(10)} = \frac{1}{137.50987} \times 12^3 \tag{5}$$

The ratio of the Bohr radius, a_B, and "the dimensioned quantity of length, L, which is derived from the Rydberg constant, $R_\infty (= 1.0973732 \times 10^{7}_{(10)} \times 2\pi\text{rad/m})$",

$$L = 2\pi\text{rad}/R_\infty = 0.91126705 \times 10^{-7}_{(10)}\text{m} \tag{6}$$

is

$$\frac{a_B}{L} = \frac{\alpha}{4\pi}(\text{strict}) = 1.0034581 \times 12^{-3}_{(10)} \qquad (!!) \tag{7}$$

2.1.3 The electron mass and the atomic mass unit

The ratio of the mass of an electron, m_e, and "the dimensioned quantity of mass, M, which is derived from L, the speed of light in a vacuum, c_0, and the quantum of action, $\hbar$",

$$M = \frac{\hbar}{c_0 L} \tag{8}$$

is

$$\frac{m_e}{M} = \frac{4\pi}{\alpha^2}(\text{strict}) = 0.94835932 \times 12^5_{(10)} \tag{9}$$

The ratio of the atomic mass unit, u, and the mass of an electron, m_e, is

$$\frac{u}{m_e} = 1822.8885_{(10)} = \frac{\alpha^2}{4\pi} \times 1.0004359 \times 12^8_{(10)} \tag{10}$$

The deviations of ratio (9) and ratio (10) from multiples of an integer power of 12 are nearly of the same magnitude, but in opposite directions. Therefore,

$$\frac{u}{M} = 1.0004359 \times 12^8_{(10)} \qquad (!!!) \tag{11}$$

2.1.4 The Planck length

The ratio of the general expression of the Planck length, $\sqrt{\frac{G\hbar}{c_0^3}}$, and L is close to 2, when factors of multiples of an integer power of 12 are eliminated.

$$\frac{\sqrt{\frac{G\hbar}{c_0^3}}}{L} = 2 \times 1.0150587 \times 12^{-26}_{(10)} \tag{12}$$

Taking the expression $\sqrt{\frac{G\hbar}{c_0^3\alpha}}$, which has been adjusted(2) by the fine structure constant, α, in order to express the tensile force in a superstring in terms of the Planck length, the ratio of the Planck length and L then becomes

$$\frac{\sqrt{\frac{G\hbar}{c_0^3\alpha}}}{L} = 2 \times 0.9902098 \times 12^{-25}_{(10)} \qquad (!) \tag{13}$$

2.2 Advantages of the dozenal system in mathematical expressions

We will consider the advantages of the dozenal system in mathematical expressions from the general viewpoint (the number of factors and factorials) and from the viewpoint of individual mathematical constants.

2.2.1 Number of factors

The number 12 has more factors (1, 2, 3, 4, 6, and 12) than does 10 (1, 2, 5, 10), so the dozenal system offers the following two advantages over the decimal system.

1. Many fractions can be expressed as finite dozenals.
2. Multiplication is simple.

2.2.2 Factorials

A little-mentioned fact is that factorials are more easily represented in the dozenal system than in the decimal system because of the large number of trailing zeroes. Of the numbers from 1 to n, one of every 2^k numbers have k times factor 2. Thus, the number of times that 2 appears as a factor in n factorial is

$$\sum_{k=1}^{\infty}[\frac{n}{2^k}] \sim \sum_{k=1}^{\infty} \frac{n}{2^k} \sim n - O(\log n) \tag{14}$$

Of the numbers from 1 to n, one of every 3^k numbers have k times factor 3. Thus, the number of times that 3 appears as a factor in n factorial is

$$\sum_{k=1}^{\infty}[\frac{n}{3^k}] \sim \sum_{k=1}^{\infty} \frac{n}{3^k} \sim \frac{n}{2} - O(\log n) \tag{15}$$

The reason is that, $12(= 2^2 \times 3)$ contains, on the average, the prime factors 2 and 3 in just the right ratio for expressing n factorial. For this reason, the dozenal number system is also generally convenient for calculating permutations, combinations, and so on.
(reference)

$$\begin{aligned} & \text{The order of the largest sporadic simple finite group} \\ = \ & 2^{46}3^{20}5^{9}7^{6}11^{3}13^{3}17^{1}19^{1}23^{1}29^{1}31^{1}41^{1}47^{1}59^{1}71^{1}_{(10)} \\ = \ & 888\_8191\_6727\_3964\_1634\_7510\_5895\_4578\_8183\_2706\_3298\_0480\_0000\_0000_{(10)} \\ = \ & 992\_4B98\_B225\_2AB9\_530B\_A466\_1487\_B0A8\_0000\_0000\_0000\_0000\_0000_{(12)} \end{aligned}$$

2.2.3 Mathematical constants

These constants, too, can be approximated by relatively simple dozenal fractions. I comment on a few interesting examples below.

1. $4\pi(\approx 10_{(12)})$
   The surface area of a sphere is approximately one order of magnitude larger than the area of a square that has sides equal in length to the radius of the sphere. Because of that, the conversion of non-rationalized units and rationalized units for the electromagnetic quantity units explained

Table 1: Mathematical constants (expressed in the dozenal system)

$\sqrt{\pi}$	=	1.9329_72A1	2^{-8}	=	0.0069	0!	=	1
2π	=	6.3494_16A0	2^{-7}	=	0.0116	1!	=	1
4π	=	10.6968_3170	2^{-6}	=	0.0230	2!	=	2
e	=	2.8752_3607	2^{-5}	=	0.0460	3!	=	6
$1/e$	=	0.44B8_4216	2^{-4}	=	0.0900	4!	=	20
γ	=	0.6B15_1888	2^{-3}	=	0.1600	5!	=	A0
ϕ	=	1.74BB_6773	2^{-2}	=	0.3000	6!	=	500
$\sqrt{2}$	=	1.4B79_170A	2^{-1}	=	0.6000	7!	=	2B00
$\sqrt{3}$	=	1.894B_9800	2^{+4}	=	14.0000	8!	=	1_B400
$\sqrt{5}$	=	2.29BB_1325	2^{+8}	=	194.0000	9!	=	15_6000
$\log_e 2$	=	0.8399_1248	2^{+14}	=	3_1B14.0000	A!	=	127_0000
$\log_2 3$	=	1.7029_9480	2^{+28}	=	9_BA46_1594.0000	B!	=	1145_0000
$z = \log_2 10$	=	3.7029_9480	2^{+37}	=	BA08_A990_A0A8.0000	10!	=	1_1450_0000

later (see Appendix B, "A method of organizing the dimensions of electromagnetic quantities") is accomplished by a correction of almost exactly a factor of 12.

2. $\sqrt{2}$ ($\approx 1.5_{(12)}$)
   Because of this relationship, it is possible to set good bounds for the heights and widths the standard sheet paper sizes. There is further discussion of paper sizes later in this paper (see Appendix D.2, "Standard sheet paper sizes").
3. The golden ratio $\phi = (1+\sqrt{5})/2$ ($\approx 1.75_{(12)}$)
   It is known that the ratio of adjacent numbers in the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, $\cdots$) rapidly converges on the golden ratio. The 12^{th} number in the sequence happens to be 12^2. It thus so happens that one of the fractional series that best approximates the golden ratio can be represented by a two-digit dozenal fraction in the dozenal system.
4. The 12-tone chromatic scale of music $\log_2 3$ ($\approx 1.7_{(12)}$)[4]
   The properties of a musical scale can be evaluated by whether combinations of sounds whose frequencies are simple integer ratios can be approximated any number of times with good accuracy. The 12-tone chromatic scale of music is excellent in this respect.
   (a) The smallest ratio of primes, 2 : 1
   This corresponds to one octave in the scale, so it must have a strict representation. Accordingly, the common ratio of a musical scale frequency must be $\sqrt[n]{2}$ (where n is a suitable natural number).
   (b) The next-smallest ratio of primes, 3 : 1
   To efficiently approximate this ratio while satisfying condition (a), it can be approximated by the best approximation fraction, $\log 3/\log 2 = \log_2 3 = 1.58496\ldots$. There are n candidates for the denominator of the best approximation fraction. Expanding into a continued fraction, we get

[4] Because of the relationship $\log_2 3 \approx 1.7_{(12)}$, $2^{37}_{(12)} \approx 10^{10}_{(12)}$. This corresponds to $2^{10}_{(10)} \approx 10^{3}_{(10)}$ in the decimal number system.

$$\frac{\log 3}{\log 2} = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{2+\cdots}}}}}}}} \tag{16}$$

$$= \frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{8}{5},\frac{19}{12},\frac{65}{41},\frac{84}{53},\frac{485}{306},\frac{1054}{665},\cdots \tag{17}$$

(c) The next-smallest ratio of primes, $5:1$

Selecting from among the n candidates obtained by condition (b) those that approximate this ratio relatively well, $n=12$ and $n=53$ remain.

(d) Larger ratios of primes

The ratios of 7:1 or higher are not very significant to the human ear.

This is all to say that the only practical musical scale is the 12-tone chromatic scale of music.[5]

[5] The 12-tone scale was probably first developed at the end of the 16^{th} century by Zhū Zài-yù of the Ming dynasty.(3) In the 19^{th} century, two men, T. Perronet Thompson and R. H. M. Bosanquet, tried to make a keyboard for use with a 53-tone scale. (4) This attempt, however, cannot be said to have produced practical results.

3 The Universal System of Units Standard

As described in the introduction, the Universal Unit System is defined as a unit system that is constructed using the dozenal number system and 'the speed of light in a vacuum', 'the quantum of action', and 'the Boltzmann constant' as the defining constants such that these constants are strict multiples of integer powers of 12 of the unit quantities and 'the Rydberg constant', 'the atomic mass unit', 'the Bohr radius', and 'half the value of the Planck length' are approximated by multiples of integer powers of 12 of the unit quantities. All unit systems that satisfy this definition are 'Universal Unit System'.

However, although this concept corresponds to our world wide 'metric system', it still contains degrees of freedom. The metric system, too, includes various types of systems, such as the MKS unit system or CGS unit system, the absolute system of units or the gravitational system of units, rationalized systems of units or non-rationalized systems of units.

Therefore, in this section, I would like to attempt a proposal for a standard that equivalent to the worldwide International System of Units (SI—Système International d'Unités), as standard which I will refer to as the Universal System of Units Standard in the following, to distinguish it from simply a Universal Unit System.

One of the most important points concerning the formulation of the specifications is the selection of the dimensions for the base units.

The concept of quantity is defined axiomatically by formulas that express natural laws, so the dimensions of the base unit, too, are selected on the basis of the ease of deriving units while considering their mutual relations, somewhat as in solving simultaneous equations.(5,6,7) Accordingly, it is difficult to explain the process of selecting the dimension for each base unit in a systematic way. Therefore, taking the International System of Units (SI) as an example, I would like to explain it in the form of explaining the discrepancies with the Universal System of Units Standard.

The dimensions of the base units of the International System of Units (SI) are length, time, mass, thermodynamic temperature, electrical current, amount of substance, and luminous intensity; the units of plane angle and solid angle were classified as supplementary units that are vague in character.

As opposed to that, the Universal System of Units Standard employ **impedance, plane angle, logarithmic quantity, amount of substance, length, time, energy, and thermodynamic temperature** as the base unit dimensions. The first four of these have natural units that are employed as base units just as they are. We create quantities of the remaining four dimensions that serve as base units by multiplication or division of the Rydberg constant,[6] the speed of light in a vacuum, the quantum of action, and the Boltzmann constant, which are fundamental physical constants used in deriving units.

1. Replacement of mass with energy
   The reason for selecting energy instead of mass is that it is more suitable as a starting point for the derivation of units such as force, work, pressure, and electrical charge. Selecting energy makes it easier to understand the meaning of quantities when the dimensions of various quantities are represented by the multiplication or division of the dimensions of the base units. The unit for pressure, for example, is $\mathrm{kg/(m \cdot s^2)}$ in the International System of Units (SI). It is probably not intuitively understandable why there is one m in the denominator. In contrast to this, in the Universal System of Units Standard, when the unit of pressure is expressed by the multiplication

[6] The reason for selecting the Rydberg constant as a defining constant is made clear in the section 3.3 "Defining constants and base units".

and division of base units, we have J_u/m_u^3. [7] This clearly indicates that pressure is energy divided by volume.

2. Replacement of electrical current with impedance
   Impedance is selected in place of electrical current to emphasize the symmetry of the units of electrical quantities and magnetic quantities (see Appendix B), "A method of organizing the dimensions of electromagnetic quantities"). With the unit of electrical current as a base unit, there is no symmetry, and a confusing collection of units whose methods of derivation are difficult to understand systematically, such as C, V, Ω, F, H, T, and Wb, become necessary. In the International System of Units (SI), how these units are derived from the unit of electrical current, A, is not ordinarily an issue and so there is a perception that they are used as completely independent units. By taking the approach described in Appendix B, it was possible to minimize the number of derived units that have a characteristic symbol.
3. Elimination of luminous intensity
   Luminous intensity is omitted because it is a quantity that is dependent on human biological characteristics. No particular optical unit is established and the unit of radiant flux, Wu, (which has the same dimension as work) is used as the unit of luminous flux. That is to say, the light that has the maximum relative luminosity factor that produces the same visual effect is converted to radiant flux and expressed in terms that unit. Other optical units are derived from the unit of luminous flux. The method of deriving the unit of luminous intensity from the unit of luminous flux is selected because it is considered to be more natural than the reverse (in the International System of Units, too, the unit of luminous intensity, cd, which is a base unit, is actually defined in terms of radiant flux). In the Universal System of Units Standard, the unit of luminous intensity is W_u/rad^2. The work equivalent of luminous flux thus becomes the dimensionless quantity, $K_m^{-1} = 0.002644_{(12)}$.
4. Dealing with supplementary units
   In the Universal System of Units Standard, the supplementary units are treated as units of clearly independent dimensioned quantities.
   (a) Plane angle is counted as one of the dimensions of the base unit.
   (b) Solid angle is regarded as the squared quantity of a plane angle (explained later).

   Furthermore, although the logarithmic quantity is ignored in the International System of Units (SI), in the Universal System of Units Standard, it is recognized as a base unit dimension. The same dimension is used for the quantity of information as well.

The only 'units' in the Universal System of Units Standard that have characteristic symbols are the 24 types that are listed below (base units that are natural units, supplementary constants that are not coherent but can be used according to natural units, defining constants, base units that are derived from the defining constants, derived units of dynamical quantities, and derived units of electromagnetic quantities – see Table 2). The constants that are classified as supplementary constants are not coherent with respect to natural units, but cannot be ignored for practical reasons.

[7] For now, I will use letters prefixed to the corresponding SI unit symbols to use in place of the new symbols required by the Universal System of Units Standard: n (natural prefix) or u (universal prefix). This is true for the following as well.

3.1 Natural units

Units of the following four dimensions are base units for which natural units are used just as they are. Concerning impedance, see Appendix B, "A method of organizing the dimensions of electromagnetic quantities"; concerning plane angle and logarithmic quantity, see Appendix A.2, " 'Mathematical' units".

Natural unit of impedance	—	$\Omega_n = \mathrm{sr}\sqrt{\frac{\mu_0}{\epsilon_0}} = 29.9792458\Omega(\mathrm{strict})$
Natural unit of plane angle	—	$\mathrm{rad} = \frac{2}{\pi}\sin^{-1} 1$
Natural unit of logarithmic quantity	—	$\mathrm{neper} = \log \mathrm{e}$
Natural unit of amount of substance	—	$\mathrm{mol}_n = N_A^{-1}$(inverse of the Avogadro constant)

3.2 Supplementary constants

The following four series of quantities are not coherent with respect to natural units, but they may be positioned as supplementary constants and used as units.

Elementary electrical quantity	$e = \sqrt{\alpha\hbar/\Omega_n}$	=	$1.0374\_43\mathrm{B}6 \times 10^{-14}_{(12)}\mathrm{C}_u$
Total solid angle of the surface of a hypersphere	$\Omega_k(k = 1, 2, ..)$	=	$\frac{2\pi^{\frac{k+1}{2}}}{\Gamma(\frac{k+1}{2})}\mathrm{rad}^k$
Logarithm of an integer	$\mathrm{B}_k(k = 1, z, ..)$	=	$\log_{\mathrm{e}} 2^k\mathrm{neper}$
Universal mole	$\mathrm{mol}_u = 10^{20}_{(12)}(N_A^{-1})$	=	$10^{20}_{(12)}\mathrm{mol}_n$

3.2.1 The elementary electrical quantity

Because the fine structure constant, α, is a dimensionless quantity, when the natural unit of impedance, Ω_n, is taken as a base unit, we cannot construct a coherent unit system with both the quantum of action, $\hbar$, and the elementary electrical quantity e as defining constants. The reason for not using the elementary electrical quantity e in place of the quantum of action, , in the defining constants is that the fine structure constant, α, should appear only in various quantities that represent the nature of an electron. [8]

To make it possible to use the elementary electrical quantity e as a unit, the elementary electrical quantity e is positioned as a supplementary constant. We take the elementary electrical quantity to be the positive value of the electrical quantity of an electron, so the sign is the opposite that of the International System of Units (SI).

3.2.2 Total solid angle of hyperspherical surfaces

The solid angle of a hypershperical surface is an extension of the concept of a plane angle into a multi-dimensional space, so the 'area' for when a section of unit 'area' on the surface of a sphere of unit radius is 'seen' from the center of the sphere is expressed as rad^k (the International System of Units (SI) sr when $k = 2$). Written as $rmrad^2$, it should be spoken as 'steradian'. Because the surface area of a sphere of radius r is $4\pi r^2$, the <u>total</u> solid angle of a sphere is $4\pi\mathrm{sr}(= \Omega_2)$.

In the Universal System of Units Standard, a hyperspherical solid angle is regarded as an integer power of a plane angle. The area, S, of a spherical square (a figure on the surface of a sphere that has four

[8] Of the supplementary constants, only the elementary electrical quantity contains in its definition a quantity that is obtained through measurement. I expect that the fine structure constant, too, may sooner or later become a mathematical constant that can be calculated strictly.

sides of equal length that meet at equal angles) that has sides of length θ is then

$$S = 4\text{sr}/\text{rad} \times \sin^{-1}\tan^2\frac{\theta}{2} \tag{18}$$

and so the solid angle, θ^2, for plane angle θ can be 'defined' as

$$\theta^2 = \lim_{n\to\infty} n^2 \times (\text{the solid angle of a spherical square which side length are } \frac{\theta}{n}) \tag{19}$$

(see Appendix A.3, "Coherent unit system").

By deriving solid angles from plane angle in this way, it is possible to avoid an unbounded increase in units when considering high-dimensional hyperspheres in general. The total solid angle of a k dimensional hypersphere, Ω_k, is [8]

$$\Omega_k = \frac{2\pi^{\frac{k+1}{2}}}{\Gamma(\frac{k+1}{2})}\text{rad}^k \tag{20}$$

In particular,

$\Omega_1 = 2\pi\text{rad}$ (which should be written as Ω_1 and spoken as "cycle". Also, $10^{-1}_{(12)}\Omega_1 = 30\text{degrees}$, $10^{-2}_{(12)}\Omega_1 = 2.5\text{degrees}$, $10^{-4}_{(12)}\Omega_1 \approx 1$ minute.)

$\Omega_2 = 4\pi\text{rad}^2$ (which shoud be written as Ω_2 and spoken as "turn".)

$\Omega_3 = 2\pi^2\text{rad}^3$

Solid angle becomes paired with impedance and takes on a symmetrical role in deriving 'field' quantities from 'charge' quantities for dimensions of electromagnetic quantities (see Appendix B, "A method of organizing the dimensions of electromagnetic quantities").

3.2.3 Logarithm of an integer

The same dimension is used for quantity of information as is used for logarithmic quantity. The reason is that because information is something that limits disorder, it is a quantity that is measured by the logarithm of the 'number of cases' that are limited by the information. The value k, which may actually be used as the base of logarithmic quantity and quantity of information, is assumed to be $1, z(=\log_2 12), 4, 8, \ldots$ (generally, except for z, integer powers of 2). $\text{B}_1 = \text{bit}, \text{B}_z = \text{digit}_{(12)}$. A semitone (half-step) of the 12-tone chromatic scale of music is expressed as $0.1_{(12)}\text{B}_1$.

3.2.4 Universal mole

Although there is a natural unit of amount of substance, the inverse of the Avogadro constant, $\text{mol}_n = N_A^{-1}$, the base unit of amount of substance in the International System of Units (SI), mol, is defined using the atomic mass unit, u and the unit of mass, g, as

$$\text{mol} = \frac{\text{g}}{u}N_A^{-1} \tag{21}$$

The quantity that is obtained in the same way in 'the Universal System of Units Standard' is the universal mole,

$$\text{mol}_u = \frac{\text{g}_u}{u}N_A^{-1} \tag{22}$$

The dimensionless quantity $\frac{\text{g}_u}{u}$ is essentially extremely close to $10^{20}_{(12)}$, [9] and with some degree of arbitrariness, may also be strictly $10^{20}_{(12)}$.

[9] That is to say, also, to a certain extent, close to $2^{72}_{(12)}$. $\log(\text{mol}_u/\text{mol}_n) = 72.057694_{(12)}\text{B}_1$.

3.3 Defining constants and base units

The defining constant of wavelength(R_∞) and the base unit of length	($\mathrm{m}_u = 10^6_{(12)}\Omega_1/R_\infty$ $= 27.21028842\mathrm{cm}$ $= 38999.753\mathrm{km}/(4 \times 12^7_{(10)})$)
The defining constant of speed(c_0) and the base unit of time	($\mathrm{s}_u = 10^8_{(12)}\mathrm{m}_u/c_0 = 390.2675219\mathrm{ms}$)
The defining constant of action($\hbar$) and the base unit of energy	($\mathrm{J}_u = 10^{26}_{(12)}\hbar/\mathrm{s}_u = 64.1433465\mathrm{mJ}$)
The defining constant of entropy(k_B) and the base unit of temperature	($\mathrm{K}_u = 10^{-18}_{(12)}\mathrm{J}_u/k_B = 1.211831\mathrm{K}$)

Derived units that have no characteristic symbol (examples)

area	(m_u^2	=	$740.39980\mathrm{cm}^2$		)
volume	(m_u^3	=	$20.146492\mathrm{dm}^3$		)
speed	($\mathrm{m}_u/\mathrm{s}_u$	=	0.697221442m/s	= 2.50999719km/h	)
frequency	(Ω_1/s_u	=	2.562344915Hz	= 30.748139Hz/12	)
molar concentration	$\mathrm{mol}_u/\mathrm{m}_u^3$	=	$6.552393\mathrm{mol/dm}^3$		)

The dimensions of the base units were selected by the process that is described at the beginning of this section. Next, it is necessary to select the defining constants from among the fundamental physical constants. From the definition of the Universal Unit System, the use of 'the speed of light in a vacuum', 'the quantum of action', and 'the Boltzman constant' is already settled. Although one more fundamental physical constant is needed for the definition of the remaining four base units, which are not natural units, that constant is selected from among 'the Rydberg constant', 'the atomic mass unit', 'the Bohr radius', and 'half the value of the Planck length'.

While 'the atomic mass unit' is by all means a desirable defining constant for the field of chemistry, there are many unsettled requirements, such as which chemical element to base it on and whether to select a particular nuclear species or to use an average of elements. Because any value within a certain range can be selected, this constant is not suitable as a defining constant of the Universal System of Units Standard.

The constant 'half the value of the Planck length' has a relative error of close to 1% with respect to the other three candidates and also has the practical problem of insufficient measurement accuracy for the constant.

'The Rydberg constant' and 'the Bohr radius' do not involve the vagueness of 'the atomic mass unit'. In addition, because these constants are deeply involved with the electron, they have the advantage that the fundamental physical constants that are closely related to the electron (charge, mass, the classical electron radius, and the Bohr magneton) as well as the Josephson constant, K_J, (of the Josephson effect) which is used in the standard representation of voltage, and the von Klitzing constant R_K (of the quantum hole effect), which is used in the standard representation of electrical resistance, can all be represented accurately if the fine structure constant can be strictly determined.

Using 'the Rydberg constant' for a defining constant, puts the derived unit of mass in the range (about equal to the mean mass of nucleons in the aluminum nucleus) where it can be used as 'the atomic mass unit' without modification. That cannot be said for 'the Bohr radius'. Also, 'the Rydberg constant' is related to optical measurements and is the only of the fundamental physical constants that are not dimensionless quantities that has a reproducible accuracy of more than 10 decimal places, and so is the most practical for use as a defining constant. Therefore, 'the Rydberg constant' was finally selected as the fourth defining constant. The base units comprise the base units just defined and the following base units that are created by means of new defining constants.

Finally, there remains the problem of how many integer powers of $10_{(12)}$ to append as a factor to give the base unit an appropriate. Concerning this problem, for $c_0 = 10^P_{(12)}\mathrm{m}_u/\mathrm{s}_u$, $e \approx 10^Q_{(12)}\mathrm{C}_u$,$u = 10^R_{(12)}\mathrm{g}_u$, we adopt the approach of selecting the multiple of integer powers of $10_{(12)}$ so as to make the greatest common divisor of P, Q, and R as large as possible. Currently, the greatest common divisor is 8, and as far as selecting a base unit of appropriate scale is concerned, this seems to be the maximum value (because of the relationship $e = \sqrt{\frac{\alpha\hbar}{\Omega_n}}$, there are surprisingly large restrictions on selecting an appropriate scale. See Table 2's comment column).

3.4 Derived units of dynamical quantities

Derived units of mass	(g_u	=	$\mathrm{J}_u\mathrm{s}_u^2/\mathrm{m}_u^2$	=	131.950228g	=	$19.000833\mathrm{kg}/12^2_{(10)}$)
Derived unit of work	(W_u	=	$\mathrm{J}_u/\mathrm{s}_u$	=	164.357378mW	=	112.256089lm)
Derived unit of force	(N_u	=	$\mathrm{J}_u/\mathrm{m}_u$	=	235.731961mN	=	24.037970gf)
Derived unit of pressure	(P_u	=	$\mathrm{J}_u/\mathrm{m}_u^3$	=	3.18384692Pa	=	$\mathrm{atm}/1\_6500_{(12)}$)

Derived units that have no characteristic symbol (example)

Torque ($\mathrm{J}_u/\mathrm{rad} = 64.1433465\mathrm{mN}\cdot\mathrm{m/rad}$)

Taking a quantity of the dimension energy as a base unit, there is probably no objection to the selection of these units. As we can see, the unit of mass is $\mathrm{g}_u = 131.950228\mathrm{g}$. On the other hand, the relationship between the unit of 'amount of substance' of the International System of Units, mole (from the measured value of the Avogadro constant), and the supplementary constant, mol_u, of the Universal System of Units Standard is $\mathrm{mol}_u = 132.007729\mathrm{mol}$. It would be surprising if this were a completely accidental coincidence. This represents the fact that the atomic mass unit can be approximated with good accuracy at an appropriate scale.

3.5 Derived units of electromechanical quantities

Derived unit of electrical quantity	(C_u	=	$\sqrt{\mathrm{J}_u\mathrm{s}_u\Omega_n^{-1}}$	=	28.8965943mC	)
Derived unit of electrical current	(A_u	=	$\sqrt{\mathrm{J}_u\mathrm{s}_u^{-1}\Omega_n^{-1}}$	=	74.0430416mA	)
Derived unit of field strength	(O_u	=	$\mathrm{A}_u/\mathrm{m}_u$	=	272.114137mA/m	)
Derived unit of flux density	(G_u	=	$\mathrm{C}_u/\mathrm{m}_u^2$	=	$390.283662\mathrm{mC/m}^2$	)

Derived units that have no characteristic symbol (examples)

Magnetic pole	$(\mathrm{C}_u\Omega_n/\mathrm{rad}^2$	$=$	$10.8862230\mathrm{Wb}/\Omega_2$	)
Magnetic flux	$(\mathrm{C}_u\Omega_n = \sqrt{\mathrm{J}_u\mathrm{s}_u\Omega_n}$	$=$	$0.86629810\mathrm{Wb}$	)
Magnetic potential	$(\mathrm{A}_u\mathrm{rad}^2$	$=$	$5.8921580\mathrm{mA}\Omega_2$	)
Magnetic field strength	$(\mathrm{O}_u\mathrm{rad}^2$	$=$	$21.6541550\mathrm{mA}\Omega_2/\mathrm{m}$	)
Magnetic flux density	$(\mathrm{G}_u\Omega_n$	$=$	$11.7004098\mathrm{T}$	)
Inductance	$(\mathrm{s}_u\Omega_n$	$=$	$11.6999260\mathrm{H}$	)
Magnetic permeability	$(\Omega_n/\mathrm{rad}^2 c_0$	$=$	μ_0	)
Electric flux	$(\mathrm{C}_u\mathrm{rad}^2$	$=$	$2.2995179\mathrm{mC}\Omega_2$	)
Electrical potential	$(\mathrm{A}_u\Omega_n = \sqrt{\mathrm{J}_u\mathrm{s}_u^{-1}\Omega_n}$	$=$	$2.2197545\mathrm{V}$	)
Electric field strength	$(\mathrm{O}_u\Omega_n$	$=$	$8.1577766\mathrm{V/m}$	)
Electric flux density	$(\mathrm{G}_u\mathrm{rad}^2$	$=$	$31.0577870\mathrm{mC}\Omega_2/\mathrm{m}^2$	)
Electrical capacitance	$(\mathrm{s}_u/\Omega_n$	$=$	$13.0179233\mathrm{mF}$	)
Permittivity	$(\mathrm{rad}^2/\Omega_n c_0$	$=$	ϵ_0	)

Because a natural unit is used as the unit of impedance, the unit of electrical quantity(charge) is derived from units that are already defined. Looking at the formula for the force between electrical quantities in Appendix B, "A method of organizing the dimensions of electromagnetic quantities", we have

$$\frac{\text{energy}}{\text{length}} = \text{impedance}\frac{\text{length}}{\text{time}}\frac{\text{charge}^2}{\text{length}^2} \tag{23}$$

Solving this equation for charge, the dimension of electrical quantity(charge) is

$$\text{charge} = \sqrt{\frac{\text{energy} \times \text{time}}{\text{impedance}}} \tag{24}$$

The constant of proportionality of Coulomb's law is represented by the product of the natural units of impedance and the speed of light in a vacuum, which is a feature of the set of formulas that are employed in the Universal System of Units Standard (see Appendix B, "A method of organizing the dimensions of electromagnetic quantities"). We should note the symmetry of electrical quantities and magnetic quantities. Because of that symmetry, made it possible to not assign a symbol to the unit of electrical potential.

In the Universal System of Units Standard, the use of supplementary constants is permitted as an exception to coherence, so either rad^2 or the supplementary constant Ω_2 may be used as the unit of solid angle (in computations, however, it is necessary to use either one or the other unit). The conversion value for International System of Units (SI) that uses rad^2 is shown above because it happens that the International System of Units (SI) is coherent with Ω_2. For the conversion value for when Ω_2 is used, the derived units of charge, electrical current, flux density, and field strength may be used as they are. The so-called rationalized units are coherent with Ω_2 and the non-rationalized units are coherent with rad^2.

4 Summary of the units of the Universal System of Units Standard

Summarizing the above, the units of the Universal System of Units Standard that have characteristic symbols are listed in Table 2.

Table 2: The units of the Universal System of Units Standard that have characteristic symbols

Category	Dimension / Item	Symbol	Value		Comment			
	wave number	R_∞				$\equiv$	12^{-2}	Ω_1/D
Defining constants	speed	c_0						
	action	$\hbar$						
	entropy	k_B						
Non-coherent	elementary electrical quantity	e						
supplementary	total solid angle of a hypersphere	Ω_k	$\Omega_1 = 2\pi\mathrm{rad}$,	$\Omega_2 = 4\pi\mathrm{sr}$	k	$=$	$1, 2, ..$	
constants	logarithm of an integer	B_k	$\mathrm{B}_1 = \mathrm{bit}$,	$\mathrm{B}_z = \mathrm{digit}_{(12)}$	k	$=$	$1, z, ..$	
	universal mole	mol_u	132.007729	mol				
Base units	impedance	Ω_n	29.9792458	Ω				
that are	plane angle	rad	57.2957795	degree				
natural units	logarithmic quantity	neper	4.34294482	dB				
	quantity of substance	mol_n	1					
Base units	length	m_u	27.21028842	cm	12^8	$\times$	1	D
that are not	time	s_u	390.2675219	ms	12^{16}	$\times$	1	D/c_0
natual units	energy	J_u	64.1433465	mJ	12^{16}	$\times$	12^{-2}	$\hbar c_0/D$
	thermodynamic temperature	K_u	1.211831	K	12^{-4}	$\times$	12^{-2}	$\hbar c_0/Dk_B$
	mass	g_u	131.950228	g	12^{32}	$\times$	12^{-2}	$\hbar/c_0 D$
Derived units of	work	W_u	164.357378	mW	1	$\times$	12^{-2}	$\hbar c_0^2/D^2$
dynamical quantities	force	N_u	235.731961	mN	12^8	$\times$	12^{-2}	$\hbar c_0/D^2$
	pressure	P_u	3.18384692	Pa	1	$\times$	12^{-2}	$\hbar c_0/D^3$
	charge	C_u	28.8965943	mC	12^{16}	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_\mathrm{n}}$
Derived units of	electrical current	A_u	74.0430416	mA	1	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_\mathrm{n}} c_0/D$
electromagnetic quantities	field strength	O_u	272.114137	mA/m	12^{-8}	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_\mathrm{n}} c_0/D^2$
	flux density	G_u	390.283662	$\mathrm{mC/m^2}$	1	$\times$	12^{-1}	$\sqrt{\hbar/\Omega_\mathrm{n}}/D^2$

A Basic approach to units

(This Appendix is part of the Universal System of Units Standard.)

A.1 Classification of quantities

In the Introduction, the term 'of the same type' (the same dimension, in the terminology of units) was introduced out of the blue. Actually, however, there is no standard for objectively distinguishing whether or not quantities are of the same type. Whether or not quantities are of the same type should be determined by agreement. It should be noted that what we can measure directly is limited to pure numbers. What we call 'measuring length' is actually no more than reading the numbers on the scale of a ruler.

For example, it wouldn't matter at all if length in the vertical direction (height) and length in the horizontal direction (horizontal distance) were represented with different units. Actually, the height of Mt. Everest[10] is not expressed as 8.848 km; nor is the distance of a marathon course expressed as 42,195 m. This can be said to show that height and horizontal distance are recognized as different types of quantities.

The type of a quantity, however, is not entirely arbitrary. Where arbitrariness enters is for the most part in the decision to classify quantities coarsely or in detail. These concepts of quantity are actually defined axiomatically within a network of natural laws. In other words, the concepts of quantity can be seen as defined by the formulas that express natural laws themselves. In natural laws, there is no need for humans to distinguish between height, horizontal distance and other such quantities, so they are all lumped together in the category of length.

A.2 'Mathematical' units

A 'mathematical' unit is a suitable example when one is considering the classification of quantities.

In the following, I attempt a discussion of 'mathematical' units from the viewpoint that they are different from pure numbers.

Because a unit is "a quantity of the same type that serves as a standard for measuring and representing a given quantity", it is also possible discover units when we restrict ourselves to mathematical objects rather than the objects of physics.

For example, $\log_{10} 2$ is a pure number that has the value 0.3010... . So, then, (from the beginning, without omission) let's introduce the baseless logarithm $\log 10$. By axiomatically defining addition, subtraction, multiplication, and division, this is easily made an object of mathematical consideration. In this case, $\log 10$ becomes the unit for the quantity 'baseless logarithm', and can be used as follows.

$$\log 2 = 0.3010..\log 10 \tag{25}$$

The two sides of this equation are the quantity 'baseless logarithm' and cannot be reduced to numbers.

The baseless logarithm probably does not appear anywhere else, but what results if we replace this log with $\sin^{-1}$? In the theory of analytic functions of complex variables, logarithmic functions and inverse

[10] Tibetan name is Chomo Lūngma.

trigonometric functions are the same type of thing. [11] Actually, this is none other than 'the plane angle' that we know so well.

That is to say, taking the principal values,

$$\begin{aligned} \text{rad} &= \frac{2}{\pi}\sin^{-1}1 && (26) \\ \text{degree} &= \frac{1}{90}\sin^{-1}1 && (27) \end{aligned}$$

It is clear that these relationships are completely parallel to Eq. (25).

In this way, it is possible to assume units that cannot be reduced to pure numbers for entirely mathematical quantities as well. Of course, no conflict results if 'mathematical' units are regarded as pure numbers as they are ordinarily considered. This does not mean that one viewpoint or the other is the correct one, but rather that this is a problem that should be settled by agreement. In this paper, we take the position that 'mathematical' units are a kind of unit that cannot be reduced to a pure number without being divided by the same type dimensioned quantities.

In the International System of Units (SI), the issue of whether plane angle is a base unit or a derived unit involving length divided by length (pure number) had not been settled. Previously, a separate classification referred to as supplementary units was established. I quote from the SI document, "Le Système International d'Unités " (2nd Ed., 1973), as translated by the Japanese National Research Laboratory of Metrology.[12]

"Although it is possible to consider an SI unit to be either a base unit or a derived unit, the 11th Conférence Générale des Poids et Mesures (1960) recognized a third class of units referred to as supplementary units. It was thus not settled whether supplementary units are base units or derived units".

"The General Conference on Weights and Measures did not settle (or, rather, has not yet settled) the matter of whether a number of SI units belong to the category of base units or the category of derived units. Those SI units are placed in a third class of units that is referred to as 'supplementary units'. One is free to choose whether to treat the supplementary units as base units or derived units".

A.3 Coherent unit system

A unit system in which a number of base units and equations that express relationships that describe natural laws are used to define ('derive', in the terminology of units) all other units in the system is called a coherent unit system.

In a coherent unit system, there is only one unit for each quantity. Thus, a coherent unit system is one for which that group of defining relationship equations is the simplest (specifically, this is to say that the equations have the simplest set of coefficients). As can also be understood from the fact that "the concept of quantity is defined by the formulas that represent natural laws" as described in the previous section, the term coherence begins with the specification of the set of relationship equations that describe natural laws.

For example, denoting the area of a triangle as S, the length of the base as a, and the height as h, we

[11] $\sin^{-1}x = -\imath\log(\imath x \pm \sqrt{1-x^2})$.

[12] In a later revision, the category of supplementary units was removed.

calculate

$$S = \frac{1}{2}ah \tag{28}$$

However, this is really very clear?

$$S = ah \tag{29}$$

Is there a problem with writing it as follows? [9]

Actually, even if written as Eq. (29), all of the coefficients of the formula that expresses the area are simply doubled, so no logical problem arises. The area of a circle is expressed as $2\pi r^2$, but if we think of this as "length of perimeter × radius", it would be more natural than our formula. This coefficient of 1/2 seems to have been handed down from heaven, but actually it is simply a human convention.

As we can see from this example, when we denote the unit of length as m, although it is convenient to attach the label of m^2 to the coherent unit of area, there is no concrete specification that definition cannot be decide unless a corresponding relationship equation is specified. This is not limited to area, but is clearly true for all derived units.

Of course, Eq. (29) is intentionally and extreme example, and so may not be suitable for use in practice. In mathematics, however, it is an everyday occurrence to have quantities of the same concept, but of the opposite sign and differing by a factor of 2π. Also, in electromagnetics, the fact that the same coherent metric system of units can have units of the same quantity that differ by a factor of 4π in rationalized units and non-rationalized units is indeed this same kind of phenomenon.

The concept of unit coherence is extremely important, but not absolute. Even the International System of Units (SI), which features coherence of units, is filled with problems.

1. Celsius temperature
   In the International System of Units (SI), Celsius temperature is defined in the following way, with ℃ classified as a derived unit.
   "Celsius temperature, t, is defined as t $= T - T_0$, the difference between the two thermodynamic temperatures T and T_0, where $T_0 = 273.15\mathrm{K}$. The temperature interval or temperature difference may be expressed using either Kelvin or Celsius degree. The unit 'Celsius degree' is equal to the unit 'Kelvin'. "
   Because a unit is "a quantity <u>of the same type</u> that serves as a standard for measuring and representing a given quantity", K and ℃ are algebraically the same. Thus there are two units for the quantity whose dimension is temperature, which is still an exception to the principle of one unit for one quantity, even if the ratio of the two units is 1.
   From the definition of Celsius temperature it is correct to say that "the normal human body Celsius temperature is 37.00K", but that the converse expression, "the normal human body temperature is 37.00 ℃", is incomplete to surely understand.[1]
2. Frequency
   Hertz (Hz) is a unit of frequency that is defined as an inverse of seconds, but carries the warning that it should only be used for periodic phenomena. For example, wind speed must be represented as 30m/s, not as 30 mHz. Because frequency is a quantity that has the dimension [period number(=phase)/time], according to the principle of coherence, it should be Hz = rad/s. Assuming from various formulas, however, it is Hz = Ω_1/s, with $\Omega_1 = 2\pi\mathrm{rad}$. If a unit is not coherent, it is very natural that its range of use is limited.

As can be understood from the above examples, coherence is something that is rather difficult to accomplish. There are also times when the coherence of units should be sacrificed in order to reduce the number of formulas. Another way we can put this is that multiple units can be recognized for the same type of quantity in order to reduce the number of concepts.

Let's consider the decay of elementary particles.

An elementary particle's lifetime is the mean time until decay, and can also be said to be the time until the number of particles becomes 1/e due to decay. The half-life, on the other hand, is the time until the number of particles becomes 1/2 due to decay. We consider these two to be quantities of different concepts that both have the dimension time. However, by recognizing the two units $\log \mathrm{e} (= \text{neper})$ and $\log 2 (= \mathrm{B}_1)$ for the logarithmic quantity, these can be interpreted as a single quantity (dimension: time/logarithmic quantity) that represents the slowness of decay with two units (for example, a half-life of 7 seconds represents roughly the same thing as a lifetime of 10 seconds. Therefore, this can be expressed as "the decay slowness is 7 seconds/ B_1" or "the decay slowness is 10 seconds/neper").

The same can be said for frequency and angular frequency as well. The relationship of the quantum of action, $\hbar$, and the Planck constant, h, should be considered to be as follows.

$$h = \hbar/\mathrm{rad} = 2\pi\hbar/\Omega_1 \tag{30}$$

Of course, this kind of lack of coherence should be limited to cases in which the ratios of multiple units can be strictly determined, such as they can be for 'mathematical' units. [13]

[13] This is also why it is desirable that the fine structure constant is a mathematical constant that can be strictly calculated.

B A method of organizing the dimensions of electromagnetic quantities

(This Appendix is part of the Universal System of Units Standard.[10])

B.1 Introduction

It has been suggested that one reason that electromagnetism is difficult to understand is the complexity of the unit system. Some of the unit systems that have been proposed in the past are listed in Table 3, but because the International System of Units (SI) based on the MKSA system of units has recently become widely adopted, the kind of confusion seen in the past has disappeared. Although the era of proposing unit systems for the real world has ended, and the viewpoint of reorganizing the relations between rationalized unit systems and non-rationalized unit systems and the relations between ternary unit systems and quaternary unit system can be considered educationally significant even at this time. In this paper, we take the position of regarding solid angle as a physical quantity that has an independent dimension, consider a reorganization of the relationships among various unit systems and dimensions of electromagnetic quantities. Although this is necessary for a reorganization of the relationship between rationalized unit systems and non-rationalized unit systems, one can understand that it is also useful for reorganizing the relationship between the dimensions of electromagnetic quantities as shown in Figure 1. (This standpoint does not conflict with the International System of Units (SI). Strangely, however, according to Table 3[9] this has not been discussed deeply in the past.)

Table 3: Unit systems that have been proposed in the past

No. of dimensions	Name	Physical quantities that have independent dimensions
3	CGS electrostatic	Length, mass, time
	CGS electromagnetic	Length, mass, time
	CGS Gaussian	Length, mass, time
4	CGS-Fr	Length, mass, time, and electrical quantity
	CGS-Bi	Length, mass, time, and electrical current
	MKSμ	Length, mass, time, magnetic permeability
	MKSϵ	Length, mass, time, permittivity
	MKVA	Length, mass, voltage, electrical current
	MKSΩ	Length, mass, time, and electrical resistance
	MKSC	Length, mass, time, electrical quantity
	MKSA	Length, mass, time, electrical current
	VAMS	Voltage , electrical current, length, time
5	LMTQP*	Length, mass, time, electric flux, and magnetic flux
	LMTIϕ*	Length, mass, time, electrical current, and magnetic flux
	LMTIγ	Length, mass, time, electrical current, and electric and magnetic coupling coefficient
	LMT$\epsilon\mu$	Length, mass, time, permittivity, and magnetic permeability

B.2 Introducing the solid angle

The 4π difference that appears in the coefficients of the formulas of rationalized unit systems and non-rationalized unit systems is, as is well known, a geometrical value. What demonstrates the origin of that in the most straightforward way is probably Gauss' theorem (integration form).

Rationalized unit system Non-rationalized unit system

$$\int\int \mathbf{D}\cdot\mathbf{n}dS = Q \qquad \int\int \mathbf{D}\cdot\mathbf{n}dS = 4\pi Q$$

In a rationalized unit system, the unit electric flux is considered to be the electric flux created by the unit electrical point charge in all of space; in a non-rationalized unit system the unit electric flux is considered to be the electric flux created by the unit electrical point charge in 1 steradian (sr). Therefore, if we rewrite this taking solid angle to be an independent dimension, we have

$$\int\int \mathbf{D}\cdot\mathbf{n}dS = \Omega_2 Q \tag{31}$$

Here, Ω_2 is the total solid angle of a sphere. Performing a dimension analysis considering $\mathbf{D} = \epsilon_0\mathbf{E}$, and using the constant Ω_n, which has the dimension of impedance, and the speed of light in a vacuum c_0,

Permittivity of a vacuum $\epsilon_0 = \dfrac{\mathrm{sr}}{\Omega_n \cdot c_0}$

Magnetic permeability of a vacuum $\mu_0 = \epsilon_0^{-1}c_0^{-2} = \dfrac{\Omega_n}{\mathrm{sr}\cdot c_0}$

Characteristic impedance of a vacuum $Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = \dfrac{\Omega_n}{\mathrm{sr}} = 4\pi\dfrac{\Omega_n}{\Omega_2}$

We must note that the dimension differs for the ratio of voltage and electrical current [Ω] and the ratio of electric field and magnetic field [Ω/sr].

B.3 Formula set that takes solid angle into consideration

In the following, I regard solid angle as an independent dimension and try to rewrite the set of formulas of electromagnetism. We can confirm that Ω_2 appears in the places where it should appear, geometrically (see section B.4). The rationalized unit system is a unit system in which Ω_2 is regarded as the pure number 1; the non-rationalized unit system is a unit system in which sr is regarded as the pure number 1. Comparing the formula for the force between electrical currents and the definitions of meter and ampere, we get $\Omega_n = 29.9792458\Omega$ (strict).

Force between electrical quantities	$f = \frac{1}{\epsilon_0}\frac{\Omega_2 Q}{4\pi r^2}Q' = \Omega_n c_0 \frac{QQ'}{r^2}$
Force between electrical currents	$df = \mu_0 \frac{\Omega_2 I}{2\pi r}I' = \frac{2\Omega_n}{c_0}\frac{II'}{r}$
Lorentz force	$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
Energy density of an electromagnetic field	$u = \frac{1}{2\Omega_2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})$
Poynting vector	$\mathbf{S} = \frac{1}{\Omega_2}\mathbf{E}\times\mathbf{H}$
Electromagnetic induction law	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
	$\nabla\times\mathbf{H} = +\frac{\partial\mathbf{D}}{\partial t} + \Omega_2\mathbf{J}$
Gauss' theorem (differential form)	$\begin{cases}\nabla\cdot\mathbf{D} = \Omega_2\rho \\ \nabla\cdot\mathbf{B} = 0\end{cases}$
Charge conservation law	$\nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$
Scalar potential	$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}$
Vector potential	$\mathbf{B} = +\nabla\times\mathbf{A}$
Equation that satisfies the potential	$\begin{cases}\Delta\phi - \epsilon_0\mu_0\frac{\partial^2\phi}{\partial t^2} = -\Omega_2\frac{\rho}{\epsilon_0} \\ \Delta\mathbf{A} - \epsilon_0\mu_0\frac{\partial^2\mathbf{A}}{\partial t^2} = -\Omega_2\mu_0\mathbf{J}\end{cases}$

B.4 Relationships among the dimensions of electromagnetic quantities

By performing a dimension analysis based on the set of equations described above, a diagram that illustrates the relations among the dimensions of the electromagnetic quantities can be constructed (Figure 1). The quantities that are related to 'charge' lie in the middle, with the quantities that are related to electric 'field' and magnetic 'field' arranged symmetrically on either side. Also impedance and solid angle take on symmetrical roles in generating the 'field' quantities from the 'charge' quantities. The concepts that are to be distinguished are arranged so that their dimensions are all mutually different, and the electromagnetic quantities, which are complex at first sight, are seen to be orderly and systematic.

A result of the dimension analysis is that the magnetic potential equals the product of the electrical current and solid angle, as shown in Figure 1. The geometrical grounds for that are explained below.(11)

Assuming that.

1. the principle of superimposition is established for the magnetic potential and
2. the magnetic potential is 0 when a circuit is seen directly from the side,

the magnetic potential at viewpoint O in Figure 2 is the sum of the magnetic potentials due to the three sides, which is to say 0. On the other hand, this is also the sum of the magnetic potentials due to circuit ABC and circuit FED.

Accordingly, we can say that the magnetic potential due to a triangular circuit is proportional to the product of the current that is flowing in the circuit and the solid angle that the circuit makes. Because any circuit can be represented by a combination of triangles, the same is also true for any circuit.

The solid angle made by one trip around the circuit from the viewpoint varies only with Ω_2. n trips around a one-turn circuit and one trip around an n-turn circuit are equivalent in terms of phase geometry, so the 'turn' of ampere-'turn' can be regarded as Ω_2.

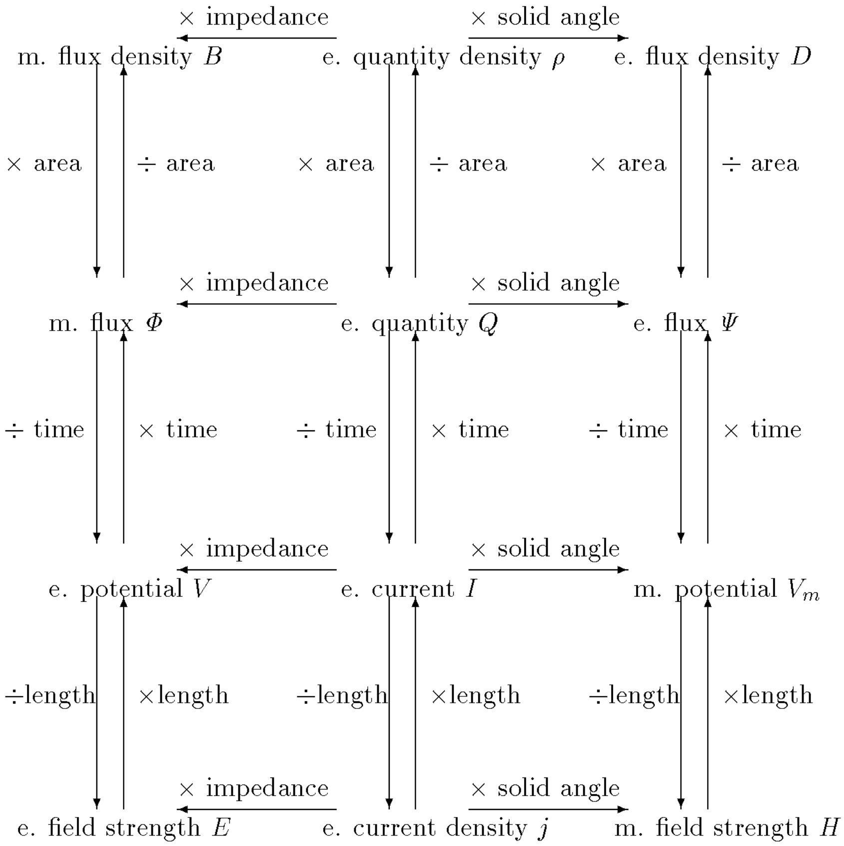

Figure 1: Relationships among the dimensions of electromagnetic quantities

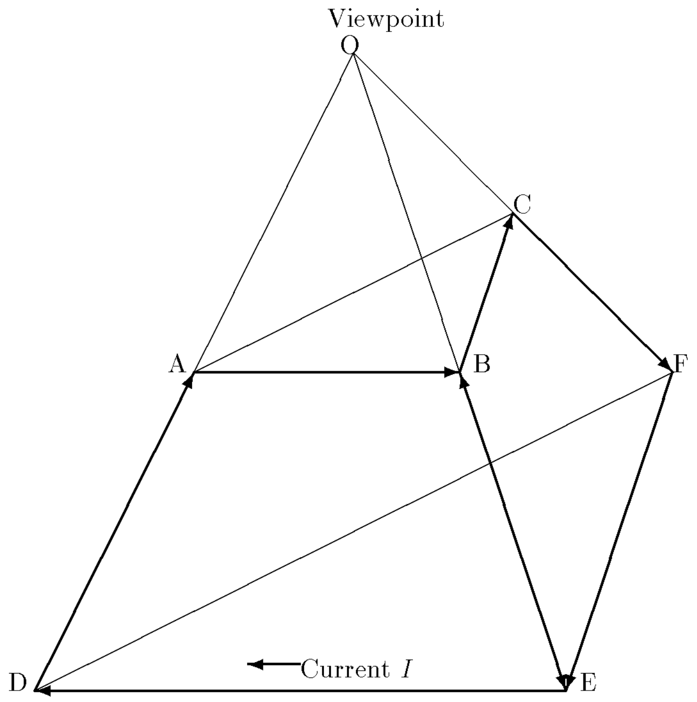

Figure 2: Explanation of magnetic potential

B.5 Electrostatic, electromagnetic and symmetrical unit systems

Looking at Figure 1, deriving the dimension of impedance from length and time, we can see that it is possible to reduce the number of independent dimensions by one by reorganizing the dimensions of electromagnetic quantities.(12) Electrostatic, electromagnetic, and symmetrical unit systems can be positioned as unit systems for which this kind of reorganization has been carried out.

1. Electrostatic unit system
   This is the unit system in which Ω_n is set to c_0^{-1} so that the coefficient on the right side of the formula for the force acting between electrical quantities becomes the pure number 1, and the $(\Omega_n c_0)^{1/2}$ multiples of the quantities of the center column and right-hand column in Figure 1 become the newly defined quantities of the center column and right-hand column, and the $(\Omega_n c_0)^{-1/2}$ multiples of the quantities of the left column become the newly defined quantities of the left-hand column. Thus, the electric flux density/electric field strength = solid angle. The formula set of the electrostatic unit system is the formula set of section B.3 in which $\epsilon_0 = \text{sr}$. Thus, if sr is set to the pure number 1, then ϵ_0 becomes the pure number 1.
2. Electromagnetic unit system
   This is the unit system in which Ω_n is set to c_0 so that the coefficient on the right side of the formula for the force acting between electrical currents becomes the pure number 2, and the $(\Omega_n/c_0)^{1/2}$ multiples of the quantities of the center column and right-hand column of Figure 1 become the newly defined center and right-hand column quantities, and the $(\Omega_n/c_0)^{-1/2}$ multiples of the quantities of the left-hand column become the newly defined left-hand column quantities. Thus, the magnetic

flux density/magnetic field strength = solid angle^{-1}. The formula set of the electromagnetic unit system is the formula set of section B.3 in which $\mu_0 = \mathrm{sr}^{-1}$. Thus, if sr is set to the pure number 1, then μ_0 becomes the pure number 1.

3. Symmetrical unit system
   This is the unit system in which, in the electrostatic unit system, the magnetic flux and magnetic flux density are replaced by the c_0 multiples of the magnetic flux and magnetic flux density and the magnetic potential and magnetic field strength are replaced by the c_0^{-1} multiples of the magnetic potential and magnetic field strength, [14] so that electric flux density/electric field strength = solid angle and magnetic flux density/magnetic field strength = solid angle^{-1}. The formula set of the symmetrical unit system is the formula set of section B.3 in which $\epsilon_0 = \mathrm{sr}$ and A and B are everywhere replaced by $c_0^{-1}A$ and $c_0^{-1}B$, and H is everywhere replaced by $c_0 H$. Thus, if sr is set to the pure number 1, then ϵ_0 and μ_0 become the pure number 1.

B.6 Conclusion

The positioning of the existing unit systems, when starting from the standpoint of regarding solid angle as a physical quantity that has an independent dimension is summarized in Table 4. According to that positioning, the relationship between the rationalized unit system and the non-rationalized unit system and the relationship between the ternary unit systems and the quaternary unit system are concisely organized. In addition, the understanding of the relationships among the dimensions of the electromagnetic quantities brought about by Figure 1 is probably useful from an educational standpoint.

Table 4: Positioning of the existing unit systems

No. of dimensions	Name	Position
3	CGS electrostatic	sr = the pure number $1, \Omega_n = c_0^{-1}$
	CGS electromagnetic	sr = the pure number $1, \Omega_n = c_0$
	CGS Gaussian symmetrical	sr = the pure number $1, \Omega_n = c_0^{-1}$, However the upper left and lower right of Figure 1 are corrected by c_0
4	MKSA system	Ω_2 = the pure number 1

[14] That is to say, dimensioned quantities of the electromagnetic unit system are used for these.

C Gravitation

(This Appendix is part of the Universal System of Units Standard.)

C.1 The gravitational constant and the gravity field equations

When representing the mass of a celestial body by means of the Universal System of Units Standard, the gravitational radius (half the Schwarzshild radius) is used rather than using mass directly. Because the accuracy of measuring the Newton constant is poor, representing the mass of a celestial body directly in terms of mass results in poor accuracy, but the gravitational radius can be measured to an accuracy of more than 10 decimal places. The reason for the poor accuracy of measuring the Newton constant is that the quantities that are required for astronomical calculations almost always appear in the form of the product of the Newton constant and mass and the Newton constant seldom appears alone, so it is difficult to construct observations and experiments for measuring the bare Newton constant with high accuracy. The gravitational radius has an appropriate scale and so is convenient.[13]

If we define a quantity that has the dimension 'force' as 'the gravitational constant', there is a good chance that the geometrical parts can be separated from the coefficients in the formula. Make the gravitational constant $N_G = c_0^4 G^{-1} = \frac{c_0 \hbar}{4\alpha l_P^2} = 2\text{A.B33B} \times 10^{34}_{(12)}\mathrm{N}_u$, then

$$\begin{aligned}
\text{gravitational radius } r_m &= \frac{Gm}{c_0^2} = \frac{mc_0^2}{N_G}(\text{ half the Schwarzschild radius}) && (32)\\
\text{gravitational force } f &= N_G \frac{r_m r_{m'}}{r^2} = c_0^2 \frac{r_m m'}{r^2} && (33)\\
\text{gravitational acceleration } g &= c_0^2 \frac{r_m}{r^2} = \frac{r_m}{(r/c_0)^2} && (34)\\
\text{gravity field equation } T_{ik} &= \frac{N_G}{2\Omega_2}(R_{ik} - \frac{1}{2}\delta_{ik} R) && (35)
\end{aligned}$$

In the gravity field equation (35), because R is the curvature tensor, it has the dimension of solid angle/area. T, on the other hand, is the energy-momentum tensor, and so has the dimension of energy density. Therefore, the denominator of the coefficient $\frac{N_G}{2\Omega_2}$ must be the dimension of solid angle (It is interesting to compare this to the equation for the energy density of an electromagnetic field).

C.2 The Planck length

The Planck length, a distinguishing feature of superstring theory, can also be represented in well-bounded form in the Universal System of Units Standard.

That is to say, half the value of the Planck length is

$$\frac{1}{2}\sqrt{\frac{G\hbar}{c_0^3}} = 1.022031 \times 10^{-28}_{(12)}\mathrm{m}_u \qquad (36)$$

In order to represent the tensile force in a superstring, half the value of the Planck length, l_P, adjusted by the fine structure constant, α,[2] becomes

$$l_P = \frac{1}{2}\sqrt{\frac{G\hbar}{c_0^3 \alpha}} = 0.\text{BA70BB} \times 10^{-27}_{(12)}\mathrm{m}_u \qquad (37)$$

D Units outside the Universal System of Units Standard

(This Appendix is for reference only. It is not part of the Universal System of Units Standard.)

D.1 Time units based on the earth's rotation

universal minute	=	$100.17_{(12)}\mathrm{s}_u$	(strict)
clock	=	$10_{(12)}$ universal minutes	
day	=	128 clocks	
year	=	365 days and 31 clocks	
universal century	=	64 years	
	=	$10\_0513\_16A2.8_{(12)}\mathrm{s}_u$	

The time units can be seen as both as units of time and as units of the angle of rotation of the earth in space.

For activities on the earth, year and day cannot be ignored as time units. However, year and day cannot be expressed as s_u multiples of integer powers of 12, nor is their ratio a simple value. Therefore, are the following possible within small integer powers?

1. The ratio of the largest unit of local earth time and s_u is approximately an integer power of 12.
2. The ratio of the largest unit of local earth time and year is exactly an integer power of a certain integer n.
3. The ratio of the smallest unit of local earth time and day is exactly an integer power of same integer n.
4. The ratio of the smallest unit of local earth time and s_u is approximately an integer power of 12.

Actually, for $n = 2$, this kind of unit can be constructed within small integer powers.

universal century	=	the largest unit of local earth time	=	2^6 years	$\approx$	$12^9\mathrm{s}_u$
clock	=	the smallest unit of local earth time	=	2^{-7} days	$\approx$	$12^3\mathrm{s}_u$

The relations of clock to day and year to universal century are completely binary, but clock and universal century are both approximately s_u multiples of integer powers of 12, [15] and so connect smoothly to the Universal System of Units Standard. People have a proclivity for using large units for large quantities and using small units for small quantities, so the inconvenience of having to exclude year and day from the Universal System of Units Standard is reduced by connecting both units smoothly to the Universal System of Units Standard.

Accidentally one clock is equal to the difference between one Julian year and one tropical year. For the earth at this time, the relationship

$$1 \text{ tropical year} = 365\frac{2^5-1}{2^7} \text{ mean sun days} \tag{38}$$

holds to a high degree of accuracy (error on the order of 10^{-8}). Moreover, not only are leap years every 2^2 years and leap year corrections every 2^7 years, for integer power of 2 years, there is some interesting coincidences, as shown in Table 5. Therefore, if we make one universal century 64 years, the same positional relationship among the sun, Venus, the earth and Mars recurs successive universal centuries (because the rotation of Venus is in the reverse direction of the revolution, in 8 earth tropical years, Venus makes exactly 25 rotations with respect to the sun).

[15] Because 12 is $2^2 \times 3$, All factor 3 appeares between year and day as a factor of the ratio of a universal century and

Table 5: The revolution of the earth and Venus and Mars

Planet	Rotations and revolutions	Earth mean sun days	Earth tropical years	Power of two
Venus	3 rotations	729.06 days	1.9961 years	1
	13 revolutions	2921.16 days	7.9979 years	3
Mars	17 revolutions	11678.77 days	31.9754 years	5

The ratios of s_u to clock is made to become strict; the error with respect to the actual rotation of the earth is adjusted at the end of each universal century by means of one negative leap clock (from the current trend, one clock will be deleted just about every universal century, and only to that extent will no deviation arise). In the far future the error with respect to the actual ratio of tropical year to day will be adjusted by omitting a leap day at the end of each universal century.

D.1.1 Calendar 1

A normal year has 365 days(=30+31+30+31+30+31+30+31+30+31+30+30 days) and a leap year has 366 days(=30+31+30+31+30+31+30+31+30+31+30+31 days). The leap days are inserted at the end of every 4 years except at the end of every 2 universal centuries. Four universal centuries have 93502 days. This length is almost equal to 13 Mayan katuns(=93600days). If we make the epoch of 'the Universal Unit System Calendar' December 21st, 2012, [16] the end of every 4 centuries roughly coincides with the end of 'Katun 4 Ahau' of the Mayan calendar.

There are following relationships between clock and other time units.

$$
\begin{aligned}
1 \text{ clock} &= 11 \text{ minutes and } 15 \text{ seconds} \\
4 \text{ clocks} &= 45 \text{ minutes} \\
10_{(12)} \text{ clocks} &= 2 \text{ hours and } 15 \text{ minutes} \\
100_{(12)} \text{ clocks} &= 27 \text{ hours} \qquad = 1 \text{ day and } 3 \text{ hours}
\end{aligned}
$$

If a day begins at 0 o'clock AM, then 9 o'clock AM is $40^{\text{th}}_{(12)}$ clock, 6 o'clock PM is $80^{\text{th}}_{(12)}$ clock, and 3 o'clock AM of the next day is $100^{\text{th}}_{(12)}$ clock.

D.1.2 Calendar 2

I describe an alternative calendar which covers not only tropical year but also anomalistic year. In this calendar the months which consist of 31 days are continuously set around aphelion point. The complete rule of this calendar is defined in the next page. The leap days are inserted at the end of every 4 anomalistic years except at the end of universal centuries whose remainder of order number divided by 27 is odd number. The epoch of this calendar is same as calendar 1.

Instead of clock, this calendar uses universal minute and hour. There are following relationships between universal minute, hour and other time units.

---

a clock which is approximately 12^6. Actually, $3^6/2 = 364.5$. Surprisingly, this value of $3^6/2$ was used in the Tài Xúan calendar of Yáng Xíong(53BC–AD18).(14) Incidentally, the Tài Xúan calendar was never used in real history.

[16] This date is the winter solstice of year 2012 for almost all time zones, and according to G.M.T.'s 2nd modification, the date is the end of 'Baktun 4 Ahau' of Mayan Calendar whose long count is 13.0.0.0.0.

1	universal minute	=	56.25 seconds
1	hour	=	64 universal minutes
1	day	=	24 hours
$23_{(12)}$	hours	=	1 day and 3 hours

The relationship year - universal century - 27 universal centuries is parallel with the relationship universal minute - hour - 27 hours. So, the beginning of a day should be 3 o'clock in this calendar.

We can consider about more alternative calendar which uses universal minute exactly equal to $100_{(12)}s_u$. In this case, the last hour of a day have $1.7_{(12)}$ leap universal minutes.

```
** The Sample Rule and Perl Program of the Universal Unit System Calendar **

1. The date notation
      The date notation is made C/Y/M/D. where
      D: day                0 <= D <  31_(10) =  27_(12)
      M: month              0 <= M <  12_(10) =  10_(12)
      Y: year               0 <= Y <  64_(10) =  54_(12)
      C: universal century 0 <= C < 324_(10) = 230_(12)
     (* valid range is 20736_(10)(=10000_(12))years)

2. Calendar Epoch
      Calendars Epoch 121/0/0/0 is December 21st, 2012 (JDN=2456283).

3. Month(days and arrangement)
     3.1 The months which consist of 31 days:
        Continuous 5 or 6 months sequence whose start month number is equal
        to the quotient of C divided by 27.
     3.2 The months which consist of 30 days:
        The other months.

4. The definition of the normal year/leap year
     4.1 Normal year
        When the sequence of 3.1 clauses consists of 5 months, the year
        which contains the first month is defined as the normal year.
     4.2 Leap year
        When the sequence of 3.1 clauses consists of 6 months, the year
        which contains the first month is defined as the leap year.
     (* When the 6th month of the sequence belongs in the next year,
        the days of the leap year are 365 days though it is contrary to
        the etymology of 'leap'.)

5. The arrangement of the normal year/leap year
    5.1 The year when the remainder of Y divided by 4 is not 3 is a normal
        year.
    5.2 The year of the end of universal century when the remainder of
```

```
        C divided by 27 is odd number is a nomal year.
    5.3 The other years are leap years.

#! /usr/bin/perl

 # month ofset tables for nomal year
 @MM = (0, 1, 2, 3, 4, 5, 5, 5, 5, 5, 5, 5, 5);

 # usage
 die "Usage: cal c/y/m/d" if ($ARGV[0] eq undef);

 # get date form
($cc,$yy,$mm,$dd) = split('/',$ARGV[0]);

 # date normalization
 $cl  =  $cc      % 27;
 $ct  = ($cc-$cl) / 27;
 $cm  =  $ct      % 12;
 $ch  = ($ct-$cm) / 12;
 $mm -=  $cm;
($yy,$mm)     = ($yy-1,$mm+12  ) if ($mm < 0);
($cl,$yy)     = ($cl-1,63      ) if ($yy < 0);
($cm,$cl,$mm) = ($cm-1,26,$mm+1) if ($cl < 0);
($ch,$cm)     = ($ch-1,11      ) if ($cm < 0);
 $yl =   $yy      %  4;
 $yh =  ($yy-$yl) /  4;

 # conversion to Julian Day Number
 $jdn =  $ch *(((365*4+1)*16*27-13)*12-5)
       + $cm * ((365*4+1)*16*27-13 +30)
       + $cl *  (365*4+1)*16       - int($cl/2)
       + $yy *   365               + $yh
       + $mm *    30               + $MM[$mm]
       + $dd +  (2526409-2898564);

 # adjustment of leap day
 $jdn++ if ($yl ==  3 && $mm > 5 &&
          !($yh == 15 &&($cl % 2 > 0)));

 print "$ARGV[0]:  $jdn";
```

D.2 Paper size specifications

Although this cannot be called a unit, we take it up as an interesting coincidence.

An An-size sheet is "a rectangle whose long side and short side are in the proportion of $\sqrt{2}$: 1 and whose area is $2^{-n} \times 1\mathrm{m}^2$".

A Bn-size sheet is "a rectangle whose long side and short side are in the proportion of $\sqrt{2}$: 1 and whose area is $2^{-n} \times 1.5\mathrm{m}^2$".

Therefore, if we define the system of paper size specifications as "rectangles whose long side and short side are in the proportion of $\sqrt{2} : 1$ and whose area is $2^{n/2} \times (\mathrm{m}_u/12)^2$", then

n=22	A0	$2^{5.3}_{(12)} \times 2^{5.9}_{(12)}$	n=23	B0	$2^{5.6}_{(12)} \times 2^{6.0}_{(12)}$
n=20	A1	$2^{4.9}_{(12)} \times 2^{5.3}_{(12)}$	n=21	B1	$2^{5.0}_{(12)} \times 2^{5.6}_{(12)}$
n=18	A2	$2^{4.3}_{(12)} \times 2^{4.9}_{(12)}$	n=19	B2	$2^{4.6}_{(12)} \times 2^{5.0}_{(12)}$
n=16	A3	$2^{3.9}_{(12)} \times 2^{4.3}_{(12)}$	n=17	B3	$2^{4.0}_{(12)} \times 2^{4.6}_{(12)}$
n=14	A4	$2^{3.3}_{(12)} \times 2^{3.9}_{(12)}$	n=15	B4	$2^{3.6}_{(12)} \times 2^{4.0}_{(12)}$
n=12	A5	$2^{2.9}_{(12)} \times 2^{3.3}_{(12)}$	n=13	B5	$2^{3.0}_{(12)} \times 2^{3.6}_{(12)}$
n=10	A6	$2^{2.3}_{(12)} \times 2^{2.9}_{(12)}$	n=11	B6	$2^{2.6}_{(12)} \times 2^{3.0}_{(12)}$
n= 8	A7	$2^{1.9}_{(12)} \times 2^{2.3}_{(12)}$	n= 9	B7	$2^{2.0}_{(12)} \times 2^{2.6}_{(12)}$
n= 6	A8	$2^{1.3}_{(12)} \times 2^{1.9}_{(12)}$	n= 7	B8	$2^{1.6}_{(12)} \times 2^{2.0}_{(12)}$
n= 4	A9	$2^{0.9}_{(12)} \times 2^{1.3}_{(12)}$	n= 5	B9	$2^{1.0}_{(12)} \times 2^{1.6}_{(12)}$
n= 2	A10	$2^{0.3}_{(12)} \times 2^{0.9}_{(12)}$	n= 3	B10	$2^{0.6}_{(12)} \times 2^{1.0}_{(12)}$
n= 0	A11	$2^{-0.3}_{(12)} \times 2^{0.3}_{(12)}$	n= 1	B11	$2^{0.0}_{(12)} \times 2^{0.6}_{(12)}$

The size of a B5 sheet would then be 181.40mm $\times$ 256.54mm, which is almost the same as the actual size of 182.06mm $\times$ 257.47mm. The same can be said for the entire B series of paper sizes (the length of the upper side of a B5 size book is almost exactly $2/3^{\mathrm{rd}}\mathrm{m}_u$).

E Table of physical and astronomical constants

(This Appendix is for reference only. It is not part of the Universal System of Units Standard.)

Finally, fundamental physical constants, material constants, and astronomical constants expressed by means of the Universal System of Units Standard are presented.[17]

* a constant that is entirely linked to the fine structure constant.

Table 6: Fundamental physical constants

Characteristic impedance in a vacuum	1		Ω_n/rad^2	$(\sqrt{\mu_0/\epsilon_0})$
Avogadro constant	1		mol_n^{-1}	(N_A)
Rydberg constant	1	$\times 10^{6}_{(12)}$	Ω_1/m_u	(R_∞)
Speed of light in a vacuum	1	$\times 10^{8}_{(12)}$	$\mathrm{m}_u/\mathrm{s}_u$	(c_0)
Quantum of action	1	$\times 10^{-26}_{(12)}$	$\mathrm{J}_u\mathrm{s}_u$	$(\hbar)$
Boltzmann constant	1	$\times 10^{-18}_{(12)}$	$\mathrm{J}_u/\mathrm{K}_u$	(k_B)
Gas constant	1	$\times 10^{4}_{(12)}$	$\mathrm{J}_u/(\mathrm{mol}_u\mathrm{K}_u)$	(R)
Atomic mass unit	1.0009_051B_6	$\times 10^{-20}_{(12)}$	g_u	$(m^{12}\mathrm{C}/12)$
Bohr radius	1.005B_859A_5	$\times 10^{-9}_{(12)}$	m_u	$*(\alpha\Omega_1/4\pi R_\infty)$
Fine structure constant	0.0107_3994_38	$_{(12)}$		$*(\alpha = e^2\Omega_n/\hbar)$
Charge of an electron	1.0374_43B6_4	$\times 10^{-14}_{(12)}$	C_u	$*(\sqrt{\alpha\hbar/\Omega_n})$
Mass of an electron	0.B469_2178_0	$\times 10^{-23}_{(12)}$	g_u	$*(m_e = 4\pi R_\infty\hbar/\Omega_1\alpha^2 c_0)$
Classical electron radius	1.1368_3609_A	$\times 10^{-11}_{(12)}$	m_u	$*(\alpha^3\Omega_1/4\pi R_\infty)$
Bohr magneton	0.659A_AB66	$\times 10^{-17}_{(12)}$	$\mathrm{A}_u\mathrm{m}_u^2$	$*(e\hbar/2m_\mathrm{e})$
Proton/electron mass ratio	1090.19B5_78	$_{(12)}$		$(m_\mathrm{P}/m_\mathrm{e})$
Gravitational constant	2A.B33B	$\times 10^{34}_{(12)}$	N_u	$(N_G = c_0^4/G)$
Half the value of the Planck length	0.BA70BB	$\times 10^{-27}_{(12)}$	m_u	$(l_P = (1/2)\sqrt{G\hbar/c_0^3\alpha})$
Planck mass	5A.B223	$\times 10^{-8}_{(12)}$	g_u	$(\sqrt{\hbar c_0/G})$
Stephan-Boltzmann constant	0.1B82_B282	$\times 10^{-6}_{(12)}$	$\mathrm{W}_u/(\mathrm{m}_u^2\mathrm{K}_u^4)$	$(\pi^2 k_B^4/60\hbar^3 c_0^2)$
Josephson constant	0.3ABA_1394	$\times 10^{12}_{(12)}$	$\Omega_1/(\mathrm{C}_u\Omega_n)$	$*(K_J = 2e/h = (\Omega_1/\pi)\sqrt{\alpha/\hbar\Omega_n})$
von Klitzing constant	5.B903_2B9B	$\times 10^{2}_{(12)}$	Ω_n/Ω_1	$*(R_K = h/e^2 = 2\pi\Omega_n/\Omega_1\alpha)$

[17] This table doesn't take account of the latest (1998) values of the constants. If you are interested in these constants, see http://physics.nist.gov/constants.

Table 7: Material constants

Black-body radiation at the ice point	BA.2482_6	$_{(12)}$	W_u/m_u^2	
Molar volume of ideal gas	102.A553_0	$_{(12)}$	m_u^3/mol_u	(standard state)
Density of air	0.2451_8	$_{(12)}$	g_u/m_u^3	(standard state)
Speed of sound in air	337.479	$_{(12)}$	m_u/s_u	(standard state)
Density of water	108.817B_A6	$_{(12)}$	g_u/m_u^3	(maximum density)
Density of ice	B8.0	$_{(12)}$	g_u/m_u^3	(0 ℃)
Buoyancy of saltwater	6	$\times 10^{2}_{(12)}$	N_u/m_u^3	(specific gravity of 1.03)
Buoyancy of saltwater	6	$\times 10^{2}_{(12)}$	P_u/m_u	(pressure / water depth)
Ice point	169.49BA_9	$_{(12)}$	K_u	(1 atmosphere)
Boiling point of water	217.B09B_0	$_{(12)}$	K_u	(1 atmosphere)
Specific heat of water	0.6052_24	$\times 10^{4}_{(12)}$	$J_u/(g_u K_u)$	(by the definition of calorie)
Viscosity of water	1.2A29	$\times 10^{-3}_{(12)}$	$P_u s_u$	(25 ℃)
Kinematic viscosity of water	1.207B	$\times 10^{-5}_{(12)}$	m_u^2/s_u	(25 ℃)
Surface tension force of water	0.BB64_8	$\times 10^{-1}_{(12)}$	N_u/m_u	(25 ℃)
Enthalpy of the formation of water	1.4500_1	$\times 10^{8}_{(12)}$	J_u/mol_u	(25 ℃)
Gibbs energy of the formation of water	1.1757_B	$\times 10^{8}_{(12)}$	J_u/mol_u	(25 ℃)
Maximum sensitivity light wavelength	611	$\times 10^{-8}_{(12)}$	m_u/Ω_1	(by the definition of candela)
Maximum sensitivity photon energy	1.01	$_{(12)}$	$eA_u\Omega_n/mol_n$	(by the definition of candela)
Maximum sensitivity photon energy	1.05	$\times 10^{-14}_{(12)}$	J_u/mol_n	

Table 8: Astronomical Constants

Standard gravitational acceleration	5.5A54_B	$_{(12)}$	m_u/s_u^2	
Standard atmosphere	165.0086	$\times 10^{2}_{(12)}$	P_u	
Earth's geoid potential	0.3719_A81	$\times 10^{8}_{(12)}$	m_u^2/s_u^2	(square of the escape velocity)
Earth escape velocity	0.669B_3217	$\times 10^{4}_{(12)}$	m_u/s_u	(square root of the potential)
Gravitational radius of the earth	241.B898_22	$\times 10^{-4}_{(12)}$	m_u	(including the atmosphere)
Equatorial radius of the earth	7A2.4AAB	$\times 10^{4}_{(12)}$	m_u	
Astronomical unit	8A6.7575_4	$\times 10^{8}_{(12)}$	m_u	(distance of the sun)
Mean sun day	A8.14A7_261	$\times 10^{3}_{(12)}$	s_u	
Tropical year	0.230B_59A6_37	$\times 10^{8}_{(12)}$	s_u	
Universal century	10.0513_16A2_8	$\times 10^{8}_{(12)}$	s_u	(64 years)
Gravitational radius of the moon	4.1A76_416	$\times 10^{-4}_{(12)}$	m_u	
Equatorial radius of the moon	218.04	$\times 10^{4}_{(12)}$	m_u	
Mean distance of the moon	3.3513_B	$\times 10^{8}_{(12)}$	m_u	
Synodical month	222B.AB7A	$_{(12)}$	clock	
Nodical month	2023.1B61	$_{(12)}$	clock	
Gravitational radius of the sun	3182.870A_56	$_{(12)}$	m_u	
Equatorial radius of the sun	5.B475	$\times 10^{8}_{(12)}$	m_u	
Radiation of the sun	25.57	$\times 10^{20}_{(12)}$	W_u	
Luminous intensity of the sun	0.40	$\times 10^{20}_{(12)}$	W_u/rad^2	
Sun constant	435.1B	$_{(12)}$	W_u/m_u^2	
Luminance of a magnitude 5 star	1	$\times 10^{-A}_{(12)}$	W_u/m_u^2	
Universe expansion constant	5.3∼7.0	$\times 10^{14}_{(12)}$	s_u	(inverse of the Hubble constant)

F References

Table 9: References

No.	Author/Editor	Title	Publisher (year)
(1)	M.L.McGlashan	Physicochemical Quantities and Units (2nd Edition)	The Royal Institute of Chemistry (1971)
(2)	E. Witten	Reflections on the fate of spacetime	Physics Today Vol.49(1996) No.4
(3)	Xìa Yě	A Short History of Chinese Music	Gāo děng jìao yù chū bǎn shè (1991)
(4)	A. Wood and J. M. Bowsher	Physics of Music	Methuen(1965)
(5)	F. de Saussure	Course in General Linguistics	Open Court Publishing Co.(1988)
(6)	R.L.Wilder	Introduction to the foundations of mathematics	John Wiley & Sons, Inc.(1965)
(7)	D.R.Hofstadter	Gödel, Escher, Bach	Basic Books, Inc. (1979)
(8)	Shigekazu Moriguchi, et al.	Mathematical Formulas III	Iwanami shoten(1960)
(9)	Seiji Takada	Units and Unit Systems	Kyōritsu Shuppan(1980)
(10)	Takashi Suga	A Method of Organizing the Dimensions of Electromagnetic Quantities	Parity Vol.12(1997) No.4
(11)	Hidetoshi Takahashi	Electromagnetics	Shōkabō (1959)
(12)	Osamu Aono	Electromagnetic Unit System	Maruzen(1990)
(13)	Hiromasa Hirakawa	Relativity Theory (2nd Edition)	Kyōritsu Shuppan (1986)
(14)	Hideki Kawahara	The Thought of Science in China	Sōbunsha(1996)

The author is deeply grateful to F.J. Kurdyla and Associates Co., Ltd. (URL – http://www.kurdyla.com/) for their cooperation in the translation of this paper.