

Table 1: Mathematical constants (expressed in the dozenal system)

$\sqrt{\pi}$	=	1.9329_72A1	2^{-8}	=	0.0069	$0!$	=	1
2π	=	6.3494_16A0	2^{-7}	=	0.0116	$1!$	=	1
4π	=	10.6968_3170	2^{-6}	=	0.0230	$2!$	=	2
e	=	2.8752_3607	2^{-5}	=	0.0460	$3!$	=	6
$1/e$	=	0.44B8_4216	2^{-4}	=	0.0900	$4!$	=	20
γ	=	0.6B15_1888	2^{-3}	=	0.1600	$5!$	=	A0
ϕ	=	1.74BB_6773	2^{-2}	=	0.3000	$6!$	=	500
$\sqrt{2}$	=	1.4B79_170A	2^{-1}	=	0.6000	$7!$	=	2B00
$\sqrt{3}$	=	1.894B_9800	2^+4	=	14.0000	$8!$	=	1_B400
$\sqrt{5}$	=	2.29BB_1325	2^+8	=	194.0000	$9!$	=	15_6000
$\log_e 2$	=	0.8399_1248	2^+14	=	3_1B14.0000	$A!$	=	127_0000
$\log_2 3$	=	1.7029_9480	2^+28	=	9_BA46_1594.0000	$B!$	=	1145_0000
$z = \log_2 10$	=	3.7029_9480	2^+37	=	BA08_A990_A0A8.0000	$10!$	=	1_1450_0000

later (see Appendix B, “A method of organizing the dimensions of electromagnetic quantities”) is accomplished by a correction of almost exactly a factor of 12.

2. $\sqrt{2}$ ($\approx 1.5_{(12)}$)

Because of this relationship, it is possible to set good bounds for the heights and widths the standard sheet paper sizes. There is further discussion of paper sizes later in this paper (see Appendix D.2, “Standard sheet paper sizes”).

3. The golden ratio $\phi = (1 + \sqrt{5})/2$ ($\approx 1.75_{(12)}$)

It is known that the ratio of adjacent numbers in the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...) rapidly converges on the golden ratio. The 12th number in the sequence happens to be 12². It thus so happens that one of the fractional series that best approximates the golden ratio can be represented by a two-digit dozenal fraction in the dozenal system.

4. The 12-tone chromatic scale of music $\log_2 3$ ($\approx 1.7_{(12)}$)⁴

The properties of a musical scale can be evaluated by whether combinations of sounds whose frequencies are simple integer ratios can be approximated any number of times with good accuracy. The 12-tone chromatic scale of music is excellent in this respect.

(a) The smallest ratio of primes, 2 : 1

This corresponds to one octave in the scale, so it must have a strict representation. Accordingly, the common ratio of a musical scale frequency must be $\sqrt[n]{2}$ (where n is a suitable natural number).

(b) The next-smallest ratio of primes, 3 : 1

To efficiently approximate this ratio while satisfying condition (a), it can be approximated by the best approximation fraction, $\log 3 / \log 2 = \log_2 3 = 1.58496\dots$. There are n candidates for the denominator of the best approximation fraction. Expanding into a continued fraction, we get

⁴ Because of the relationship $\log_2 3 \approx 1.7_{(12)}$, $2_{(12)}^{37} \approx 10_{(12)}^{10}$. This corresponds to $2_{(10)}^{10} \approx 10_{(10)}^3$ in the decimal number system.