2.2 Advantages of the dozenal system in mathematical expressions

We will consider the advantages of the dozenal system in mathematical expressions from the general viewpoint (the number of factors and factorials) and from the viewpoint of individual mathematical constants.

2.2.1 Number of factors

The number 12 has more factors (1, 2, 3, 4, 6, and 12) than does 10 (1, 2, 5, 10), so the dozenal system offers the following two advantages over the decimal system.

1. Many fractions can be expressed as finite dozenals.
2. Multiplication is simple.

2.2.2 Factorials

A little-mentioned fact is that factorials are more easily represented in the dozenal system than in the decimal system because of the large number of trailing zeroes. Of the numbers from 1 to n, one of every 2^k numbers have k times factor 2. Thus, the number of times that 2 appears as a factor in n factorial is

$$\sum_{k=1}^{\infty}[\frac{n}{2^k}] \sim \sum_{k=1}^{\infty}\frac{n}{2^k} \sim n - O(\log n) \tag{14}$$

Of the numbers from 1 to n, one of every 3^k numbers have k times factor 3. Thus, the number of times that 3 appears as a factor in n factorial is

$$\sum_{k=1}^{\infty}[\frac{n}{3^k}] \sim \sum_{k=1}^{\infty}\frac{n}{3^k} \sim \frac{n}{2} - O(\log n) \tag{15}$$

The reason is that, $12(= 2^2 \times 3)$ contains, on the average, the prime factors 2 and 3 in just the right ratio for expressing n factorial. For this reason, the dozenal number system is also generally convenient for calculating permutations, combinations, and so on.

(reference)

$$\begin{aligned}
& \text{The order of the largest sporadic simple finite group} \\
= \; & 2^{46}3^{20}5^{9}7^{6}11^{3}13^{3}17^{1}19^{1}23^{1}29^{1}31^{1}41^{1}47^{1}59^{1}71^{1}_{(10)} \\
= \; & 888_8191_6727_3964_1634_7510_5895_4578_8183_2706_3298_0480_0000_0000_{(10)} \\
= \; & 992_4B98_B225_2AB9_530B_A466_1487_B0A8_0000_0000_0000_0000_0000_{(12)}
\end{aligned}$$

2.2.3 Mathematical constants

These constants, too, can be approximated by relatively simple dozenal fractions. I comment on a few interesting examples below.

1. $4\pi(\approx 10_{(12)})$
   The surface area of a sphere is approximately one order of magnitude larger than the area of a square that has sides equal in length to the radius of the sphere. Because of that, the conversion of non-rationalized units and rationalized units for the electromagnetic quantity units explained