B.2 Introducing the solid angle

The 4π difference that appears in the coefficients of the formulas of rationalized unit systems and non-rationalized unit systems is, as is well known, a geometrical value. What demonstrates the origin of that in the most straightforward way is probably Gauss' theorem (integration form).

Rationalized unit system	Non-rationalized unit system
$\int\int \mathbf{D}\cdot\mathbf{n}dS = Q$	$\int\int \mathbf{D}\cdot\mathbf{n}dS = 4\pi Q$

In a rationalized unit system, the unit electric flux is considered to be the electric flux created by the unit electrical point charge in all of space; in a non-rationalized unit system the unit electric flux is considered to be the electric flux created by the unit electrical point charge in 1 steradian (sr). Therefore, if we rewrite this taking solid angle to be an independent dimension, we have

$$\int\int \mathbf{D}\cdot\mathbf{n}dS = \Omega_2 Q \tag{31}$$

Here, Ω_2 is the total solid angle of a sphere. Performing a dimension analysis considering $\mathbf{D} = \epsilon_0\mathbf{E}$, and using the constant Ω_n, which has the dimension of impedance, and the speed of light in a vacuum c_0,

Permittivity of a vacuum	$\epsilon_0 = \dfrac{\mathrm{sr}}{\Omega_n \cdot c_0}$
Magnetic permeability of a vacuum	$\mu_0 = \epsilon_0^{-1}c_0^{-2} = \dfrac{\Omega_n}{\mathrm{sr}\cdot c_0}$
Characteristic impedance of a vacuum	$Z_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = \dfrac{\Omega_n}{\mathrm{sr}} = 4\pi\dfrac{\Omega_n}{\Omega_2}$

We must note that the dimension differs for the ratio of voltage and electrical current [Ω] and the ratio of electric field and magnetic field [Ω/sr].

B.3 Formula set that takes solid angle into consideration

In the following, I regard solid angle as an independent dimension and try to rewrite the set of formulas of electromagnetism. We can confirm that Ω_2 appears in the places where it should appear, geometrically (see section B.4). The rationalized unit system is a unit system in which Ω_2 is regarded as the pure number 1; the non-rationalized unit system is a unit system in which sr is regarded as the pure number 1. Comparing the formula for the force between electrical currents and the definitions of meter and ampere, we get $\Omega_n = 29.9792458\Omega$ (strict).