A Basic approach to units

(This Appendix is part of the Universal System of Units Standard.)

A.1 Classification of quantities

In the Introduction, the term 'of the same type' (the same dimension, in the terminology of units) was introduced out of the blue. Actually, however, there is no standard for objectively distinguishing whether or not quantities are of the same type. Whether or not quantities are of the same type should be determined by agreement. It should be noted that what we can measure directly is limited to pure numbers. What we call 'measuring length' is actually no more than reading the numbers on the scale of a ruler.

For example, it wouldn't matter at all if length in the vertical direction (height) and length in the horizontal direction (horizontal distance) were represented with different units. Actually, the height of Mt. Everest[10] is not expressed as 8.848 km; nor is the distance of a marathon course expressed as 42,195 m. This can be said to show that height and horizontal distance are recognized as different types of quantities.

The type of a quantity, however, is not entirely arbitrary. Where arbitrariness enters is for the most part in the decision to classify quantities coarsely or in detail. These concepts of quantity are actually defined axiomatically within a network of natural laws. In other words, the concepts of quantity can be seen as defined by the formulas that express natural laws themselves. In natural laws, there is no need for humans to distinguish between height, horizontal distance and other such quantities, so they are all lumped together in the category of length.

A.2 'Mathematical' units

A 'mathematical' unit is a suitable example when one is considering the classification of quantities.

In the following, I attempt a discussion of 'mathematical' units from the viewpoint that they are different from pure numbers.

Because a unit is "a quantity of the same type that serves as a standard for measuring and representing a given quantity", it is also possible discover units when we restrict ourselves to mathematical objects rather than the objects of physics.

For example, $\log_{10} 2$ is a pure number that has the value 0.3010... . So, then, (from the beginning, without omission) let's introduce the baseless logarithm $\log 10$. By axiomatically defining addition, subtraction, multiplication, and division, this is easily made an object of mathematical consideration. In this case, $\log 10$ becomes the unit for the quantity 'baseless logarithm', and can be used as follows.

$$\log 2 = 0.3010..\log 10 \tag{25}$$

The two sides of this equation are the quantity 'baseless logarithm' and cannot be reduced to numbers.

The baseless logarithm probably does not appear anywhere else, but what results if we replace this log with $\sin^{-1}$? In the theory of analytic functions of complex variables, logarithmic functions and inverse

[10] Tibetan name is Chomo Lūngma.